Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
2000 Volume V: Sound and Sensibility: Acoustics in Architecture, Music, and the Environment

Discovering the Mathematics of Sound

Curriculum Unit 00.05.10
by Lewis L. Spence

Sound is the form of energy which most of us seem to take for granted. Maybe the prevailing impression which encourages this attitude is the acceptance that it is as natural and abundant as air. For people who experience temporary or permanent loss of hearing, the perception of sound and acoustics do have significant implications which create limitations and demands for sizeable adjustments to their other senses of perceptions and to the environment in which they operate. Acoustics, which is the science of sound, provides animals with added awareness of their environment. It facilitates a useful form of communication. For humans, this is most significant in the development of the spoken language, an art which is common only to mankind.

I do have a great interest in exploring the mathematics of acoustics and the acoustical effects on architectural designs. This knowledge will allow students to practice mathematical solutions to acoustical problems. Hopefully this exposure might instill a keener awareness of sound quality in the general environment.

I am an eighth grade mathematics teacher at the Betsy Ross Arts Magnet Middle School in the city of New Haven, Connecticut. The school has a diverse ethnic population of African-Americans, Asians, Afro-Caribbeans, Caucasians, and Hispanics. The Afro-American population accounts for about sixty percent of the total population. The curriculum allows students to complete their studies in the basic academic subject areas and still retain a strong focus on the visual and performing arts.

There is this constant complaint among our student population that mathematics is boring. One of the suggested solutions to this problem is the integration more of relevance in the presentation of the subject matter. An approach that investigates the acoustical effects on architecture and the environment might somehow provide that missing relevance. The proposed units of development will be geared for the students in the pre-algebra and algebra groups since the science requires some form of algebraic manipulations. The primary objectives are:

1) To provide practice in the basic skills of algebraic manipulations
2) To provide practice in the application of basic mathematics in science
3) To offer an interdisciplinary approach to mathematics
4) To determine the nature of good acoustics
5) To develop proposals for providing good acoustics

Implementation

Students should be involved in identifying types of sound. They should be required to relate these sounds to some useful purposes. From this discussion the sounds which might not be categorized as useful could then be given a different category. This could possibly introduce the topic of noise. The nature of sound does come to play. Here the production of sound should provide adequate class participation. Students will be required to produced different sounds and comparisons can be drawn. There are some basic observation which can be made. For example: What is common about these different sounds? What is dissimilar about these sounds? What process is involved in the production of sound? What is responsible for the variation in sound intensity? Here a common instrument like a guitar can be introduced to demonstrate the production of sound. The visual demonstration can give rise to elements of vibration, pitch and amplitude.

The phenomenon of wave propagation should require some form of visual presentation. Maybe the use of a rope, a delicate coil spring, a large container of water, could serve to demonstrate types of waves and the nature of their behavior.

The introduction of formulas to determine the speed, wavelength, frequency and intensity of a sound should provide practice in evaluating an equation. Also the students should be allowed to discover the effects of manipulating the formulas to get a required value. For example, in the formula of frequency, wavelength, and speed, what happens if a particular value is increased or decreased? The same manipulation should be applied in the formula dealing with sound intensity. From this students should get a clearer understanding of the variables which control basic acoustics.

The nature of materials and their effects on sound propagation requires some attention. Careful attention should be paid to shapes and designs with regard to how they affect sound. What constitutes noise? What constitutes good acoustics? What noise is noise? What are the basic parameters that affect good acoustics? This knowledge base should allow the students to make reasonable proposals as to what measures to take in order to provide a suitable acoustical environment.

The Nature of Sound

Sound is the form of energy which allows us to hear and communicate through speech. It is responsible for acquainting us with our environment through the remarkable organ called the ear. Like light and some other forms of energy, sound travels in the form of waves. They all have energy and their waves can be reflected, refracted, diffracted or absorbed. One of the other common properties of waves is that they transmit energy and information through a medium but the medium itself is not moving with the wave. The disturbance occurs in the medium and is passed on from one point to the next. In the case of sound waves, the disturbance is in the form of a change in pressure and density which is sometimes described as a vibration in the medium.

Light waves and radio waves travel at a speed of 3 x 10 meters (186,000 miles) per second, while sound waves travel at 344 meters (1100 ft) per second at 70 F. Unlike the other mentioned waves, sound waves do not travel in a vacuum; they require a material medium in the form of a solid, liquid or gas. In water sound will travel at a speed of about 1500 meters per second (5000 ft/sec), which is about five times its speed in air. While in a steel material the speed could be more than three times faster than in water.

The Features of a Sound Wave

When waves are propagated in a material, the disturbance allows temporary displacement in portions of the material as the disturbance continues from point to point through the medium. A measure of the length of this temporary displacement is called the wavelength. The wavelength is the distance from the crest of one wave to the next or from one trough of one wave to the next.

The frequency, f, is the number of vibrations or oscillation per second occurring in the material. The unit of measurement is Hertz (Hz). One thousand Hertz is called one kilohertz (kHz)

Period, ((Greek, tau) is the measure of the time for one oscillation. Since the frequency tells how many oscillations occur in one second, and the period tells the time for one oscillation, one could find the value of the other if the value of one of the features is known. For example: if the frequency is 200Hz, that is 200 oscillations in one second, then the period, the time for one oscillation would be 1/200 = 0.02 seconds. Likewise, if the period, (, is 0.01second, the frequency would be 1/0.01 = 100 Hertz.

We could determine the speed or the velocity of the wave. Since speed is the distance divided by the time, which can be classified as rate. In this instance, the rate is the velocity of the sound, c = (/(, where ((lambda) is the wavelength. However, since 1/(= f, the velocity can be expressed as c = (f.

The velocity of sound in air is constant, 1100 ft/sec. Therefore it makes it fairly easy to determine the wavelength of a sound wave if the frequency is known. Since the velocity (1100 ft/sec) is constant which is the product of the wavelength and the frequency, it argues that if the value of the frequency is large, the value of the wavelength must be small. If the value of one is double the value of the other is halved. For example, if the frequency is1100 Hz, then the wavelength would be 1 ft. If the wavelength is doubled, 2200 Hz, then the wavelength is 0.5 ft. It will be the same effect on the frequency if the wavelength were doubled or halved.

The ear is the organ in humans which perceives sound. Reasonable acoustics is somewhat subjective; it depends on individual and cultural norms and standards which are not necessarily permanent but acceptable for the age, time and setting. Sound is actually the result of “an organized disturbance of pressure in the air. The human ear is capable of perceiving sounds which are so weak that they cause the ear drum to displace by less than the size of a hydrogen atom which has a diameter of about a billionth of an inch. Such a faint sound has a pressure disturbance of about a billionth of one atmospheric pressure (which is about 14.7 pounds per square inch or 0.1 MPa). Extremely loud sounds which normally produce pain, have sound pressures of about one thousandth of an atmosphere.

The sound we hear is largely dependent on the frequency. It is this particular feature which determines the pitch or timber. Humans perceive sounds with tones as low as 20 Hz and high pitched as 20,000Hz. This implies that the human ear perceives sounds below the lowest tone on the piano scale which has a tone of 27.5 Hz (the note is A), and above the highest tone which has a fundamental frequency of 4156 Hz, (the note is C).

How Do We Hear

The outer portion of the ear called the pinna directs sound waves into the ear canal. The sound waves cause a displacement in the very thin delicate membrane called the eardrum. On the inner side of the eardrum is attached a chain of three bones (the hammer, anvil and stirrup) which has the other end attached to the oval window. This area constitutes the middle ear. The small displacement of the eardrum is transferred along this chain of bones which creates a much larger displacement in the membrane of the oval window. On the other side of this membrane is the inner ear where the cochlea, a coiled device sits in an aqueous solution. Here the vibrations are transformed into electrical impulses which are transmitted to the brain by route of the auditory nerve. The brain interprets the signals.

The human ear is most sensitive to frequencies within the range of 2 and 4 kHz, which to some extent is the result of the ear canal which resonates within this frequency range. We perceive speech mainly in the frequency range of 500Hz to 3000 Hz. A man's voice which is generally lower than a female’s voice because of the lower frequency band due to the size of the vocal tract, is still perceived over the telephone even though most of the low frequencies are filtered out. The recognition is still possible because “our perception mechanism tends to fill them back in”.

How Does the Ear Detect the Source of a Sound?

There are two ways by which we determine the direction of a sound source. This is dependent on whether the wavelength of the sound is much larger or smaller than the distance between one’s ears. If a high frequency sound (shorter wavelength) is coming from the right, the head will block most of the waves effectively preventing more sound from getting to the left ear which in effect producing a louder sound in the right ear as opposed to the left ear. However, in the case of a low frequency sound (larger wavelength), the wave washes over and around the head. In this case the sound intensity in both ears is the same. However, the ear which is closer to the source will receive the sound first which is enough to determine the direction of the sound source. Our ear is capable of discriminating between a time difference of as small as 0.34 milliseconds which corresponds to a difference in distance of 4.5 inches (=11.4 cm). This is based on the fact that the speed of sound in air is 1100 feet per second (344 m/sec.).

Sound In Intensity Level (Sound Pressure Levels)

The unit used for the measurement of sound intensity level is called decibel, dBA sometimes dB for a change in decibel levels. A sound which doubles in perceived loudness, means that there is a change of about10 decibel. This implies that a sound with an intensity change of 20 dB has increased its loudness by a multiple of four. Similarly, a drop of 10 dB in intensity of the sound means that the loudness has dropped to half its original level. If a sound source 100 feet away has an intensity of 90 dB and you were to move to a distance 200 feet from the source, the intensity of the sound would drop by 6 dB (if the distance from the source is doubled, there is a drop of 6 dB). This is true for the behavior of sound outdoors.

Source Level (in dBA)

Faintest audible sound 0

Whisper 20

Quiet residence 30

Soft stereo in residence 40

Cafeteria 80

Cafeteria kitchen 90

Loud crowd noise 100

Accelerating motorcycle 110

Hard rock band 120

Jet engine (75 feet away) 140

Acoustics in Architecture

A measure of good acoustics is demonstrated in the ability to enhance the sound quality in the environment where people come to listen, for example: concert halls, lecture theaters, music halls churches, synagogues or mosques. The primary goal is to enhance the reflected sound. On the other hand good acoustics is evident in areas where the primary goal is to suppress and control the reverberating sound energy in places like, a library, an office, a classroom or a residential area. These are some of the basic requirements for good acoustics in spaces where one enhances the reflected sound:

1. The sound should be loud enough everywhere. The room should not absorb most of the sound waves.

2. The sound should be adequately distributed around the room. This enhancement could be achieved by having the appropriate reflecting surfaces placed at the proper angles.

3. There should enough clarity. This can be achieved by ensuring that the room does not experience excessive reverberation. The proper reflecting surfaces should provide the solution. Especially for speech, the reverberating sound waves should have only a set acceptable life span.

4. The room should be free of echoes. If a strong the reflected sound reaches the listener 1/10 of a second or greater after the listener receives the direct sound, that could distort the sound which could result in a speech which is unintelligible. The sound could also be perceived as an echo.

5. The room should be free of extra noise- traffic, playground, air conditioning units, etc.

Lesson #1: The production of sound

Aim: To produce and measure the intensity (loudness) of a sound

Students are expected to draw from their own experience in describing sound and how it affects their lives and the lives of others.

Describe your world without sound for a week.

How would it affect the people around us?

How do we communicate without sound?

How do we produce sound?

How do we measure the intensity or perceived loudness of a sound?

Activity: Students can explore differences in sound by banging on different objects.

They could experiment on a set of the same type of objects varying in size or length. They should be allowed to make some inferences from the results. From this demonstration the word “pitch” could be introduced.

The use of a sound meter will allow students to produce different sounds at different volumes. They can use a sound meter to measure the loudness from varying distances.

They can be assigned the task of designing a room or a box of a given dimension using plywood for the outer walls. Their primary task should be to use an insulating material on the inside to insulate the sound from getting to the out side environment. To test the effectiveness, a sound source (an alarm clock) is set off inside the closed room or box and the emitting sound is measured from a set distance using a sound meter. This process can be repeated using different insulating materials.

Mind teaser: Which famous musician in his adult life was deaf yet was able to write classical music? How was he able to accomplish such a task?

Lesson # 2

Aim: To be able to calculate frequency, period, wavelength and velocity

Students should be familiar with the method by which sound waves are propagated. The feature of a longitudinal wave can be helpful in demonstrating amplitude, crest and trough of the wave. In addition, a slinky could be helpful in showing how the wave travels through the material. Students need to note the forward and backward movement of the coils as the disturbance or vibration moves from one point to the other with each section returning to its original position. This forward and backward movement represents one cycle. A repetition of this movement represents another cycle. The definition of frequency, f, could then be introduced as the number of times the wave goes through a cycle in one second (frequency, f, is the number of cycles completed in one second), measured in the unit of Hertz, Hz (1000 Hz = 1 kilohertz). What is one megahertz?

The time taken for the completion of one cycle is the period, (, in seconds. The distance on a wave from one crest to the next is called the wavelength, or the distance from one trough to the next (the wavelength determines the frequency of the wave).

The speed or velocity of a moving object is the distance traveled divided by the time it takes the object to move that distance. This is also referred to as rate (where velocity, v, is equal to the distance, d, divided by the time, (: c = d/().

In the case of a wave, the velocity, c, is equal to the wavelength, (, divided by the period, t (c = (/().

Examples: What is the velocity of a wave that has a wavelength of 10m and a period of 2 seconds?

Solution: c = (/(

c = 10/2 = 5 m/sec

If a wave has a period, (, of 0.01sec, what is the frequency, f (frequency tells the number of cycles per second. We are given the information that in 0.01 seconds there was only one cycle). The task is to find how many cycles there are in one second.

Solution: f = 1/(and (= 1/f

In the equation where c = (/(, it can also be expressed as c = (f.

Since the speed of sound in air is a constant, 1100 feet per second, we can apply the formula to determine the wavelength if the frequency or the wavelength is known. Activity: What is my note?

Each student can measure his/her height in feet. This measurement should be used in the formula (c = (() to calculate the frequency. A chart with the frequency of the notes on the piano scale would be useful in assisting the student in identifying the particular key. The student should strike the key to acquaint himself/herself with that tone.

Lesson #3

Objective: To determine time delay of a sound

When someone speaks in a room, the speech is intelligible if there is an appropriate time delay for the reflected sound. This delay is the difference between the time it takes for the direct sound and the reverberating sound to get to the listener. A time delay longer than 0.075 seconds (75 milliseconds) produces echoes which distort the listening process.

Activity:

Jennifer sits 30 feet from the speaker in an auditorium which is140 feet long. What is the delay time, T?

Solution: Distance from Jennifer to the back of the room is 140-30 = 110 ft.

Total distance of the path of the reflected sound from the speaker to the back of the room reflected to Jennifer: 140 + 110 = 250 ft.

Distance of the path of the direct sound from the speaker to Jennifer = 30ft.

Difference in path of the distance of the reflected sound and the direct sound: 140 + 110 – 30 = 220ft.

(220 ft.is the extra distance the reflected sound had to travel compared to the direct sound)

What is the delay time, T? (The time taken to travel 220 ft.)

Sound travels 1100 feet per second. Therefore the time for 220feet is, T = 220/1100 = 0.2 seconds

Would this be an acceptable time delay?

If this time delay is unacceptable, what can be done to improve the situation?

Project: Students cold use cardboard material to design the four walls of the room. A single piece of polystyrene (that white packing material) can be used for the floor. Toothpicks with cutouts can be used to show the location of the speaker and the listener. Using a scale of one inch being equivalent to ten feet, the students can show varying adjustments from which they can calculate several time delays. To show the path of the sound wave, the students could use pieces of thread of different colors to indicate the direct wave to the listener and the reflected wave from the back of the room leading to the listener.

Evaluation:

What is time delay?

What is an acceptable time delay?

How does excessive time delay affect sound quality?

How is time delay calculated?

Lesson # 4

Aim: To determine the distance of a sound source

Since light travels faster than sound we can use this situation to measure the distance of a sound. Whenever there is a thunderstorm (if we are within a reasonable distance) we might be able to see the lightening then afterward we hear the thunder- light travels faster than sound. The same is true for a fireworks display in the distance. If we could measure the time the flash is seem, then measure the time it takes for us to hear the sound, we could use this information to determine how far away the fireworks is. The same is true for the thunderstorm – we could tell how far away the storm is located

Activity:

Fireworks: After the flash we hear the sound 2 seconds later. How far away is it?

Since we know that sound travels at 1100 ft per seconds, therefore the distance the sound travels in 2 seconds is, 2 x 1100 = 2200ft

If a person is standing 6600ft away, how long should it take for him or her to hear the same sound? 66600/1100 = 6 seconds.

Students can be asked to find the solutions for similar problems, varying the time and distance. They could be asked to design an activity that could demonstrate the speed of sound in air.

Suggested Readings

Deaf Architects & Blind Acousticians? A guide to the Principles of Sound Designs; Robert E. Apfel Architectural Acoustics: Principles and Practice; William J. Cavanaugh Noise Control Manualfor Residential Buildings; David A. Harris

Student Reading List

Noise and Fumes (What Can We Do); Donna Bailey Noise Control; A Primer; Alberto Behar Noise Pollution: Earth's Conditions Series; Zachary Inseth

Bibliography

Apfel, E. Robert, Deaf Architects and Blind Acousticians, Apple Enterprises Press, New Haven, CT, 1999 Rossing, D. Thomas, The Science of Sound – Second Edition, Addison-Wesley Publishing Company, USA, 1990