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The Relationship between Addition and Subtraction in Problem Solving

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In my fourth year of teaching second grade, I noticed a pattern with students and word problems. Students see a mathematics word problem and feel overwhelmed and nervous. They tend to read the words they know and use the numbers that they see. They tend to guess the operation, solve the problem, and move on. Students do not check their work.

Problem solving and word problems seem to be a universal concern at all grade levels. Students do not have the tools needed to solve word problems. Students need to learn steps in problem solving. They need steps that can be used to solve both simple and complex problems.

It is important that students understand how to solve word problems. This unit is designed for my second grade class, but can be used and modified for grades two through three. I will focus on the relationship between addition and subtraction in word problems. I want students to have the tools to solve word problems. Problem solving is a “life skill.” It is a skill that students will use in their everyday activities and careers.

According to New Haven Public Schools curriculum standards, students must relate operations to real word experiences and problem solving activities. Students must develop strategies for solving word problems and be able to write their own story problems. In the second grade curriculum, students are required to solve a word problem each day. This problem is solved through a guided class discussion. They are also asked to solve a similar problem for homework. These problems tend to be single or two-digit addition or subtraction problems.

My students for this unit are second graders. My classroom is composed of 23 students, 14 male students and 9 female students. We are a very diverse classroom made up of 14 Hispanic students, 6 African American students, and 3 White students. 8 of the 23 students are LTSS, Language Transition Support Services. They are LTSS students because they were in bi-lingual classrooms for thirty months. After the thirty months, the students are exited and forced into mainstream classrooms, whether they are proficient or not.

My students vary in levels from very low to very high. Almost half the class is below level in reading. Unfortunately, their reading affects all subject areas. I have noticed that the students that are below level in reading, tend to display difficulty in math because they can not read the problems. My LTSS students seem to be my students who score low in math. On the other hand, the other 15 students in the class seem to really

enjoy and excel in math. The students are comfortable and have been doing Saxon Mathematics now for two years.

Saxon Mathematics is the math program that has been adopted at our school. The program is designed so that the students practice math during a morning meeting for twenty minutes and a math lesson for sixty minutes. During the meeting, students practice answering questions about attendance/lunch count, the calendar (days of the week and months of the year), finish patterns (shapes, numbers, etc.), tell time, count money, graph weather/temperature, and complete the given daily count (begin at 49 and count by 2's). The students always seem to be actively engaged in this meeting. I try to make it unpredictable by asking different questions, switching the order, etc. Later in the afternoon, we continue with math. The lessons are scripted for the teachers and tell the teachers what materials are needed to do the lesson. After the lesson, we complete one side of two sheets, fact practice and guided practice. The fact practice gives students the opportunity to master the given facts. The guided practice has about eight or nine different problems on it. As a class, we discuss each problem. All of the problems are related to previous lessons, so that the students are practicing what they learned. At the top of the guided practice sheet is a word problem. For homework each night, students complete the other side of the work sheets, fact practice and guided practice.

Saxon is a very thorough program. The students are constantly assessed. Every five lessons, students take a fact assessment and written assessment (similar to the guided practice sheets). Students also take an oral assessment every ten lessons. A math portfolio is kept on each student. In the portfolio, I analyze the students strengths and weaknesses. I use this information in my focus groups.

In the beginning of the year, we focused on addition facts and fact families. A fact family is composed of two addition facts and two subtraction facts, for example, 4+5=9, 5+4=9, 9-5=4, and 9-4=5. At the time, my class seemed to grasp the concept of fact families, relating addition and subtraction facts. Now, we are focusing on two-digit subtraction facts. Some students are doing well, some are having difficulty. Sadly, I noticed that they have forgotten about the relationship between addition and subtraction when solving problems. They struggle to solve 14-7 because they forgot to relate the "doubles fact" 7+7=14. Instead of relating this, they try to solve the problem by counting with their fingers. This also carries over to two-digit subtraction problem solving. Students complete a problem like 95-33=62 but they cannot check their work or relate the addition fact 62+33=95.

Somewhere along the line, students lost the important lesson that addition and subtraction go "hand in hand." Therefore, I hope that this unit will reinforce the concept that addition and subtraction relate. The principal objective of this unit is to have creative, critical thinkers who are able to solve word problems. Students will learn strategies, tools to solve word problems and check their work. I want students to not only "know how" but also "why."

For many of the ideas about teaching problem solving I have relied on my reading of *Teaching Problems and the Problems of Teaching* by Magdalene Lampert. I have also used *Knowing and Teaching Elementary Mathematics* by Liping Ma and *Children's Mathematics* by T. Carpenter. I found it enlightening that Liping Ma compares and contrasts Chinese teachers to American teachers. All of these books opened my eyes to new ideas.

In *Knowing and Teaching Elementary Mathematics* , Liping Ma discusses what is called the "knowledge package." She suggests that there is not one piece that the teacher is presenting, rather a group of ideas that are related. In this unit I have composed a "knowledge package." These are the ideas that I feel relate to my topic and my students. Keep in mind that some ideas may not relate to you and your students, make

adjustments as necessary.

There are many ideas that should be discussed in this unit. These ideas scaffold. If a student is having trouble in one area, it is because the student is lacking in another area. The topics that I suggest are as follows:

Place Value

Simple Addition and Subtraction (within 10)

Fact Families

Addition and Subtraction (within 20)

Addition and subtraction without regrouping

Addition and Subtraction with regrouping

Strategies for solving problems

> What facts are given?
> What do I need to know?
> How do I solve it?
> Is my answer right? CHECK IT.

Place Value

Liping Ma suggests that this concept should be taught over a long period of time. If students do not understand place value then they will not be able to add and subtract with regrouping. Students need to know the difference between one and two digit numbers and what that really means. Students should see the relationship between the tens and ones place (one ten is equal to ten ones). They should understand, for example, that in the number 79, the 7 is in the tens place which means that it is equal to 70+9. Place value is an important concept because when students are asked to add or subtract two two-digit numbers they must be able to line up the numbers to solve the problem.

Addition and Subtraction (within 10)

These basic skills are important to master. In this area students should be taught the *Commutative Property.* The Commutative Property says that when two numbers are added, the sum is the same no matter what the order of the numbers. For example, 6+3=3+6. This property helps students to see the relationship between numbers and also prepares them for the understanding of "fact families."

Fact Families

Once students understand the Commutative Property, they should be able to grasp fact families. For example, if they understand 6+3=9 and 3+6=9, then teachers should relate the subtraction problems 9-3=6 and 9-6=3. A fact family is composed of four facts that relate to three numbers. Understanding fact families will help in addition and subtraction of larger numbers.

Addition and Subtraction (within 20)

Liping Ma discusses the importance of working with numbers to 20. Chinese teachers really focus on the understanding of numbers to 20. They focus on understanding place value, making a 10 out of 10 1's. For example, 8+5 would become 8+ (2+3). Then make a ten (8+2) +3 which is 10+3=13. Chinese really think of place value, as a "foundation." Again, it is part of the "knowledge package." If students don't understand addition and subtraction within 20, how will they understand larger numbers?

Addition and Subtraction with and without regrouping

Liping Ma discusses the "concept" rather than the "procedure" of regrouping. For example, 12+9=21 students must see that there is more than 9 in the ones column so that ten must be put over the tens column to make 2 or 20. We must examine 2+9+11 and relate it to the problem. Also this is a great opportunity to show a member of the fact family, 21-12=9. In adding two-digit numbers, the basic step is to add the 10s together and add the 1s together. Sometimes this will produce more than 10 1s, in which case 10 of the ones should be grouped into a ten and then joined to the other 10s. In a subtraction problem, this procedure is reversed. If a 10 was formed during the addition, it will have to be broken up again to do the subtraction.

Strategies for solving problems

It is important for students to set up a framework. What should I do first? Next? Last? Depending on what grade level you teach you may want to make these steps simple or more complex. I like to use four questions to lead students in the right direction. These four important questions are as follows:

What facts are given?

What do I need to know?

How do I solve it?

Is my answer right?

In *About Teaching Mathematics,* Marilyn Burns discusses problem-solving procedures or strategies. She talks about how students naturally solve problems with their own strategy. But she stresses the importance of being familiar with a collection of strategies. Some strategies can be used alone or in combination with other strategies. Teachers should model strategies and give students time to apply them. Burns says that this will give students the tools to solve problems and broaden their problem-solving abilities. Burns suggests the following:

Look at the Pattern

Construct a table

Make an organized list

Act it out

Draw a picture

Use objects

Guess and check

Work backward

Write and equation

Solve a simpler (or similar) problem

Make a model

It is important to give students different avenues to solve problems. Students may use the strategies that they are comfortable with. Teachers could choose one or a combination of the strategies to solve a problem.

Lesson 1

Focus on four basic problems that relate. Discuss the problems. Students should be given manipulatives to solve the problems (counters, blocks, etc.). Discuss how these problems are the same and how they are different.

Objectives

To have students use the four basic strategies to solve a set of word problems.

To have students communicate using mathematical terms and vocabulary.

Problem 1

6 students are reading in the library. 3 more students come along. How many students are reading in the library?

Problem 2

9 students were reading in the library. 3 students go home. How many students are still reading?

Problem 3

Some students are reading and 6 more students came. Then there were 9 students all together. How many students were reading at the start?

Problem 4

3 students were reading, and some more students joined them. Then there were 9 students reading. How many students were in the group that came?

In this set of problems, I arranged them so that the operations alternate, addition then subtraction. Is there a pattern that you notice? Then I would ask the students how these problems are the same? I hope that they would notice that these problems use the same numbers, therefore are the same fact family. Finally, I would ask, “How are these problems different?” It seems that students find it easier to detect the differences. Some of the differences are the operation that is used (addition or subtraction) and the wording of the problems.

I would spend more time discussing Problem 3. This problem seems to give students more trouble than Problems 1, 2, and 4. Carpenter and authors discuss this type of problem in their book, *Children’s Mathematics* . They classify subtraction problems into several groups. The groups are as follows:

Take away

Part-part-whole, second part unknown

Part-part-whole, initial unknown

Comparison

Carpenter and authors would classify Problem 3 as part-part-whole, initial unknown. They describe these problems as difficult to children because of that “unknown.” Therefore, I think it is important to discuss this problem with the students. How is this problem different from the others? How would you solve it? Some students may solve this by subtracting, while others may write an addition problem to try to find the “unknown.”

Lesson 2

Focus on four basic problems that relate. Discuss the problems. Give students the opportunity to use manipulatives and draw pictures to solve the problems. Discuss how these problems are the same and how they are different.

Objectives

To have students use the four basic strategies to solve a set of word problems.

To have students communicate using mathematical terms and vocabulary.

Problem 1

2 children are playing. 9 more children come to play. How many children are playing in all?

Problem 2

2 children were playing and some more children came. Then there were 11 children all together. How many children came?

Problem 3

Some children were playing and 9 more children came to join them. Then there were 11 children all together. How many children were there at the start?

Problem 4

11 children are playing. 2 children go home. How many are still playing?

Again, I would discuss Problem 3 with the students. Have you ever seen a problem like this? How is this problem different from the other two problems? Carpenter and authors refer to this problem as “initial unknown.” In the book, *Children’s Mathematics* , they discuss the fact that children seem to have the most trouble with these types of problems because of the “unknown.” Some children would think ___+ 9=11 while others would subtract to get the answer, 11-9=2. Both strategies, they say come naturally to children. Children do not have to be taught that a particular strategy goes with a particular type of problem. In *Children’s Mathematics* , the authors focused on students using manipulatives (counters or blocks) to solve a word problem. Lastly, the authors state that in an environment that encourages children to use procedures that are meaningful to them, they will construct these strategies for themselves.

The teacher should discuss how these problems are the same? Students should see that they are using the same numbers and are the same fact family. This is a great time to compare these problems, how are they different? Students should see that the problems are worded differently. They may also comment on different operations, for some problems addition is used while others use subtraction.

It is important to give students the opportunity to communicate. Students should have the opportunity to lead the discussion. They may want to show students how they solved a particular problem and explain why they did what they did. Students will learn terms and vocabulary from one another. Most importantly, students will learn different strategies from other students. For example, one student may show the class how he or she drew a picture to solve a word problem. So students will see the different methods that can be used to solve word problems.

Posing problems of all types based on the same fact family will help teachers understand how well students grasp when subtraction is called for. It will also help students understand that better, give them practice with calculation, and reinforce the idea of fact families.

Lesson 3

Focus on four basic problems that relate. Discuss the problems. Students may use manipulatives to solve the problems. Discuss how these problems are the same and how they are different.

Objectives

To have students use the four basic strategies to solve a set of word problems.

To have students understand and discuss the relationship between this set of problems.

Problem 1

Dave ate 4 cherries. Sal ate 2 cherries. How many cherries did they eat altogether?

Problem 2

If Dave ate 4 cherries and Sal ate 2 cherries, how many more did Dave eat?

Problem 3

Dave and Sal together ate 6 cherries. If Sal ate 2 cherries, how many did Dave eat?

Problem 4

Two boys ate a total of 6 cherries. If Dave ate 4, how many did Sal eat?

In this set of problems, I threw them a “curve ball.” I wanted to give my students a problem that doesn’t belong to the fact family. This will help me understand how well they know what a fact family is. Problem 2 is

different from the problems given in the two previous lessons. I would begin by asking the students the following questions:

Which problem does not belong? Why?

How is this problem different?

Why is this problem different from the other three problems? Why doesn't it "fit in?"

Have you ever seen a problem like this before?

This problem is not from the same fact family. It is not a difficult problem but it is different. I would ask students how they would solve this problem and have students share this with one another. Then I would ask students how we could change this word problem or write a new word problem to fit in with the existing fact family. Students may wish to discuss this with a partner and present their ideas to class. Teachers may find it interesting to see how difficult or easy this task may be.

Lesson 4

Now that students have seen the relationship between addition and subtraction using single-digit numbers, move to two-digit numbers. Discuss how these problems differ from the previous lessons. Discuss how the problems relate to each other. Discuss two-digit fact families. Focus on regrouping in addition and subtraction.

Objectives

To have students use the four basic strategies to solve a set of word problems.

To have students communicate using mathematical terms and vocabulary.

Problem 1

18 apple trees are in a field. 42 peach trees are in the same field. How many trees are there altogether

Problem 2

18 apple trees are in the field. 42 peach trees are in the field. How many more peach trees are there than apple trees? ? (Technically, the answer to this question is not in the same fact-family as the other problems in this lesson.)

Problem 3

There are 60 trees in the field. If 18 of them are apple trees, how many are peach trees?

Problem 4

Altogether there are 60 trees in a field. If 42 of them are peach trees, how many are apple trees?

These problems are more complex because students are asked to regroup. In Problem 1, students are adding the two numbers. In that problem students must regroup. In Problem 2, the students are subtracting the two numbers. In this problem students do need to regroup. Teacher should ask how these two problems are the same and how are they different? Students should explain that they use the same numbers but in one problem there is addition and in other problem there is subtraction. Most importantly, in Problem 1 and Problem 2 students must regroup.

In Problems 3 and 4, students are subtracting with regrouping. This may be a good time to introduce a sequence of problems that cross the regrouping divide. Some problems involve regrouping and some do not. An example of these problems is as follows: 69-34, 70-34, 71-34, and 72-34. I would also do problems like this for addition, for example, 34+45, 34+46, 34+47, and 34+48. Teachers may wish to discuss these problems as is or discuss them as word problems. If you choose to discuss them as word problems you may wish to extend the lesson period or revisit this at the beginning of the next lesson. It is important to note that the subtraction problem requires regrouping exactly when the associated addition problem does. In giving these problems, students may need to do each one from scratch while others may see how to modify the answer to get another.

Lesson 5

Students have seen the relationship between addition and subtraction using single-digit numbers and have worked on two-digit numbers. This lesson is to reinforce the previous lesson. My hope is that students will make a connection as if to say, “Didn’t we do this same problem yesterday.” But my feeling is that practice makes perfect, the more students are exposed to word problems, the better their understanding will be. Students will also feel more confident and comfortable because they are familiar with the problems.

Discuss how these problems differ from the previous lessons. Discuss how the problems relate to each other. Discuss two-digit fact families. This lesson focuses on regrouping in addition and subtraction.

Objectives

To have students use the four basic strategies to solve a set of word problems.

To have students communicate using mathematical terms and vocabulary.

Problem 1

14 brown baskets are on the truck. 26 red baskets are on the same truck. How many baskets are there?

Problem 2

14 brown baskets and 26 red baskets are on the truck. How many more red baskets are there than brown baskets? (Technically, the answer to this question is not in the same fact-family as the other problems in this lesson.)

Problem 3

There are 40 red and brown baskets. If 26 are red baskets, how many are brown baskets?

Problem 4

A total of 40 red and brown baskets are on the truck. If there are 14 brown baskets, how many red baskets are there?

In discussing these problems, I would begin by asking how the problems are alike and how they are different. Now that we have been working on word problems, I expect students to use mathematical terms. When comparing and contrasting these problems, I hope students would say that there is only one addition problem (Problem 1). Problem 2, 3, and 4 are subtraction problems with regrouping. Most importantly, this is a fact family because all of the problems use the numbers 14, 26, and 40.

In the previous lesson, we discussed a sequence of problems that cross the regrouping divide. During this lesson, teachers may want to revisit that set of problems or a similar set of problems. By working on and examining these sets of problems, students will see where they do not need to regroup and where they do need to regroup. Students may even be able to follow the pattern to find the solutions.

Lesson 6

Students have seen the relationship between addition and subtraction using single-digit numbers and two-digit numbers. This last lesson will again practice two-digit word problems. In my four years as a second grade teacher, I have noticed that working with two-digit numbers is an area of weakness for many students, whether they are adding or subtracting.

Discuss how these problems differ from the previous lessons. Discuss how the problems relate to each other. Discuss two-digit fact families. Focus on regrouping in addition and subtraction.

Objective

To have students use the four basic strategies to solve a set of word problems.

To have students communicate the relationship of the problems.

Problem 1

60 oranges were in a basket. 25 were sold. How many oranges were not sold?

Problem 2

If 25 oranges were sold yesterday and 35 oranges were sold today. How many oranges were sold on both days together?

Problem 3

There were 60 oranges. If 35 are still in the basket, how many were sold?

As always, I think it is important to discuss how these problems are similar and different. This discussion will give teachers the opportunity to hear mathematical communication. By now students should be using key terms and vocabulary in their explanations. Students should notice that in all three of these word problems there is regrouping and that this is a fact family. A few students may notice that this lesson only has three problems while all the other lessons had four problems.

This is a great time to give students an opportunity to write a word problem for this set. When students are constructing their own sets of problems, I think it is important to give them the opportunity to work with a partner. Students may even want to create their own set of problems, similar to this lesson. Teachers may want to use one class period to write and construct the problems and another class period to solve them. Students can switch problems with a partner, solve them, discuss them, and give feedback.

In the Future

Where can you go when you have completed this unit? I think that giving students the opportunity to construct a set of word problems and solve them would be a great extension of this unit. Students will take pride in doing this. Teachers can even take all of the results and put them together in a workbook. The students can solve a set of problems for morning work.

It is also important to have students think about fact families with larger numbers. There are some complex CMT questions that would fit in nicely here.

Problem 1

If you are given that 34+48=82, which of the following statements relates to the given:

a) 82+32=48

b) 82-34=48

c) 35+47=82

d) 48-34=82

Students would need to not only find the answer but explain why that answer is correct and logical. Some students may immediately notice that the solutions a and d have incorrect totals. Other students may notice that answer b uses the same numbers as the given and is part of the same fact family.

You may even want to introduce three-digit number problems like this. One of my goals that I wish to achieve with my students is recognizing fact families. I want students to recognize fact families, whether they are single-digit, two-digit or even three-digit fact families. More importantly at the end of this unit students should really see the connection between addition and subtraction, no matter how big the numbers are.

Mathematics Standards

New Haven Public Schools has Mathematics Standards for all grade levels. These standards can be found on-line on the New Haven Public Schools website. The standards (PK-4) are related to the Connecticut Mastery Test. There are main “Content Standards” and “Performance Standards” listed as part of the content standards. This unit has touched upon four of the content standards. The standards are as follows:

Content Standard 1.0Number Concepts, Arithmetic, and Operation Concepts

Content Standard 5.0Problem Solving and Mathematical Reasoning

Content Standard 6.0Mathematical Skills and Tools

Content Standard 7.0Mathematical Communication

Content Standard 1.0

This unit focuses on two performance standards in this content. Performance Standard 1.1 states that students will add and subtract. Students will analyze problems in order to figure out when to add or subtract and relate problems to one another. Performance Standard 1.2 states that students will understand place value and use it to solve problems.

Content Standard 5.0

This standard focuses on problem solving. Hence, this problem solving unit touches upon all four of the performance standards. Students will use a variety of strategies to solve problems. Students will make connections, explain patterns, and relate problems.

Content Standard 6.0

This standard focuses on skills and tools. This unit relates to Performance Standard 6.1. Students will use a variety of strategies to solve problems and learn strategies that others use.

Content Standard 7.0

In this standard students mathematically communicate. This unit focuses on three of the performance standards in this content. Performance Standard 7.1 states that students use mathematical terms and vocabulary. In Performance Standard 7.2, students show ideas in a variety of ways, including words, numbers, and pictures. And Performance Standard 7.3 states that students will explain solutions to problems orally and written.

Bibliography for Teachers and Students

Burns, Marilyn. *About Teaching Mathematics.* California: Math Solutions Publications, 2000. This book addresses the toughest issues teachers face in mathematics teaching. It discusses replacing traditional mathematics teaching with strategies that focus on students' thinking and reasoning. It emphasizes teaching mathematics through problem solving. It is an essential teaching source.

Carpenter, T., Fennema, E., Franke, M., Levi, L. and Empson, S. *Children's Mathematics* . New Hampshire: Heinemann, 2003. This book was written to help you understand children's intuitive mathematical thinking and use that knowledge to help children learn math with understanding. The authors offer a detailed explanation and numerous examples of the problem-solving and computational processes that virtually all children use. They also describe how classrooms can be organized to foster that development.

Hatfield, M., Edwards, N. and Bitter, G. *Mathematics Methods for Elementary and Middle School Teachers* . Massachusetts: Allyn & Bacon, 1997. This book provides teachers of grades K-8 with ideas, techniques, and approaches to teaching mathematics so that students will be prepared for later study. There is an emphasis on manipulatives, problem solving, and mathematical connections.

Lampert, M. *Teaching Problems and the Problems of Teaching.* Connecticut:Yale University Press, 2001. In this book an experienced classroom teacher and noted researcher takes us into her fifth grade math class. She shows how classroom dynamics are critical in the process of bringing students to a deeper understanding of mathematics (or any subject). Lampert looks at the common problems of teaching and arrives at an original model of teaching.

Ma, Liping. *Knowing and Teaching Elementary Mathematics* . New Jersey: Lawrence Erlbaum Associates, 1999. The author compares American to Chinese teaching. Ma's key point is that American teachers are not trained to think mathematically. Ma brings forth interesting information about Chinese educational practices.