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Multi-Sensory Manipulatives in Mathematics: Linking the Abstract to the Concrete

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by Judith L. Bellonio

"I hear, and I forget. I see and I remember. I do, and I understand"

-Chinese Proverb

Experiential education is based on the idea that active involvement enhances students' learning. Applying this idea to mathematics is difficult, in part, because mathematics is so 'abstract'. One way of bringing experience to bear on students' mathematical understanding, however, is the use of manipulatives. Manipulatives are small, usually very ordinary objects that can be touched and moved by students to introduce or reinforce a mathematical concept. Manipulatives come in a variety of forms, from inexpensive, simple buttons or empty spools of thread to tangrams and pattern blocks. Typically, it has been the primary grades' teachers who have generally accepted the importance of manipulatives.

"Both Pestalozzi, in the 19th century, and Montessori, in the early 20th century, advocated the active involvement of children in the learning process. In every decade since 1940, the National Council of Teachers of Mathematics (NCTM) has encouraged the use of manipulatives at all grade levels. Every recent issue of the "Arithmetic Teacher" has described uses of manipulatives". "Arithmetic Teacher" articles feature monthly articles describing the usefulness (as well as the why, when, what, how and with whom) of manipulatives.

Research indicates that manipulatives are particularly useful in helping children transition from the concrete to the abstract level. It is key for the teacher, however, to select the appropriate activities/manipulatives to support the transition. The transition often reflects the developmental process. Further research, in a review of activity-based mathematics learning, indicates that mathematics achievement increased when manipulatives were used.

"Children learn best when they are active rather than passive learners". According to Spikell most learners, whether adults or children, will master mathematical concepts and skills more readily if they are presented first in the concrete/pictorial or with symbols. Manipulatives are concrete objects used to teach a concept. By using manipulatives, pictures and symbols to model or represent abstract ideas, the stage is set for learners to understand the abstractions they represent.

Just recently, while teaching an Extended Day 5th grade Mathematics class, I presented each student with a set of tangrams (ancient Chinese 7-piece tile set). The students had an opportunity to explore the tangrams,

reconfiguring the pieces to create new shapes and designs. After 'playing' with the tangram pieces one student commented that she felt as though she was back in Kindergarten. I felt sad to think that a 5th grade student's memory of working with manipulatives in a Mathematics class took her all the way back to Kindergarten. Research also shows that long-term use of manipulatives was more effective than short-term use.

Math Anxiety

Many students (and adults) have math anxiety. These students are constantly frustrated by poor grades, poor math related experiences and lack of understanding of mathematics. These students do not enjoy mathematics and do not see the need for its usefulness or its applicability in their lives. This feeling of math anxiety often leads students to pursue careers that limit the use of mathematics. As our society advances so does the ever-present need for mathematics skills. Mathematics is an element present in virtually every career (especially if you are looking to make money).

Types of Learners

Research by Howard Gardner has indicated eight distinct "intelligences" which are the essential ways by which people learn. In general, "only two of the intelligences are taught ". They are the linguistic intelligence and mathematical/logical intelligence. The linguistically oriented learners have an innate love of language. They often have well-developed vocabularies and their fluent use of language includes a richness of expression and elaboration. Reading, writing, editing, listening and speaking opportunities are enjoyed. Working with books and diverse printed materials, records, tapes, lectures, word processors, etc. are pleasurable activities for these learners. The logically-mathematically-oriented learners enjoy forming concepts, looking for patterns and relationships, and doing activities in a sequential manner. They like to arrange the steps of a project into a sensible order or timeline and desire time to complete each component. These learners enjoy opportunities to problem solve, learn quantities or facts and have time to explore new concepts. They often ask many questions and desire logical and clear explanations. Problem solving, working with games, kits and puzzles, collecting, classifying, organizing information tasks and people are enjoyable pastimes. According to Gardner there is no uniform way of teaching and testing is patently unsatisfactory when everyone is so different.

The Kinesthetic Learner

Another one of Gardner's eight distinct "intelligences" is the kinesthetic learner. The kinesthetic learners are individuals who learn best by moving, touching and doing. They are usually not attentive to visual or auditory instruction, but are eager to attack problems physically and with great activity. Manipulatives, role-play, stimulations, physical exercise, games; competitive sports and action-packed stories are enjoyed. These learners will remember best what they have physically done . Manipulatives appeal to the kinesthetic learners because they are able to actually touch the objects. For this type of learner manipulatives are important in the teaching and understanding of mathematics.

Manipulatives help relieve boredom in students, they offer a change from the textbook (abstract) method of learning allowing students to explore and use their imagination. Manipulatives provide a picture of a math concept that appeals to visual/spatial learners. Visualization is the natural way one begins to think. Before words, images emerge. Manipulatives can also be placed within cooperative groups, which is appealing to the interpersonal learners.

It is beneficial for educators to be aware of the different types of learners. It is necessary that lessons be presented in a variety of ways to reach the different types of learners. If a student is always being taught in a style that does not ever comply with his or her learning style, this will cause the student great anxiety toward a subject and not optimize the learning process. It is, therefore, important that math classes be taught in different ways. The use of manipulatives enables the teacher to more easily reach all of the various learners. The use of games such as Math Bingo where the teacher asks a variety of review-type questions such as duration of time, measurement, squared numbers, and the calculation of money to name just a few is one way in which a teacher can reach students through alternative methods. Teachers can incorporate a Quizmo-type card game where student are each given a card containing an answer and a question and they are to listen for the question that fits their answer. Students then ask the question listed on their card (some students may even enjoy creating their own Quizmo-type game to share with the class). Activities that involve the measuring of actual containers or items are also very useful. Various activities using pattern block is extremely useful in reinforcing fractions, patterns, symmetry, and geometric shapes. Art projects that incorporate graphing or creating designs using geometric shapes will also help connect some students. The use of computer technology, both the Internet and commercial software is also beneficial for many students.

"Almost every mathematics idea, except simple arithmetic facts, consists of three components: linguistic, conceptual and skill/procedural" . The conceptual component deals with the center of mathematical concept. According to Sharma students needs a model to conceptualize the mathematical idea.

Levels of Mastery

Sharma feels there are six levels of mastery of mathematical concepts: intuitive, concrete, representation (pictorial), abstract, applications and communication. According to Sharma, ideally, each mathematics concept should be introduced beginning at the communication level. However, almost all mathematics-teaching activities take place at the abstract level. It is believed that students have a tendency to forget when taught only at the abstract stage. Students therefore become frustrated because mastery was never fully attained. Students will then have difficulty in learning mathematics.

Types of Manipulatives

Manipulatives come in many shapes and forms; they vary in price and complexity. Some manipulatives include calculators, money, two-color counters, buttons, paper clips, tooth picks, string, playing cards, rulers, number cubes (dice), graph paper, empty egg cartons, capacity containers (measuring cups), spinners, drinking straws, thermometers, pattern blocks, dominoes, cuisenaire rods or strips, geo-boards, tangrams, and pentominoes to name only a few of the commonly available manipulatives that can be successfully used in the classroom. These manipulatives can be used to teach such concepts such as angles, decimals, factoring,

estimation, fractions, measurement, counting, percent, prime numbers, probability, geometry, place value and whole numbers.

Games with manipulatives are also a great way to allow students to apply what they learned to the real world. Using board games and card games along with cooperative learning are ways that students can become involved in a positive mathematical environment. Games are highly motivational to students and can be used effectively to practice specific skills. "Using games in the classroom and at home will maximize students' problem-solving competence, ability to communicate and reason mathematically, perception of the value of mathematics, and self-confidence in their ability to apply mathematical knowledge to new situations" .

Incorporating the Computer

When using manipulative and games to introduce and to teach math concepts it is also beneficial to reinforce the math concept. With the increase in computer use both in the home and at school it makes perfect sense to incorporate the computer via Internet and commercial software to reinforce math concepts. There are many exceptional Internet sites as well as commercial software packages. For example, the Internet site <http://math.rice.edu/~lanius/Lessons> has numerous excellent links to such topics as ratios, fractions with pattern blocks and graphing. The Internet site <http://best.com/~ejad/java> has three links: pattern blocks, Cuisenaire rods, and base ten blocks. A teacher may have to spend some time preparing a computer lesson to be used at this site but all the on-line manipulatives are ready to be used. It is helpful for many students to be given a zip-lock bag of actual manipulatives (pattern blocks, Cuisenaire rods or base ten blocks) to go along with the lesson. Another excellent Internet site is <http://www.enchantedmind.com>, which has numerous links, one of which is to tangrams. This link enables students to use their mouse to arrange the tangram pieces into a variety of shapes and designs.

There are also countless excellent math related commercial software programs on the market that can be purchased at almost any computer software store, i.e., Circuit City or CompUSA. The Math Blaster series have great math depth and come in a wide range of grade levels and math subjects. The Edmark series includes the Mighty Math Calculating Crew and Mighty Math Numbers Heroes that cover math topics such as fractions, measurement, addition, multiplication, probability, and geometry. Another excellent software program is Kaplan's Math for the Real World, which gives students problem solving and math application experience via a rock and roll band that they can really relate to and benefit from.

Why don't all Teachers use Manipulatives?

If the findings are so compelling then why don't all mathematics teachers use manipulatives? There are several possible reasons why more mathematics teachers do not use manipulatives in their lessons, namely:

- 1) Lack of training, many teachers feel that they do not know how to teach using manipulative and, therefore, are not comfortable using manipulatives and hesitate to use them in the classroom. New Haven teachers are fortunate to have access to numerous classes and workshops

to learn how to teach using manipulatives. In addition, the companies that distribute the manipulatives provide books; pamphlets and videos on ways the materials can best be used. There are also countless articles on using manipulatives in mathematics teaching journals such as the NCTM's Arithmetic Teacher magazine.

2) Availability of manipulatives, although New Haven teachers are very fortunate to have an ample and varied supply of manipulatives at their disposal. Even without resources any teacher can easily come up with a can of buttons or straws.

3) Lessons using manipulatives may perhaps be noisier and not as neat. Using manipulatives works nicely in a cooperative learning setting. It is a good idea to use plastic cups or zip-lock sandwich bags as a way to keep manipulatives organized.

4) A fear of the breakdown in classroom management. Manipulatives require a great deal of prior planning and organization.

I enjoy mathematics and I believe that the learning process should be enjoyable for all types of learners. I believe that the use of manipulatives is beneficial in all areas of mathematics. The focus of this unit is the use of multi-sensory manipulatives in the teaching/learning of mathematics. The unit is intended for 5th and 6th grade students. The area of mathematics that is being covered is fractions. I have chosen fractions because it appears to me that that is the area in which the majority of my students have not mastered through conventional (worksheets, etc.) means. When learning fractions there are so many rules to know, there are rules for adding and subtracting fractions, different rules for multiplying fractions, and then dividing fractions is another story altogether. Students have to learn about denominators, numerators, proper and improper, mixed numbers. The students probably start learning about fractions informally at a young age when simply sharing a snack with a sibling. Formal education of fractions probably begins in the 4th grade. Students then learn about fractions each year from then on, yet very few 8th graders have a good handle on fractions. There is no ownership. The concepts of fractions are very abstract. I feel that through the use of multi-sensory manipulatives I might afford my students the opportunity to buy into fractions.

This unit includes lessons that extensively use pattern blocks, tangrams, and Cuisenaire strips. Pre-lesson activities include the making of tangram sets and Cuisenaire strips. In addition, the lessons include interactive Internet activities that coincide with each manipulative type. The unit will culminate with a Family Math Night at which time the students, along with their families, will partake in mathematics activities to include pattern blocks, tangrams, various computer software programs and several Internet activities. I have also compiled a Mathematics Guide to Parents, 'Everything You Ever Wanted to Know About Math But Were Afraid to Ask' which will be made available to each family.

The problem that I see is that the students in New Haven (and Betsy Ross Arts Magnet School in particular) are substantially behind the rest of the State in Mathematics Mastery. I believe that the use of multi-sensory manipulatives will help but I can only reach the 90 or so students that I teach. I feel that in order for this unit to be fully effective it will be necessary for me to reach out to my fellow Mathematics teachers at Betsy Ross.

I am excited and motivated to use multi-sensory manipulatives in teaching mathematics. As a new teacher I have been exposed to (through recent training) and have researched various manipulatives and techniques for their using. I have also observed that many of the veteran mathematics teachers are not using manipulatives (this is not due to lack of supplies because the New Haven district Mathematics Department is very generous). Due to the extensive research that I have done using the Internet I was chosen to present a teacher workshop on 'Using the Internet to Integrate Mathematics into the Curriculum' at a recent in-service training session (see attached listing of internet sites). The workshop was well received, gaining me the respect of my fellow teachers. Since then several teachers have come to me for direction. Now that I have the support and confidence of our staff it is my hope to present future workshops. It is my intention to make this unit available to all mathematics teachers at Betsy Ross Arts Magnet School by way of a Multi-Sensory Manipulatives Mathematics Teachers' Workshop.

Lesson 1: Fraction Puzzles with Pattern Blocks

Purpose: Develop an understanding of fraction using pattern blocks as well as to reinforce geometric shapes.

Materials: Overhead projector

Overhead pattern block pieces

Pattern Block pieces - each student needs 3 red trapezoids, 3 blue rhombuses and 12 green triangles

This is a teacher-directed activity. Students will, in the following sequence:

(figures available in print form)

- 1) build a triangle that is $\frac{1}{1}$ green
- 2) build a triangle that is $\frac{4}{4}$ green
- 3) build a triangle that is $\frac{2}{4}$ ($\frac{1}{2}$) blue and $\frac{2}{4}$ ($\frac{1}{2}$) green
- 4) build a triangle that is $\frac{4}{4}$ green (helpful to repeat this step before doing #5)
- 5) build a triangle that is $\frac{3}{4}$ red and $\frac{1}{4}$ green
- 6) build a triangle that is $\frac{9}{9}$ green
- 7) build a triangle that is $\frac{1}{9}$ green, $\frac{2}{9}$ blue, $\frac{6}{9}$ ($\frac{2}{3}$) red
- 8) build a parallelogram (each pair of sides is parallel) that is $\frac{8}{8}$ green
- 9) build a parallelogram that is $\frac{6}{8}$ ($\frac{3}{4}$) blue and $\frac{2}{8}$ ($\frac{1}{4}$) green
- 10) build a parallelogram that is $\frac{6}{6}$ green
- 11) build a parallelogram that is $\frac{4}{6}$ ($\frac{2}{3}$) blue and $\frac{2}{6}$ ($\frac{1}{3}$) green
- 12) build a trapezoid (only one pair of sides is parallel) that is $\frac{12}{12}$ green
- 13) build a trapezoid that is $\frac{6}{12}$ ($\frac{1}{2}$) green and $\frac{6}{12}$ ($\frac{1}{2}$) blue

Pre-Lesson Activity: Have students stand around a table that has an ample supply of pattern blocks placed in the center of the table. The teacher, using a timer, will allow the students to create a design, when the timer sounds (says 1 minute) the students will move to their right and build upon (not destroying or taking away from) their neighbor's creation. Activity ends when students arrive back at their original location.

Pre-lesson Game: Teacher creates a design out of connected hexagons. Two students play the game by taking turns placing either a green, blue or red pattern block pieces on the design. A player loses if they place the last pattern block piece on the design.

Internet Activity: <http://math.rice.edu/~lanius/Lessons>. Click on pattern blocks (yellow hexagon)

Lesson 2 - Fractions with Tangrams

Purpose: Develop an understanding of fractions using tangrams.

Materials: Overhead projector

Overhead tangram pieces

Tangram sets (various colors) - each student needs one set

Teacher directed activity. Lesson may work best with students working cooperatively in groups of two so that they may combine their different-colored tangram sets for ease in naming pieces.

Activity: Below is a tangram set. Students are to give each of the seven pieces a fractional name ($\frac{1}{2}$, $\frac{1}{4}$, etc). Teacher may want to start by asking students the fractional names of the two larger triangles asking how many large triangles it will take to cover the completed tangram set. Then move on to the medium-sized triangle by having student first determine how many medium-sized triangles it takes to cover one of the larger triangles. Next move on to the small-sized triangles as they compare to the medium-sized triangle. Once the medium and the small-sized triangles' fractional names have been established move on to the small square and the parallelogram.

(figure available in print form)

Pre-Lesson Activity: Using colored construction paper, have students create their own person tangram set. Refer to <http://www.askdrmath.com/trscavo/tangrams/construct.html> for a lesson plan.

Internet Activity: www.enchantedmind.com. Click on Puzzles and then tangrams.

Students will use their mouse to arrange tangram pieces into various shapes.

Lesson 3 - Fractions with Cuisenaire Strips (or rods)

Purpose: Develop a further understanding of fractions and mixed numbers using

Cuisenaire strips (or rods).

Materials: Cuisenaire strips (or rods) - one set per student

Cuisenaire strip (or rods) worksheet

Students may work cooperatively in pairs. Prior to starting, students should lay the strips out from smallest to largest. There are 10 different colors in a Cuisenaire strip set; each 1" wide color strip is measured in inches starting with White (1") through Orange (10"). The colors are in the following sequence: White, red, green, purple, yellow, dark green, black, brown, blue and orange.

Activity: Cuisenaire Strips Worksheet

1. If the orange strip = 1, each colored strip is what fractional part of the orange strip? List each of the colors from white to orange.
2. If the purple strip = 1, each colored strip is what fractional part of the purple strip? List each of the colors from white to orange.
3. If the red strip = $\frac{1}{2}$, which strip = 1?
4. If the red strip = $\frac{1}{3}$, which strip = 1?
5. If the white strip = $\frac{1}{5}$, which strip = 1?
6. If the white strip = $\frac{1}{4}$, which strip = 1?
7. If the red strip = $\frac{1}{2}$, which strip = $1\frac{1}{2}$?

Pre-Lesson Activity: Have students make their own set of Cuisenaire strips. Students can:

- Work cooperatively, i.e., one student is in charge of making all of the white 1" strips and another is in charge of the red ones, etc. or
- Each student is given 10-1" wide colored strips and is to measure out/make their own set or
- Each student is given 10-1" wide white strips and is to color/measure out/make their own set.

Internet Activity: <http://best.com/~ejad/java> and click on Cuisenaire rods.

(figures available in print form)

Manipulatives that can Enhance Math Skills - Ideal for Kinesthetic Learners

Calculators can be used for:

Counting, decimals, estimations, fact strategies, fractions, number concepts, patterns, problem solving, whole numbers.

Money can be used for:

Classification, counting, decimals, equations, equivalence, fact strategies, money, probability, whole numbers

Two-Color Counters can be used for:

Counting, equivalence, equations, fact strategies, fractions, integers, number concepts, number theory - odd, even, prime, composite, place value, probability, ratio/proportion, whole numbers

Number Cubes (Dice) can be used for:

Counting, decimals, logical reasoning, mental math, number concepts, probability, and whole numbers.

Playing Cards can be used for:

Counting, number concept, probability, number theory - prime, even, odd, composite, whole numbers, place value

Rulers can be used for:

Area, perimeter, construction, fractions, decimals, estimation, measurement (especially hours), volume, whole numbers

Capacity Containers (cups, pints, quarts, gallons, liters, etc.) can be used for:

Estimation, fractions, measurement, volume

Spinners can be used for:

Counting, fact strategies, fractions, logical reasoning, mental math, number concepts, probability, whole numbers

Clocks can be used for:

Fractions, measurement, whole numbers

Cubes can be used for:

Area, classification, counting, equations, equivalence, fractions, number concepts, percent, probability,

visualization, square/cubic numbers, surface area, geometric transformations, volume, whole numbers

Games That can Enhance Math Skills - ideal for Intrapersonal, Verbal, Visual and Kinesthetic Learners

Backgammon - problem solving, communication, reasoning, patterns, algebra

Battleship - problem solving, communication, reasoning, patterns, algebra, probability, geometry, measurement, graphing

Card Games - basic math facts, problem solving, communication, reasoning, patterns

Checkers - problem solving, communication, reasoning, patterns, probability

Chess - problem solving, communication, reasoning, patterns, probability

Dominoes - problem solving, communication, reasoning, patterns, counting, equations, equivalence, fact strategies, mental math

Legos - problem solving, estimation, patterns, geometry, measurement

Life - problem solving, communication, reasoning, computation, estimation, patterns, probability

Monopoly - problem solving, communication, reasoning, computation, estimation, probability, money sense

Simon - problem solving, communication, reasoning, patterns

Sorry - problem solving, communication, reasoning, computation, estimation, algebra

Spirograph - problem solving, reasoning, patterns, geometry, measurements

Tangrams - angles, area, classification, fractions, reasoning, patterns, ratio/proportion, size/shape/color, visualization, symmetry, computation, estimation

Tic Tac Toe - problem solving, communication, reasoning, patterns

Uno - problem solving, communication, reasoning, patterns

Yahtzee - problem solving, communication, reasoning, number relationship, computation, estimation, patterns, statistics, algebra, probability

Internet Sites That Can Enhance Math Skills

It is beneficial for the students to also have the actual manipulatives to aid them while working on the following various Internet sites.

<http://www.enchantedmind.com> - This site has creative Java puzzles designed to exercise both sides of the brain. The site includes a puzzle link, which connects to a hands-on tangram activity, pentomino activity and chess.

<http://math.rice.edu/~lanius/Lessons/> - This site has numerous hands-on math-related activities to include graphing, algebra, geometry, calculus, polyominoes (dominoes/ tetrominoes), ratios, applied mathematics as well as fractions using pattern blocks.

<http://www.best.com/~ejad/java> - This site has links to three math-related activities: base-ten blocks (whole decimals, place value and algebra), pattern blocks (fractions) and Cuisenaire (Integer) rods (math skills, fractions, decimals).

<http://www.funbrain.com> - This site has many educational activities for a wide age range. Math activities include math baseball, football, soccer, racecars, money (change making, checkbook), fractions, graphing, patterns, measurement, and surveying.

<http://www.quia.com/dir/math/> - This site includes a matching game format for students to reinforce various skills such as equivalent fractions, addition, multiplication, division, fractions/decimals, geometry, angles, algebra, trigonometry, measurement, square roots, and change making.

<http://forum.swarthmore.edu/students> - This site will link students to Ask Dr. Math where they can ask any math-related question or they can view the Math Tips & Tricks for insight on techniques to sharpen their skills.

<http://www.mathgoodies.com> - This site includes interactive math lessons, which include statistics, pre-algebra, probability, percents, area and perimeter.

<http://www.aaamath.com> - This site covers all grade level and all math topics to include basic math skills, decimals, geometry, estimation, graphs, patterns, and place value. Includes interactive lessons, explanations, and challenging math games.

www.mathstories.com - This site is geared for elementary and middle schools. It focuses on math problem solving and critical-thinking skills. Grade level problems are grouped by topics such as division, estimation, fractions and time/rate.

Internet Sites to Integrate Mathematics into the Curriculum

www.wcom.com/marcopolo

www.eduplace.com/search/activity.html

www.thegateway.org/

www.learner.org/exhibits/dailymath

encarta.msn.com/schoolhouse/lessons/default.asp

www.nytimes.com/learning (search of mathematics)

ericir.syr.edu/Virtual/Lessons/ (will take you to Gateway)

www.askeric.org/cgi-bin/lessons.cgi/Interdisciplinary (Plan #AELP-INT0002)

www.teachnet.com/lesson/index.html

www.forum.swarthmore.edu/library (link to www.ncsa.uiuc.edu Stock Market lesson)

www.col-ed.org/cur/

www.siec.k12.in.us/~west/sites/online.html#Examples

www.scssi.scetv.org/cgi-bin/state/indxsrch?q_f=15

www.teams.lacoe.edu/documentation/places/lessons.html

www.falcon.jmu.edu/~ramseyil/

www.awesomelibrary.org/

www.askdrmath.com

www.bigchalk.com

www.ricksmath.com

www.superkids.com

www.astro.virginia.edu/~eww6n/math/math.html

Commercial Software Programs That Can Enhance Math Skills

Broderbund - Carmen SanDiego Math Detective - reasoning and problem solving, number sense, fractions/decimals, estimation, geometry, measurement, patterns, algebra

Davidson - The Cruncher - spreadsheets, graphing, measurement, patterns, money

Davidson - Math Blaster (Geometry) - geometry, tangrams, geoboard

Davidson - Math Blaster (Ages 6 - 9) - addition, subtraction, multiplication, division, number patterns, estimation, fractions, decimals, percents.

Davidson - Math Blaster (Ages 9-12) - addition, subtraction, multiplication, estimation, place value, adding fractions, complex patterns and attributes.

Edmark - Mighty Math Number Heroes - problem solving and logic, addition, subtraction, multiplication, division, fractions, interpreting graphs and charts, and probability.

Edmark - Mighty Math Calculating Crew - problem solving and reasoning, money transactions, estimating and rounding, 1-4 digit addition and subtraction, 1-3 digit multiplication and division, 3D Geometry (solids and nets)

Kaplan - Math for the Real World - reasoning, problem solving, measurement, fractions, patterns, applied mathematics (ideal for the musically oriented learner).

Mecc - Math Munchers Deluxe - basic math skills, geometry, prime numbers, fractions, ratios, polygons, angles

Parent Resources

Apelman, M., & King, J. *Pizzas, Pennies and Pumpkin Seeds: Mathematical Activities for Parents and Children* . Denver: Colorado State Department of Education, 1989.

Kanter, P.F., & Dorfman, C.H. (Eds.). *Helping your Child Learn Math with Activities for Children aged 5 through 13* . Washington, DC: U.S. Department of Education, 1992.

National Parent Teacher Association. *Math Matters: Kids Are Counting on You* . Chicago: The National PTA, 1989.

Teacher Resources

Burns, M. (1992). *About Teaching Mathematics: A K-8 Resource* . White Plains, NY. Math Solutions Publication

Burns, M. (1975). *The I Hate Mathematics! Book* . Covelo, CA. Yolla Bolly Press.

Curtain-Phillips, M. (1999). *Math Attack: How to Reduce Math Anxiety In the Classroom, at Work and in Everyday Personal Use* . Kearney, NE. Morris Publishing

Hatfield, M. M., Edwards, N. T., and Bitter, G. G. (1989). *Mathematics Methods for Elementary and Middle School Teachers* . Needham Heights, MA. Allyn & Bacon.

Heddens, J. Bridging the Gap between the Concrete and the Abstract. *Arithmetic Teacher* , 33 (6), 14-17, 1986.

Howden, H. The Role of Manipulatives in Learning Mathematics. *Insights into Open Education* , 19 (1), 1-11. 1986

Meers, Trevor B., *101 Best Web Sites for Kids* . Lincolnwood, IL. Publications International, Ltd., 1999

Sowell, E. Effects of Manipulative Materials in Mathematics Instruction. *Journal for Research in Mathematics Education* , 20, 498-505, 1989.

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