



Curriculum Units by Fellows of the Yale-New Haven Teachers Institute  
2000 Volume V: Sound and Sensibility: Acoustics in Architecture, Music, and the Environment

---

## **Sounding Off About Trig**

Curriculum Unit 00.05.09  
by Andrea Sorrells

### **Introduction**

---

The unit, Sounding Off About Trig, intends to take the overarching theme of sound and applies it to various aspects of algebra and trigonometry. While the unit is designed to supplement a Pre-Calculus textbook, there are parts that may be adaptable to an Algebra II/Trigonometry class. I have chosen not to center the unit on one specific aspect but instead will create short vignettes that can be applied in different places of the curriculum. This will enable teachers to do one of two things. They may either use individual lessons of this unit to enrich (or replace) a given trigonometric or algebraic concept or they can choose to continually revisit the overarching theme by using all parts of this unit throughout the school year.

In addition, these short pieces will be as “hands-on” as possible. Most students learn best when they can put their hands around what they are learning. Students will learn how trigonometry applies to science through the classroom activities. Often math is taught as a skill without being given a place to apply those skills. The activities will ask students to integrate their knowledge of mathematical skills with their understanding of sound. By having the unit’s activities focus on sound most will require students to use musical instruments as well as an oscilloscope. An oscilloscope is a piece of scientific equipment that records sound waves on a screen from a played musical note. The graphs produced will provide students with a visible depiction of an invisible concept: sound waves.

Because the unit integrates sound and mathematics, there are several specific “sound” objectives as well as “math” objectives. In a given class activity many of the sound and math objectives will both be addressed. Occasionally a few objectives will need to be introduced independently before the class activity can begin. Prerequisite skills will be clearly indicated in each activity immediately following the activity’s objective. Background information that a teacher may need to use the activity will be presented in the appropriate places.

### **Sound Objectives**

As mentioned above there are many objectives to this unit. Some are science related and others are mathematically related. The objectives listed in the next few paragraphs are followed by background information. The objectives are offset for quick reference and the material is grouped such that a teacher with knowledge of the material could skim or skip each idea. For the novice teacher or one unfamiliar with the

sound concepts there should be enough substance in the paragraphs to provide at least a working knowledge of the topics.

### *Objective 1*

At the end of the unit students should be able to state and discuss general concepts about sound waves. These topics should include how sound waves travel, and how their wavelength is determined.

Sound waves need a medium through which to travel. For the purposes of the experiments included in this unit, we will only deal with sound waves traveling through air. The sound wave travels through air by compressing air particles as it goes. It is often reflected off of objects that are bigger than the wavelength and “washes” over objects that are smaller than the wavelength. The wavelength is calculated by the formula,  $c = f\lambda$ , where  $\lambda$  stands for the length of the wave,  $c$  stand for the speed of sound, and  $f$  stands for the frequency. The speed of sound in air is 1100 ft/sec or 343 m/sec. Therefore, if we know that the frequency is 440 Hz then we can find the wavelength by dividing 1100 ft/sec by 440 Hz. This gives us a wavelength of approximately, 2.5 feet. Thus anything much larger than 2.5 feet will reflect the sound and anything much smaller than 2.5 feet will be “washed” over by the sound. Should the wavelength be approximately the same size, the sound wave will be scattered in all directions. This knowledge becomes very important in acoustics as buildings are designed for different purposes.

### *Objective 2*

At the end of the unit students should be able to stipulate how the frequency and period of a sound wave relate to one another.

The frequency,  $f$ , of a sound is defined as the number of cycles per unit of time. Sound waves that humans can hear range between 20 Hz and 20,000 Hz with the human voice ranging in the area of roughly 100 Hz and 5100 Hz. The period of the graph of a sound wave is inversely proportional to the frequency, giving us the equation,  $f = 1/p$ , where  $p$  stands for the period. Thus, if we know that the frequency is 440 Hz, then the period is  $1/440$  or .00227 seconds. Similarly, if we know the period is 3.14 seconds, then the frequency is .31846 Hz. If the period is .0314, then the frequency would be 31.846 Hz and in our hearing range. You can see that as the period gets smaller, the frequency rises. Because these go in opposite directions they are said to be inversely proportional.

### *Objective 3*

At the end of the unit students should be able to discuss what a Helmholtz resonator is and how it works.

A Helmholtz resonator is “another type of air vibrator.” It can be used to analyze musical sounds. A common type would be a bottle. Air blown across a bottle’s opening acts as a spring compressing the air in the bottle beneath the neck. There are many instruments that act as Helmholtz resonators, such as a guitar. The vibrating strings force the air inside the guitar to act as a spring as the back and front boards of the guitar vibrate. Air is then pushed through the opening on the guitar and a sound is heard.

The formula to find the frequency of a note produced from a bottle is  $f = (c/2\pi) \sqrt{a/Vl}$  where  $c$  equals the speed of sound in air (1100 ft/sec or 343 m/s),  $a$  is the area of the neck,  $l$  is the length of the neck,  $V$  is the air volume of the resonator. We can determine the frequency of a produced note from a by finding the individual values and then substitute them into the formula. So for a bottle with a neck of .25ft, a volume of air

of .097ft<sup>3</sup>, the area of the neck is .0314in<sup>3</sup>, then we get the following equation:  $f = [1100/(2*3.14)][(\text{sq. rt. } (.0314/ (.097*.25)))] = 199.32\text{Hz}$ . Then using appendix A we find that the sound produced was approximately the G note below middle C. The Helmholtz resonator provides a wonderful opportunity for students to work with formulas, musical notes, and understand frequency better. It also gives the teacher the opportunity to talk about conversions from centimeters to inches and inches to feet, bringing in multiple levels of mathematics.

## **Trigonometric Objectives**

As with the Sound objectives there are specific math objectives covered in this unit. The introduction mentioned that the math material is intended to be used with a Pre-calculus or an Algebra II/Trigonometry class. Many of the current Algebra II texts address trig issues in the later chapters of the book. Some even begin to focus on trigonometric functions as does the book I currently use, Advanced Algebra by Prentice Hall. Advanced Algebra has students graph  $y = \sin x$  and  $y = \cos x$  in the middle of the ninth chapter. Thus this unit could be used to help ground the math concepts in applications.

### *Objective 1*

At the end of the unit students should be able to use formulas such as volume, area, and others to find the missing variable.

In mathematics it is very important that students can evaluate mathematical formulas when given the values of the variables used in the formula. It is equally important that they are able to solve equations for any one of the variables used. For example, if a student is given the formula  $\text{Area} = (\pi r^2)$ , and they are told the area is  $36\pi$  and the radius is 6, then they need to be able to verify that this is correct. In addition, they need to be able to find the radius if only given the area of  $36\pi$  or the area if only given the radius is 6. These skills are so important that they are introduced in middle school mathematics and continually revisited through high school.

### *Objective 2*

At the end of the unit students should be able to graph simple sine functions.

Given a circle with radius of one and centered at (0,0) there are specific values for x and y that can be found as the point travels around the circle. The points at the four axes, starting on the positive x-axis are (1,0), (0,1), (-1, 0) and (0, -1). Other specific points can be found by using the ratios of special right triangles as an angle is formed using the positive x-axis as one side of the angle. ( \*add a source here for teachers to refer to) The x values of these points can also be found by using trigonometric functions. As cosine is known as the adjacent side over the hypotenuse, it quickly becomes apparent that we have the x value over one. This is because the hypotenuse is also the radius of the circle, which equals one. So cosine quickly becomes defined as the x value or  $x = \cos(\theta)$  where  $\theta$  is the angle formed. Along similar lines we have  $y = \sin(\theta)$ . Each of these functions can then be graphed on a new set of axes where the horizontal axis is the angle measure, and the vertical axis is either the value of x or the value of y depending on which function one chooses to graph.

By first developing the points on the circle for given angle measures, a chart of values containing the angle measures and the height of the circle (y) can be created. Then students can use this data to plot the sine curve. Some values can be found by using the special right triangles. Other values can be generated by using calculators to ensure a smooth, correct curve.

Depending on how you choose to use this unit, you may wish to develop the points and graph first. Thus, when students begin to use the oscilloscope, they have a mathematical equation in their head that defines it. I would suggest this approach if you were spreading out the unit across the curriculum. However, if you are using this unit as a separate “chapter,” then I suggest that you first model the data with the oscilloscope and then try to find the equation that accurately models it. In my classes, I plan to use the lessons to supplement and enrich the data in the chapters; so I would first have students understand  $y = \sin x$  and then move to the experiments.

### *Objective 3*

At the end of the unit students should be able to determine the period, phase shift, and amplitudes of functions when given an equation or a graph.

The period of a function is a numerical value that states how long it takes for the height of the function to repeat. If you had a fence surrounding your yard and every six feet there was a post, then the period of the fence would be six feet. A sine or cosine function repeats every 360 degrees. If you imagine the unit circle again, you can see how the points on the circle would begin to repeat after you have rotated around from the positive x-axis back to the positive x-axis. The period is represented on the function  $y = \sin ($  or  $y = \cos ($  by a value multiplied to  $($ . In their basic forms there is an understood one in front of the  $($ , so the period of the function is  $360/1$  or 360. If the function was  $y = \sin 6($  then the period is inversely shortened. Thus, the period of this new function is  $360/6$  or 60. The height of this function  $y = \sin 6($  at 60, 120, 180, etc should be the same because every 60 degrees the function begins a new period.

Traditionally, trigonometric functions are graphed in radians as well as degrees. The more advanced the course, the more likely the function will be in radians. To convert from degrees to radians you only have to multiply the degrees by  $($  and then divide by 180. Therefore the period in radians in a basic  $y = \sin ($  function is  $2($ . And multiplying the angle can change the period by some value. So if the equation was  $y = \sin 2($  then the period would be  $2($  divided by 2 or just  $($ .

As with any function in math, sine and cosine functions can be translated (moved) around in a given coordinate system. There are several ways to translate a function. To translate a function horizontally a value is added to the independent variable before any operations are performed on that variable. For example, if one has a line  $y = x$ , it can be translated right four units by subtracting four from the x. This results in the equation  $y = x - 4$ . To translate the function  $y = \sin ($  one merely has to add a value to  $($  before taking it's sine. This is modeled by the equation  $y = \sin (( + c)$  where c is any real number. In trig functions, a horizontal shift is known as a phase shift.

The amplitude of an equation affects how high and low the function goes on the vertical axis. It is mathematically calculated by taking half of the difference between the maximum height and the minimum height. Thus, as the sine wave hits a height of one and a low of negative one then  $\text{amplitude} = \frac{1}{2} (1 - -1) = \frac{1}{2} (2) = 1$ . The amplitude can be found in a trig function by taking the absolute value of the number multiplied by the  $\sin ($ . For example, if  $y = -3 \sin ($  then the amplitude is  $|-3| = 3$ .

The concepts of amplitude, period, and phase shift help students understand what changes can be made to a function in order to move it in the plane. If these are approached without some context, they tend to be very abstract for students. Thus sound is incorporated into the unit. How we change sounds directly translates into a phase shift, period change, or amplitude adjustment. The incorporation gives students a chance to see that the concepts they learn have direct application.

#### Objective 4

At the end of the unit students should be able to translate a sine or cosine function.

Objective 2 dealt with each of the possible changes as if they occurred in a vacuum. But as we all know it is rare for just one thing to occur - many things simultaneously happen in real life. As the purpose of this curriculum unit is to show uses of trig in real situations, it is necessary to look at problems that have more than one of the transformations occurring.

A general equation for a trig function is in the form  $y = a \sin b(\theta - c)$  where  $|a|$  is the change in amplitude,  $2\pi/b$  is the change in period, and  $c$  is translation left or right. It is my experience that students have difficulty dealing with multiple changes. I often have students draw several layers of their graphs from the equations, advising them to alter the function  $y = \sin \theta$  one step at a time. I have my students first deal with the new amplitude, then the new period, and finally the horizontal shift. For example, given the equation  $y = 2 \sin 4(\theta + \pi)$ , I would have my students mark that the height of the function will go to 2 and -2 instead of 1 and -1. Then I would recommend that they mark that the period will end at  $2\pi/4 = \pi/2$  instead of  $2\pi$ . Often I even recommend that they sketch a light graph given just these two changes. Finally I have them shift this whole sketch by the correct amount, which in this example would be  $\pi$  units to the left.

As students will be exploring different graphs generated on the oscilloscope, they will need to know how to do multiple transformations at once. For a sound that is introduced a split second later than a first the sine graph has been shifted right, that is a phase shift has occurred. And since period is inversely related to frequency - as the frequency increases the period shrinks and as the frequency shrinks the period is increased.

#### Objective 5

At the end of the unit students should be able to graph the sum of two or more sine functions.

Two functions can be mathematically combined by using any of the basic arithmetic operations (+, -, \*, or /). As stated earlier, very rarely is math represented in real situations in its basic forms. Musically pure notes from a flute are present, but notes produced from other instruments such as an oboe are not "pure" tones. Thus the graph of their sound waves would not be a simple sine function...not even a transformed sine wave. In fact the graph of their sound is a periodic function that is much more complicated. (See diagram below next paragraph)

The graph of the non-pure tones is created from adding several different sine waves together that have different periods because their frequencies are different. Thus the graph of these tones can be generated by finding the graphs of the different frequencies and then adding their y values together to form the new function. This is demonstrated mathematically by having the function  $y = \sin 3\theta$  and  $y = \sin 2\theta$ . If  $\theta = \pi/4$  (or 45 degrees) then we get  $y = \sin 3(\pi/4) = \sin 3\pi/4 = .707$ . We also get  $y = \sin 2(\pi/4) = \sin \pi/2 = 1$ . Thus if we add these two functions together, producing a new function  $y = \sin 3\theta + \sin 2\theta$  and use the same angle, then we get  $y = .707 + 1$  which equals 1.707. Hopefully you can see that many of the y-values will change and a new graph will occur. See diagram below to see how the two functions add together to give us the third.

*(figure available in print form)*

If students can understand how these functions can be combined then they can comprehend why the graphs of the non-pure tones do not look like plain sine curves.

## Classroom Activities

---

### Activity 1 - Creating a musical octave with water and bottles

Objectives:

- \* use mathematical formulas to calculate area, volume, and frequency
- \* determine the frequency of a sound
- \* create different sounds using the mathematical formulas

Prerequisites: Students need to know what frequency means and be given the formula for finding the frequency in a Helmholtz resonator,  $f = \frac{v}{2l}(\text{sq. root of } a/Vl)$  where  $v$  equals the speed of sound,  $a$  is the area of the neck,  $l$  is the length of the neck,  $V$  is the volume of the resonator. They will also need the formulas for the volume of a sphere or cylinder and the area of a circle. Students also need to know what a Helmholtz resonator is and how it actually works.

Materials Needed: Per group, you will need water, a measuring cup, a tape measure or ruler, a calculator and approximately 7 to 12 identical size bottles. You will also need a few bottles of varying shapes and sizes. They will also need the included chart that relates frequency to a piano scale. (see appendix A)

Activity: Students will begin by taking the different size bottles and blow across the opening to hear the sound that is created by the Helmholtz resonator. Once the students have heard the sound they have created, they need to calculate the frequency,  $f$ , for each of the empty bottles using the appropriate formulas. Since the equation given in the sound objective #5 uses the AIR volume of the bottle, students should calculate this for their bottles. To change the air volume in the bottle they will need to add different amounts of water. The total volume ( $V_t$ ) of the bottle below the neck, minus the liquid volume ( $V_l$ ), will give the students the volume of air ( $V_a$ ) in the bottle. This is demonstrated by the equation  $V_t - V_l = V_a$ . Before the students create a musical octave it might be nice practice to have them fill the bottles with varying amounts of liquid and calculate the frequency of the note created. When they are ready to create their octave, students will need to work the equation for frequency of a Helmholtz resonator backward to determine the air volume they need for each note. I would have my students actually fill their seven bottles appropriately and then test their creation.

Homework: Students could be given a worksheet that has them practice using the formulas. They could be given a note and told how to calculate the liquid needed to create the appropriate air volume. It would be a nice conclusion to ask the students to discuss what would make a note flat or sharp given the formula and a frequency.

## **Activity 2 - Relating frequency to period of a graph**

Objectives:

- \* discuss the relationship of frequency to period
- \* calculate the period of a sound wave generated from a musical note

Prerequisites: Students will need a good understanding of what frequency means. They will also to have knowledge of what a period represents in a graph...it does NOT have to be a trigonometric graph although if it is, their connection to the trigonometric concepts will be greatly increased. Finally they will need to be able to determine the frequency of a played musical note. This could come directly from the previous experiment or from the chart in Appendix A.

Materials Needed: The students will need something that will create a musical note. A keyboard, flute, or one of the bottles from the previous experiment will be necessary. They will also need Appendix A and a calculator.

Activity: This is a much shorter activity and shouldn't take the entire period. It very easily could be combined with Lesson 3 on amplitude. During this lesson, one student in the group should play a note. The other students will then determine the frequency of the note from the appendix. After having determined the frequency they then would calculate the period of the function. In a pre-calculus class I would finally have the students convert the period to radians so that we were ready to graph a function after lesson 3.

Homework: For homework I would have students practice working in both directions...that is to say I would give them two types of problems. One type would give them the period of the graph and ask them to determine both the frequency and the note. The other type of problem would ask the students to determine the period of the graph generated from a given musical note.

## **Activity 3 - Finding the amplitude of a sound wave generated from a note**

Objectives:

- \* use logarithms to solve equations
- \* use and interpret data gathered from a sound level meter
- \* calculate the amplitude of the sine wave generated from a note

Prerequisites: Students will need to have some understanding of and practice with logarithms. They will also need to know how a sound level meter works and what its data output means.

Materials Needed: A sound level meter, graph paper, and a calculator are all needed for this experiment. In addition there needs to be something present to create a musical note such as a keyboard etc. Less expensive



sound level meters only go as low as 50 decibels. If your musical instrument won't register in this range or higher, you either need a louder instrument or a more sensitive sound level meter.

Activity: This too is a relatively shorter activity and involves more pen and paper work than some of the other experiments. Students should turn on their sound level meters using the C weighting. A second student should play a note from the given musical source. A third person should hit DH on the sound level meter when they feel the note is being played consistently. The sound level meter will then "freeze" on a reading. This number is then placed in to the following equation for SPL, where SPL stands for the sound pressure level.  $SPL = 10 \log A$  and A stands for the amplitude of the generated sound wave. Using their knowledge of logarithmic functions students should then calculate A. Because they already know how to calculate the period from the frequency, I would recommend having the students graph the sine wave that models the note. For example, say I heard a musical note C (256Hz) and the sound level meter read 54dB. I could then find the period of the graph from activity 2 finding that  $p = 1/256 = .00390625$ . I could also solve the log function  $54 = 10 \log A$  finding that  $A = 251188.6432$ . Now I have enough information to graph the sine wave. Students should follow these steps and graph the sine wave for the note they played. A beautiful extension to this experiment would be to have the students use the same note only at a louder volume. They will result in a different amplitude, which in turn changes the graph. This should generate a good deal of discussion.

Homework: I think it would be wonderful to have the students write about what makes the same note produce two different graphs. In addition, I might ask my students to calculate a few examples. Some of these I would ask them to find the SPL and others I would ask them to find the amplitude.

#### **Activity 4 - Given a graph of a sound wave, finding the frequency and amplitude.**

Objectives:

- \* use a microphone, CBL unit, and a graphing calculator to create data
- \* interpret the graph of a note recorded from the materials
- \* determine the period and amplitude of a graph
- \* calculate the frequency and then determine the note analyzed
- \* compare and discuss the accuracy of experiment

Prerequisites: The student needs to understand the relationship between frequency and period. They also should be able to calculate the frequency if given the period. They will have to be introduced to the CBL unit and how it works. This can be done during the lab if they are unfamiliar with the apparatus. Students will need to have seen a sine graph before and have some knowledge of how to find the amplitude and period. Finally students will need to be able to use the graphing calculators, understanding how to use the trace key.

Materials Needed: You will need for every group a microphone, a CBL unit, tuning forks of various pitches or something that creates musical notes, TI-82 graphing calculators with the program SOUND already programmed in.



Activity: Students will link the CBL unit to the microphone and TI-82. They will go to PRGM (program) key on the calculator and run the program entitled SOUND. They should follow the directions given from the program. The students should be ready with the musical instrument or tuning fork, as they will need to be ready to play the note at the appropriate time. The musical note should be generated BEFORE the CBL unit is started. Once the CBL unit is receiving data, a graph will appear on the calculator screen. It will not be a perfect sine wave, but it will be close. The teacher should then be prepared to ask the students to calculate the period of the graph using the trace key. They should then use this data to determine the frequency (activity #2). Similarly they should determine the amplitude of the function. After they have found the frequency they need to determine the note played. Finally they should compare the note they have found from the data and the actual note of the keyboard or tuning fork. These will not always match exactly and students hopefully will have questions as to why this occurs. This inaccuracy provides a great opportunity to discuss precision in experiments. If time permits it would be nice to have them re-try the experiment with the same note a second time to compare data. You could ask them to determine which experiment they think was better/more accurate and why. As a final extension the students could use a different musical note or tuning fork and try the experiment again.

Homework: Perhaps the students could write up a one-page essay discussing the accuracy or inaccuracy of the experiment and ways to improve for the next time.

### **Activity 5 - Phase Shifts of Sine Functions**

Objectives:

- \* to determine how a phase shift occurs when dealing with sound
- \* to find the equation that represents a sound wave that has been delayed (that is shifted horizontally)

Prerequisites: The student will need to be already familiar with how to find the period and amplitude of a function from its graph. They will also need to know the general equation of a trigonometric sine wave so they will be able to insert all the appropriate constants and give an exact equation for the sound wave. The formula is explained above, but is also given again here:  $y = a \sin b(( - c)$ .

Materials Needed: Each group will need either a tuning fork or a musical instrument, a CBL unit, a microphone, and a TI-82 calculator.

Activity: This activity is very similar to Activity 4. The key difference between these two activities is that in this activity the students will start the CBL unit BEFORE they generate the note. Doing so will create a brief time when there is no noise. It is important that the students do not talk during this time, as the microphone will pick up anything noises nearby. They shouldn't wait too long to start the musical note or else the CBL unit will have completed its data collection before it has time to register the note. If the activity is done correctly there should be a straight line and the generated sine curve should start a ways into the screen. Students should measure this distance compare to the period of the function. For example, if the period is .45 and the lag distance is 1.3, then we would not say the phase shift is 1.3. They will need to calculate the remainder that occurs when .45 is divided into 1.3. The remainder is the phase shift. The mathematical reasoning behind this

is that a pure true sine wave continues on forever in both directions. When it is not shifted, the sine wave always hits the point (0,0). If the lag distance is greater than the period, then we have to imagine that the function did continue to the left as well. And if it did, every distance of .45 would contain another complete period of the sine wave. Thus we need to find out the remainder to know how much we have actually been shifted from (0,0). Students will also have to find the amplitude and period from the function as we did in Activity 4. The activity could be expanded to include multiple graphs with different lag times.

Homework: Have students practice finding the equations from given graphs. This could include both sine waves and cosine waves. Similar problems could include giving them the equations and having them practice creating the graphs.

### **Activity 6 - Addition of two or more Sine Functions**

Objectives:

- \* find the sum of two trigonometric functions
- \* relate how two pure sounds can create a new sound that no longer looks like a pure sine wave

Prerequisites: Students need to know what sine waves look like as well as how to calculate their value for a given independent value. They also need to have a basic understanding of how a pure sound can create a sine function. If the experiments are followed in order, then students should already have a nice grasp of these concepts.

Materials Needed: At this point the students will need an oscilloscope. Perhaps either the physics department or a nearby college might allow you to borrow an oscilloscope. The class will have to work as a group or they will need to come up one group at a time. You will also need calculators and graph paper.

Activity: Students should play one pure note so that the oscilloscope records the data. A sine wave will appear on the screen. Students will need to find the amplitude and period of this sine wave from the screen and record this data. Next students will need to play a different pure tone and follow the same procedures for it...recording amplitude and period. I would have them assume no phase shift. At this point I would send the students back into their groups and have them graph the two sine waves on separate graphs. I would ask them to guess what the graph would look like if the two sounds were played simultaneously. I might even hint to them that the two sounds were "added" together. Once the group has drawn a new graph that they think is represented from the two graphs I would bring them back up to the oscilloscope and play the two tones together. They would then be asked on a handout if their prediction was correct or not. If it was, I would have them explain how they know they are right. If their prediction was inaccurate I would then ask them to think about how the graph on the screen from the two notes was created from the two individual notes. They may need a little pushing at this point to figure things out. Once they think they know how the graph was created I would send them back to their work area and have them draw the graph and then come and check it against the oscilloscope. This idea could be expanded upon in two ways. One way would be for them to try more than two notes and see if they can predict the combined graph. A second method would be to show them a combined graph and see if they could determine the two individual notes that created the graph. A final question to extend the lesson would be to see if they could determine two functions that would totally cancel

each other out leaving a straight line.

Homework: I would have them practice problems that ask them to graph several combined functions.

## Appendix A

### TEMPERED CHROMATIC SCALE

American Standard pitch. Adopted by the American Standards Association in 1936.

Note	Frequency	Note	Frequency	Note	Frequency	Note	Frequency
C0	16.352	C2	65.406	C4	261.63	C6	1046.5
C#0	17.324	C#2	69.196	C#4	277.18	C#6	1108.7
D0	16.354	D2	73.416	D4	293.66	D6	1174.7
D#0	19.445	D#2	77.782	D#4	311.13	D#6	1244.5
E0	20.602	E2	81.407	E4	329.63	E6	1318.5
F0	21.827	F2	87.307	F4	349.23	F6	1396.9
F#0	23.125	F#2	92.499	F#4	369.99	F#6	1480.0
G0	24.500	G2	97.999	G4	392.00	G6	1568.0
G#0	25.957	G#2	103.83	G#4	415.30	G#6	1661.2
A0	27.500	A2	110.00	A4	440.00	A6	1760.0
A#0	29.135	A#2	116.54	A#4	466.16	A#6	1864.7
B0	30.868	B2	123.47	B4	493.88	B6	1975.5
C1	32.703	C3	130.81	C5	523.25	C7	2093.0
C#1	34.648	C#3	138.59	C#5	554.37	C#7	2217.5
D1	36.708	D3	146.83	D5	587.33	D7	2349.3
D#1	38.891	D#3	155.56	D#5	622.25	D#7	2489.0
E1	41.203	E3	164.81	E5	659.26	E7	2637.0
F1	43.654	F3	174.61	F5	698.46	F7	2793.8
F#1	46.249	F#3	185.00	F#5	739.99	F#7	2960.0
G1	48.999	G3	196.00	G5	783.99	G7	3136.0
G#1	51.913	G#3	207.65	G#5	830.61	G#7	3322.4
A1	55.000	A3	220.00	A5	880.00	A7	3520.0
A#1	58.270	A#3	233.08	A#5	932.33	A#7	3729.3
B1	61.735	B3	246.94	B5	987.77	B7	3951.1
						C8	4186.0

## Teacher Resources

---

Apfel, R. E. *Deaf Architects & Blind Acousticians?* New Haven: Apple Enterprises Press, 1998.

This source is very readable. It describes the basics of sound as well and relates it to both music and architecture.

Barrow, J. D. *Theories of Everything*. Oxford: Clarendon Press, 1991. Pg. 188-193.

This source offers a nice tie-in between mathematics and physics describing how each has led to development of parts of the other.

Bellman, A., et al. *Advanced Algebra*. Needham: Prentice Hall, 1998. Pg. 400-456.

This is a traditional textbook that covers all of Algebra II. It is student friendly and easily read. Chapter 9 deals with trigonometry directly.

Bredenweg, D., Elsner-McCall, K., Komon, A. and Lally, S. *Holt Physics: Lab Experiments*. Austin: Holt, Rinehart, and Winston, 1999. Pg. 53-58.

This lab experiment book goes hand in hand with the Holt Physics book. There are two experiments listed, one has students create their own instruments and the other deals with resonance and the nature of sound

Cutnell, J. D. and Johnson, K. W. *Physics*. Fourth Edition. New York: John Wiley & Sons, Inc., 1998. Pg. 458-493.

This is a traditional college level physics book. The book will be easily read by students, this book offers clear explanations of frequency, period, wavelength, decibels, etc. It also offers nice applications of sound.

Marson, R. *Sound: 20 Task Card Activities*. Oregon, 1990.

This is a wonderful resource with lots of further activities. It includes notes for the teacher and 20 tasks or experiments that can be given to the students.

Rossing, T. D. *The Science of Sound*. Massachusetts: Addison-Wesley Publishing Company, 1990.

This text is very involved and definitely more geared to the educator than the student. It covers extensively the concepts of music and sound, how the ear hears, and the mathematics of sound. It is very detailed and worth the read.

Ryan, M., Doubet, M. E., Fabricant, M., and Rockhill, T. D. *Advanced Mathematics: A Precalculus Approach*. Englewood Cliffs: Prentice Hall, 1993. Pg. 118-327.

This is a traditional approach to Pre-calculus ideas. It focuses heavily on trigonometry in chapters 3 through 6 with lots of practice. The practice focuses on skills as well as application. Occasionally extra explanation is needed from the instructor for students understanding.

Serway, R. A. and Faughn, J. S. *Holt Physics*. Austin: Holt, Rinehart and Winston, 1999. Pg. 437 -517.

Holt presents a very student friendly text. Chapter 12 deals with vibrations and waves with nice graphics and explanations. Chapter 13 concentrates on sound with clear explanations, giving a general background. This text also goes nicely into the mechanics of hearing. I recommend this as a nice introduction for teacher and student alike.

Tipler, P. A. *College Physics*. New York: Worth Publishers, Inc., 1987. Pg. 352 - 399.

This is an older textbook and may be harder to find. Chapter 15 offers nice mathematical as well as physical descriptions of oscillation. This is a good resource for the teacher. Chapter 16 offers the traditional background on sound but goes less into the mechanics of hearing.

Wilson, J. D. and Buffa, A. J. *College Physics*. Third Edition. New Jersey: Prentice Hall, 1997. Pg. 414-474.

It is very similar to the other Physics textbooks above, but perhaps a little less student friendly. Chapter 13 deals with waves and

vibrations while chapter 14 deals with sound directly.

## Student Resources

---

Apfel, R. E. Deaf Architects & Blind Acousticians? New Haven: Apple Enterprises Press, 1998.

This source is very readable. It describes the basics of sound as well and relates it to both music and architecture.

Bellman, A., et al. Advanced Algebra. Needham: Prentice Hall, 1998. Pg. 400-456.

This is a traditional textbook that covers all of Algebra II. It is student friendly and easily read. Chapter 9 deals with trigonometry directly.

Cutnell, J. D. and Johnson, K. W. Physics. Fourth Edition. New York: John Wiley & Sons, Inc., 1998. Pg. 458-493.

This is a traditional college level physics book. The book will be easily read by students, this book offers clear explanations of frequency, period, wavelength, decibels, etc. It also offers nice applications of sound.

Ryan, M., Doubet, M. E., Fabricant, M., and Rockhill, T. D. Advanced Mathematics: A Precalculus Approach. Englewood Cliffs: Prentice Hall, 1993. Pg. 118-327.

This is a traditional approach to Pre-calculus ideas. It focuses heavily on trigonometry in chapters 3 through 6 with lots of practice. The practice focuses on skills as well as application. Occasionally extra explanation is needed from the instructor for students understanding.

Serway, R. A. and Faughn, J. S. Holt Physics. Austin: Holt, Rinehart and Winston, 1999. Pg. 437 -517.

Holt presents a very student friendly text. Chapter 12 deals with vibrations and waves with nice graphics and explanations. Chapter 13 concentrates on sound with clear explanations, giving a general background. This text also goes nicely into the mechanics of hearing. I recommend this as a nice introduction for teacher and student alike.

## Electronic Resources

---

HYPERLINK

<http://sln.fi.edu/hotlists/physical.html>

<http://sln.fi.edu/hotlists/physical.html>

This web site lists several different places a student could go to learn more about physics. It includes more than just sound options.

HYPERLINK

[http://www.smgails.org/physics/sound\\_1.htm](http://www.smgails.org/physics/sound_1.htm)

[http://www.smgals.org/physics/sound\\_1.htm](http://www.smgals.org/physics/sound_1.htm)

This web sites gives some history to who helped with discoveries about sound waves and some applications of sound. It is not very detailed.

HYPERLINK

<http://hometown.aol.com/blutility/physicspage.html>

<http://hometown.aol.com/blutility/physicspage.html>

A student's understanding and explanation of the physics of a sub woofer is given at this web site.

HYPERLINK

<http://www.exhibits.pacsci.org/music/MusicPhysics.html>

<http://www.exhibits.pacsci.org/music/MusicPhysics.html>

This site talks about the following sound concepts: vibrations, sound waves, frequency, pitch, resonance and overtones.

HYPERLINK

[http://www.gcat.clara.net/Andy\\_Hardie/acoustpl.html](http://www.gcat.clara.net/Andy_Hardie/acoustpl.html)

[http://www.gcat.clara.net/Andy\\_Hardie/acoustpl.html](http://www.gcat.clara.net/Andy_Hardie/acoustpl.html)

This is a great site and covers: pitch, amplification, timbre, how sound travels, how the ear works, and how sound is conveyed to the brain.

HYPERLINK

<http://www.kent.wednet.edu/staff/trobinso/physicspages/PhysicsOf.html>

<http://www.kent.wednet.edu/staff/trobinso/physicspages/PhysicsOf.html>

Students are able to explore the physics of different things from sound to skating by clicking on different letters. The whole site is created by students and is therefore easy to understand and follow.

@SH:Notes Serway, R.A. and Faughn, J.S. Holt Physics. Austin: Holt, Rinehart and Winston, 1999. Pg. 481.

Rossing, T. D. The Science of Sound. Massachusetts: Addison-Wesley Publishing Company, 1990. Pg. 22.

Rossing, T. D. The Science of Sound. Massachusetts: Addison-Wesley Publishing Company, 1990. Pg. 60.

Rossing, T. D. The Science of Sound. Massachusetts: Addison-Wesley Publishing Company, 1990. Pg. 22.

A source where more practice can be found to give to students is in the Advanced Mathematics: A Precalculus Approach book. Beginning on page 126 there is text that students can read to deepen their understanding. At the end of the text for this section there are practice problems that can be assigned to give students further practice.

If you are working with a younger less experienced group of students I recommend the Advanced Algebra test listed in the student resources. The reading level of this book is a little lower than the book mentioned above and students can easily understand. The practice can be found on pages 418 and 419.

In the Advanced Algebra text you will find all of chapter 9 deals with trigonometric functions and I recommend this for more practice or as a reading assignment for the students depending on what you are working on at the time.

For the precalculus students I recommend looking at chapter 4, section 2 in the Advanced Functions: A Precalculus Approach book.

Again, for more practice for your students I recommend using pages 196 and 197 from the Advanced Functions: A Precalculus Approach book. I also recommend using practice 4.2 from the teachers supplement book.

I wrote to the TI-web site to obtain permission to include the program in this paper. They said they did not have a copy and that permission would have to be obtained from the author. I received the program from a colleague who is not sure where it originated. For that reason, the program is not included here. I am still trying to track down the author and when I have done so will be happy to send a copy to anyone who would like it. You may contact me at [HYPERLINK mailto:AndreaSorrellsaol.com](mailto:AndreaSorrellsaol.com) AndreaSorrellsaol.com to see if I am able to send you a copy and all I will be happy to respond.

See note number 6 for problems that you could assign as homework.

Rossing, T. D. The Science of Sound. Massachusetts: Addison-Wesley Publishing Company, 1990. Pg.179.

$Y = \sin 3x$   $Y = \sin x$   $Y = \sin x + \sin 3x$

---

<https://teachersinstitute.yale.edu>

©2019 by the Yale-New Haven Teachers Institute, Yale University

For terms of use visit <https://teachersinstitute.yale.edu/terms>