Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
2001 Volume V: Bridges: Human links and innovations

The Basic Mathematics of Bridges

Curriculum Unit 01.05.06
by Lewis Spence

Bridges serve one basic purpose that of connecting two points that are otherwise disconnected and difficult to access. Generally it provides the shortest distance between the two points. The art and science of constructing these structures rely heavily on the mathematics and consequently the physics of stress and load. Some of the prevailing factors that influence the designs and types of structures include the intended use and users, the available constructing materials, environmental conditions, cost, adequate manpower, the length of the span, the type of river banks, present and projected traffic load, the free height under the bridge, aesthetic considerations and the available technology. (Salvadori, 144)

The history of mankind suggests that his survival is somewhat dependent on the ability to travel from place to place whether for war, protection or in search of the resources vital to his existence. To facilitate this mobility, man has used several techniques to bridge gaps providing easy and short access routes. Over time, as the demand on human existence intensifies, coupled with the growth of technology, be it small, man has devised more efficient and elaborate means of transportation by land, water and even air. Consequently better, safer and more durable bridge structures emerged. The technology that emerged provided the world with four basic forms of bridge structures that are replicas of natural forms found in nature. "But there is still more to the bridge than just usefulness and looks. However shallow the water, the smallest stream can seem a Rubicon. Our other aim must to consider the bridge as symbolic marker, dominating a stage that the traveler has reached on a journey that may not, perhaps, be reversed."(Sweetman, 1)

I teach mathematics at the eighth grade level in a middle school in New Haven. The population is predominantly Black- about twelve percent Latino, five percent Asian and twenty percent white. There is a pressing need to improve the academic performance of our students in the area of mathematics given the recent results of our standardized testing. There is a noticeable deficiency in the area of mathematical applications - an unacceptable number of our students lack the mastery and proficiency in this area. Consequently, the development and use of a curriculum that focuses on the practical application of the basic mathematical concepts should be helpful in addressing the student's deficiencies.

Given the level of mathematics that is taught at the eighth grade level, the mathematical focus on the topic of bridges will be on proportions.

Proposed Activities

This involves determining the proportionality of force relative to a given distance from a prescribed point. A bridge must be able to withstand a certain amount of load given its intended use. A practical demonstration could incorporate the use of two scales or spring balances. A given beam is used to bridge the two balances to which is attached a sliding weight. As the weight changes positions on the beam, the result of the balances are checked to record the change in force relative to the distances moved.

2. The students will be required to construct a bridge using available materials

3. They will be expected to construct proportional models of the intended load

4. They will be expected to compare different designs and identify their advantages and disadvantages

Students will be expected to acquire information on famous bridges and the general history of their construction. Discussion will also focus on the materials used in the construction, the environmental impact of the structure and quite importantly the social and economic effect on the population within the area of the structure.

The Basic Features of Bridges

Ancient societies are known to have the experience of bridge building. However, the Romans have left a lasting legacy of durable usable structures that have lasted for more than 2000 years. Some of the remarkable features are the technology and the utilitarian diversity. Some of the designs were not only for human traffic but great aqueducts that supplied towns and plantations with great quantities of water vital to the maintenance of the city. One of the most impressive aqueducts built by the Romans is the Pont du Gard, completed in18 B. C. It carried water to a ancient city, Nemausus, a distance of more than thirty miles. The brilliance in the technology and its durability is still a testimony to the genius of the Roman engineering skill. (Dupre, 15)

There are some cardinal principles that every functioning bridge must adhere to regardless of size or utility. Every bridge must be able to tolerate and overcome the stress or forces it encounters. For it to function, it should carry its own weight, a weight which is called dead load; it should be able to carry the weight of the traffic for which it was intended, a weight which is called live or dynamic load. It must have the capability of resisting natural forces like wind, earthquake or other natural environmental stress or load.

The magnitude of a bridge is generally characterized by the length of its span, which is the distance from one end of the main support to the other. A plank across a stream is a span - the length of the plank. Longer bridges might require support from below; these columns are called piers. The major supports at the two extreme ends of the bridge are called adutments.

There are four types of forces that act on a bridge singly or in combination: tension, compression, shear, and torsion. The force that pulls or stretches apart is called tension. This is the exact opposite of compression, a force that pushes together. Shear is a sliding force while torsion produces a twisting effect. (Super Bridges)

The designs of bridges take four basic forms derived from nature: the Beam, a log across a stream, an arch that patterns rock formation, the suspension and the cable from a twig or branch hanging from a vine. Changing technology and the availability of different kinds of materials have greatly influenced the types of designs and structures over time. (Dupre, 12)

Types of Bridges

Beam Bridge

The simplest form and the most inexpensive form of a bridge is the beam or "girder'. It takes on the natural basic form of a log fallen across a stream (Dupre, 12). It consists of a horizontal beam which has at its end two support called piers or abutments. The beam must be able to bear its own weight plus the weight of the traffic without bending. The beam experiences the forces of the dead and dynamic loads which produce a compression force which forces it together, while the bottom edge is pushed apart- the tension force.

A good example is that of taking a thick piece of sponge which is about two inches thick and cutting a strip about three inches in width and a length of about six to eight inches. Get two cans of the same size to provide the support and place them about four inches apart. Use the sponge as a plank across the two supports. Cut a notch in the middle on either side. Slightly press on the middle section, then observe the notches. You will notice that the notch closes on the topside which is the evidence of compression force while on the under side, the notch tends to widen-- evidence of the tension force.

An ideal material used in beam bridge construction is pre-stressed concrete. It withstands very well the forces of compression and the steel rods imbedded in it resist the forces of tension-- pre-stressed concrete is one of the least expensive materials used in construction. However efficient the material, there is no compensation for beam bridges greatest limitation, its length. The further apart the supports are placed, it's the weaker the beam bridge. Considering this factor, beam bridges seldom span more than 250 feet, which does not imply that it cannot be designed to span great distances. In situation where it spans a great distance, the spans are daisy-chained together a connection known in the bridge world as "continuous span'. The world's longest bridge, which is 24 miles long, is a continuous span beam bridge, Lake Ponchartrain Causeway. It consists of two, two- way lane sections running parallel to each other. The Southbound Lane, which was completed in 1956, consists of 2243 separate spans, while the Northbound Lane, completed in 1969, consists of 1500 longer spans.

One big drawback of continuous spans: they are not suitable for places where there is need for the unobstructed clearance below the span given the fact that a great number of piers (support) are required.

Arch Bridge

Arch bridges pattern the natural rock formation. This type of bridge is considered one of the oldest and does posses great natural strength. In this design, the load of the bridge instead of pushing straight down, it distributes the load outward along the curve of the arch to the support at the ends of the bridge, the abutments. They support the load and prevent the ends of the bridge from spreading out.

How do the abutments support an arc bridge?

Get a 2- inch by 12- inch strip of cardboard. Bend the strip gently forming an arch. Place both ends on the table in the fashion of an inverted U. Press down gently on the top of the arch and notice the effect on the ends. The likely outcome is the spreading outward of the ends of the arch.

Place two stacks of books about six inches apart (the stacks must be lower than the height of the arch). Place the arched strip between the stacks of book in the inverted U fashion. Apply a gentle force to the top of the arch and notice how the stack of books act as abutments, preventing the ends of the arch from spreading apart. Whenever the arch bridge is supporting its dead load, its own weight, and the load of the traffic crossing the bridge, very part of the arch is under compression. Given this situation, arch bridges must be constructed from materials whish can withstand much compression force.

The Romans, who are known for their great engineering genius at building arch bridges, used stones as their primary construction material. The upper portion of the bridge is held together by mortar while the rest of the structure is held together by its own weight.

In present time, the availability of steel and pre-stressed concrete makes it possible to construct longer and more elegant arches. Generally, modern arches span a distance of 200-800 feet. In New River Gorge, West Virginia, there is a spectacular arch bridge that has a span of 1700feet.

Constructing arch bridges can be somewhat tricky due to the fact that the structure is unstable until the both spans meet in the middle. To overcome this problem, sufficient scaffolding is used to provide support for the spans until they meet in the middle, a technique known as centering. Another method of support is the use of cables that are attached to the spans at one end while the other ends anchored to the ground on the other side of the bridge. This method allows for the use of the waterway or road below the structure while it is still under construction.

The Natchez Trace Bridge in Franklin, Tennessee, which was opened in 1994, is the first American arch bridge to be constructed from pre-cast concrete. Two arches are used to support the roadway above. Conventionally arch bridges of comparable size require vertical supports called 'spandrels" to distribute the load of the roadway to the arches. This bridge is designed without the use of spandrels for aesthetic reasons. Instead, most of the live load is resting on the crown of the two arches that are slightly flattened to provide better support.(Super Bridges)

Suspension Bridge

In Asia, as early as the first century B.C., suspension bridges were constructed to provide access that averted the need for piers and other forms of centering. In the earlier designs, the structures were suspended with bamboo cables. By the sixth century A.D., the introduction of iron chains displaced the use of bamboo cables in parts of China. (Kranakis, 29)

Suspension bridges are light, strong, aesthetic and span distances from 2,000 to 7,000 feet-- much longer distances than the other types of bridges. It is the most expensive type of bridges to construct. This type of bridge as the names suggest, suspends the roadway from huge bundles of cables, which extend from one end of the bridge to the other. The cables are supported on high sturdy towers and are secured firmly at each end on the ground to firm anchors of solid rocks or massive concrete blocks.

The tower allows the cable to be draped over them for long distances. Most of the weight of the bridge is transferred by way of the cable to the anchorages. At the anchorage, the cables are spread out so that the load is evenly distributed. This provides greater security. (Super Bridges)

What are the uses of the anchorages?

Get two pieces of board an inch in thickness, 4 inches by 6 inches. Place a small nail half way down at the top of the wood. Get a length of string about 36 inches in length and to the middle of the string place a loop around each nail about 10 inches apart. Place the two pieces of wood uprightly facing each other. Allow the string to hang arching between them. Apply a weight about that of a wallet to the loop and notice the result.

Replace the pieces of wood to their original position. This time place the free ends of the string over their corresponding sides and secure each end to a separate secure anchor allowing the length of string between the anchors and the piece of wood to have just a slight arch. Then add the same weight to the loop between the pieces of wood and note the result. Notice that the anchorages on the outside of the pieces of wood help to stabilize the "bridge".

In earlier times some of the cables used were made from twisted grass. During the early part of the he nineteenth century, the cables used on suspension bridges were iron chains. Presently the cables used are made of thousands of individual steel wires bounded tightly together. Steel shows the capacity to withstand great tension making it an ideal choice for cable material.

The Humber Bridge in England, once had the world's longest center span - measuring 4,624 feet. Currently in Japan, the Akashi Kaikou Bridge, linking the islands of Honshu and Shikoku, has a center span of 6,527 feet.

Because of the length, flexibility, and lightweight feature of suspension bridges, wind is always a serious concern. The Tacoma Narrows Bridge opened in 1940, was the third longest suspension bridge in the world -- a span of 2,800 feet. It was noticeably quite unstable even in moderate wind conditions. Attempts were made to address the instability but with little success. On November 7, 1940 barely four months after being opened, it collapsed in a wind speed of 42 mph. It was designed to withstand wind speeds of up to 120 mph. Scientist believe that the wind matched the resonance frequency of the bridge and it produced a destructive displacement. (Super Bridges)

The engineer who designed the structure did anticipate some form of wind displacement, but according to others, he neglected to calculate the effects of multiple pushes of the wind force on the cables of the structure. (Kranakis,155)

Cable-Stayed Bridge

Cable-stayed bridges do look much like the suspension bridges-they all have towers and roadways hanging from cables. However, the difference lies in the manner in which the load of the roadway is supported. In suspension bridges, the cables are draped over the towers and the ends are secured to the anchorages where the load is distributed. In cable-stayed bridges, the cables are secured to the towers that are responsible for bearing the load of the bridge. There are several ways by which the cables can be attached: radial pattern -- the cables extend from several points on the road to a single point at the top of the towers. The parallel pattern attaches the cables from different heights along the tower at an angle parallel to each other.

Demonstrating a cable-stayed bridge

If you are standing up with your arms stretched horizontally, your head now becomes the tower and your outstretched arms the span or roadway of the bridge. Get a partner to tie a piece of rope to each wrist to support your arms in the horizontal position with the middle of the rope resting slightly taut on your head. Get a second piece of rope and repeat the process except this time the ends of the rope are tied to your elbows. Now you have two cable-stayed. Try bringing your arms down to your sides. Where do you feel most of the load?

The concept of cable-stayed bridges might appear to be a relatively new concept, however as early as 1595 there have been published sketches of this type of bridge. It was not until the 20th century that this design became acceptable. In Europe just after World War 11 when steel was scarce, the cable-stayed bridge designs were perfect for reconstructing bombed out bridges which still had standing foundations.

In the United States it is considered a fairly new approach to bridge construction. However, the response has been positive given the aesthetic nature of the feature and the cost effectiveness. For medium length spans between 5000 and 2,800feet, cable-stayed is fast becoming the bridge of choice. Compared to suspensions, they require less cable, can be constructed out of pre-cast concrete sections, and are faster to construct.

In 1988, the Sunshine Skyway Bridge in Tampa, Florida won the prestigious Presidential Design Award from the National Endowment for the Arts.(Super Bridges)

Activities

---

1. Students will construct models to demonstrate the following:

   a. Compression
   b. Tension
   c. Torsion

2. Students will focus on the construction of a cable-stayed bridge with the objectives of getting practice in measuring angles and linear measurements, and applying the principles of similar triangles.

3. Students will use a model of the beam bridge to discover the effect of the load on a beam at varying distances.

4. Students could choose to investigate the strength of a model cable (a thin thread) to determine the structure of a suspension bridge of a required load capacity based on the results of their findings.

Activity 1

Aim: To demonstrate Compression, Tension and Torsion

Material: Piece of foam material 3" x 3" x 10"

Procedure to demonstrate compression:

1. A weight or load is applied to the piece of foam and students are asked to describe the change in the form of the material.

2. A small portion of about a half of an inch is removed from the center on the top and bottom surface, no deeper than about one inch. Each end of the strip of foam is supported by a thick textbook. The books are placed about five inches apart. A slight pressure is applied to the top middle portion of the foam. Students should observe:

a) The change in the surface of the material in particular in the area of the slit.

What do you notice about the slit on top?

What does the force of compression do to a material?

What kind of force does your body exert on the floor?

Procedure to demonstrate Tension

a) Observe the change at the underside of the foam while a force is applied to the top.

What do you notice about the slit at the bottom while the force was applied?

Try to gently break a thin strip of pine wood, about a foot in length.

Where does the splintering start?

b) Pin two three inch pieces of thin strips of wood at one end forming an inverted V. Place it on the table in an inverted position on the desk. Put a rubber band around the opened end (slightly taut) then press down on the point (vertex) of the V.

What change or changes do you observe?

Procedure to demonstrate Torsion

Material: Strip of rubber or sponge about a foot in length

Hold one end of the strip with the right hand and the other end in the left hand. Then twist the strip in a ringing motion in opposite directions.

What effect does this turning force have on the strip?

Activity 2

Aim: To show the effect of a moving load on a bridge

Material: Two spring-balances, a meter rule

Procedure: Place the balances on a firm table. Place one end of the meter rule on one of the balances and the other end on the other balance. Get a small object that has a weight of about fifty grams starting at the end the rule.

The two balances will act as the abutments of a typical beam bridge while the weight will behave like a moving vehicular load over the bridge. The meter rule represents the beams of the bridge. To begin: places the weight at the one end of the rule, then observe and record the separate weights measured on each balance. Move the load in increments of ten centimeters and record the weights.

Observation:

a) The initial weights of the beam on balance one and on balance two
b) The changes in the weighs as the moving load changes position

Inference: If the bridge were to collapse, at what point along the beam is it most likely to occur? Why?

Activity 3

Aim: To apply the mathematics of proportion

Students will construct a beam bridge proportional to a reasonable dimension of a real beam bridge. Students could visit a local bridge and acquire the basic measurements of the width and length of the span. They can choose a their own materials to construct the bridge. Each project should have a drawn plan that matches the proposed scales. Prior to this exercise, students should be exposed to the principles of finding equivalent proportions. This bridge building exercise should serve as practice in practical applications of the mathematics of proportionality.

Activity 4

Aim: To construct a cable-stayed suspension bridge

Procedure: Students will identify a particular cable-stayed bridge and ascertain the dimension of the structure. They should draw a scaled rendition of the structure that they will use to construct their model. This activity should provide useful exercises in drawing and identifying the properties of similar right triangles and finding equivalent proportions.

The tower of the bridge forms the vertical side of the right triangle. The design could have five attached cables on each side of the tower. The distance between the points of attachment of preceding cables on the tower should be equal. Likewise, the points of attachment of the cables on the beam of the span should be equidistant.

Students should be able to calculate the length of the remaining cables after the first cable has been installed by applying the proportionality concept. For the more advanced students, the Pythagorean theorem could be utilized.

Activity 5

Aim: To evaluate the economic effect of a particular bridge.

Procedure: Students should select a particular bridge and observe for about an hour the number of vehicles that use the bridge traveling in both directions. Later they should then assume that the bridge is destroyed. They should now show the economic impact on one of the bridge user applying reasonable estimates. For example, find an alternate route to get to his/her original destination. Calculate the additional distance and the increase in the time required to travel, and the cost of the additional volume of gasoline.

Inference: In what ways will this affect the rest of the other drivers who will now share road space with the additional drivers?

Activity 6

Aim: To measure the arch of a bridge in degrees

Procedure: Students should obtain a picture of an arch bridge. They should trace the arch of the bridge on sheet of paper. They should use the arch, which is now considered an arc of a circle, to complete the missing portion of the circle. With the knowledge that a circle has 360°, they should measure the length of the arc that forms the arch of the bridge, and compare it with the length of the total circumference of the circle. They can then write a ratio of the length of the arch of the bridge to the circumference of the circle. This ratio should be considered equivalent to the ratio of the degrees in the arch of the bridge which is unknown but could be called "d ", to the total number of degrees in the full circle, 360°. The two ratios are used as equivalent fractions to calculate the degrees "d " of the arch of the bridge.

Works Sited

Dupre, Judith. *Bridges* . New York, N Y: Black Dog & Leventhal Publishers, Inc., 1997.

Kranakis, Eda. *Constructing a Bridge* . Cambridge, Massachusetts: The MIT Press, 1997.

Salvadori, Mario. *Why Buildings Stand Up* . New York, NY: W. W Norton & Co., 1980.

Super Bridges. http://www.pbs.org/nova/bridge/meetcable.html, 2002.

Sweetman, John. *The Artist and the Bridge* . Brookfield, Vermont: Ashgate Publishing Co., 1999.

Teacher and Student Resources

Calatrava, Santiago. *Dymanic Equilibrum: Recent Projects* , Zurich, Switzerland: Artemis Verlags, 1992. This text presents some of the most aesthetic and innovative structural designs in bridge.

Dupre, Judith. *Bridges* . New York, NY: Black Dog & Leventhal Publishers, Inc., 1997 This is a text which presents a great variety of outstanding bridges spanning a long time period.

K'nex Education Division. *Bridge - Educator Guide* . Hatfield, PA: K'nex Industries, Inc., 1996. It has basic bridge building projects.

Polladr, Jean. *Math Project - Building Bridges* . Paloalto, CA: Dayl Symoure Publications, 1985. This book provides bridge building projects

Salvadori, Mario. *Why Buildings Stand Up* . New York, NY: W.W. Norton & Co.,1980.

Super Bridges. http//www.pbs.org./nova/bgidg/meetcable.html, 2002 This is a site that provides information on the basic features of bridges in a format that is student friendly.