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Solving Word Problems Using Subtraction

Curriculum Unit 04.05.03
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Introduction

This unit is being written specifically to be used with my third grade class. I proposed to use subtraction as the basis for this curriculum on problem solving because the unit needed to be grounded in some specific mathematical topic or area. Therefore, I decided to focus on subtraction -- more particularly subtraction with renaming. My interest in this area comes primarily from my experiences in teaching math to third and fourth graders. It has surprised me over the years that even the very best math students I have taught still have trouble, or are confused by subtraction with renaming. The topic is usually taught in third grade but continues to confound students long after they first encounter it. While students seem to love and anticipate getting to multiplication and learning times tables, subtraction seems to be regarded as dull and laborious. For the most part I have found that whenever the math lessons find their way back to subtraction with renaming there is a kind of lapse in memory and students usually look askance at the problem until they get a quick refresher on how to do the example. This, I must suppose, is a clear indication that the concept is not really understood. If this is the case, then why and how can that be changed?

For many of the ideas about teaching subtraction that are presented in this unit I have relied on my reading of *Knowing and Teaching Elementary Mathematics* by Liping Ma, especially the chapter concerning subtraction and renaming. I have also used *About Teaching Mathematics: A K-8 Resource,* by Marilyn Burns. Burns is a well respected author and former classroom teacher who has written widely on math instruction. She has also authored a group of popular classroom resources. A series of books published by Heinemann: *Children's Mathematics: Cognitively Guided Instruction, Thinking Mathematically* and *Making Sense: Teaching Mathematics with Understanding* were also extremely helpful. These books are the work of a group from the National Center for Research in Mathematics. The purpose of the group is to promote the teaching of math in elementary schools.

I would like to begin with a conclusion gathered from the research presented in the readings -- namely, that children who have constructed mathematical knowledge are adept at using and retaining what they have learned. This seems to most of us a plausible statement, but how do we achieve this in the classroom? The focus of instruction must change from an emphasis on calculations to one that includes the recognition that doing calculations is ultimately only a tool for problem solving.

The Ma book is a presents a fascinating comparison between the mathematical knowledge and performance of American and Chinese elementary teachers.1 What seems clear in the Ma book is that the major difference between the American and Chinese teachers was the difference between procedural understanding and conceptual understanding. This is not to say that neither side was totally superior or inferior, however, the Chinese teachers seemed to be more comfortable teaching math. For the majority of American teachers teaching subtraction with renaming was more procedural. Students learn to go to the next column and cross out and add a one in front of a number but they have little understanding of what and why they are doing it. I suggest that the reason for the emphasis on the procedural is two-fold. First, I don't think that most teachers understand how the math curriculum develops below and above their grade. Most of my colleagues in the lower grades are not familiar with what types of questions are on the Connecticut Mastery test. Likewise most of us in third and fourth grade have little concrete knowledge of what is taught in fifth and sixth grade math. The Chinese teachers seemed to understand the continuum that made up what they were teaching and they had a firm idea of where students were coming from and where they needed to go. Secondly, it seemed that the Chinese teachers had time to think over what they were teaching. I almost felt as though they must be teaching only math (which is the case for many of them) because where did they get the time to do all this contemplating about their lessons and making modifications to suit their students' needs. I have to say that since most of us teach with a set curriculum and pacing chart, there is not a lot of time to expand lessons. Indeed most of the examples cited in the books are laboratory situations where the math programs are experimental. The emphasis on package curriculums is one added reason we may tend to be so procedural in teaching math. However, the curricula could develop the reasoning if it chose to do!

General Remarks on Problem Solving

When computational competence was the main goal, a procedural emphasis may have been effective. As Burns points out one of the reasons students have trouble solving problems is that the problems they are given usually follow a lesson either on some mathematical operation be it addition, subtraction, multiplication, or division.2 Then a series of word problems to be answered by using the operation just taught would be given. The problem wasn't really a problem it was just another way to practice the procedure. The students knew that if they had just studied division, then these word problems would probably be solved in the same way. In reality a problem does not come with its solution tied to it. The person trying to solve the problem has to understand it and try to figure out the best way to solve it. In my readings I ran into once again the four steps advocated by George Polya of Stanford University in his book *How to Solve It.3* The four steps are:

- *Understanding the problem*
- *Devising a plan*
- *Carrying out the plan*
- *Looking back*

In some of the readings there is a suggestion against teaching a kind of formulaic way to solve problems. While I understand the desire to make problem solving uninhibited in terms of the number and types of solutions possible for a given problem, I also know that my students do not have the profundity of language and experience and may need some helpful suggestions. I'm not beyond offering up a set of strategies that I have seen in one form or another in a few textbooks. The following are taken from Marilyn Burns:

Problem-Solving Strategies

- Look for a pattern
- Make a Table
- Make an organized list
- Act it out
- Draw a picture
- Use objects
- Guess and check
- Work backward
- Write an equation
- Solve a simpler (or similar) problem
- Make a model 4

While these are useful strategies for students to be familiar with, many like Burns do not recommend the teaching of word clues like "altogether" means add, or "how many more" means subtract. I have found these clues give students a place to start when they first read a word problem. Burns seems to feel that this only leads to giving students tricks to finding the answers. It also makes students think that the right answer is all that is important. They don't care about how they got the answer as long as they have it. I agree with her that these clues should not be overemphasized otherwise students will not develop their own problem solving abilities.

Knowledge Package for Two-Digit Subtraction

In organizing this unit I have relied on an idea gleaned from the Ma book. Here Ma compares and contrasts the mathematical knowledge of American and Chinese teachers. Ma refers to a Chinese teacher's use of the term "knowledge package."5 This term suggests that there is not one piece of knowledge that the teacher is presenting but a group of related ideas that scaffold the learning of that final concept. Interestingly there is not one correct package for teaching a given idea. This will depend on the teacher and the students. Just what do they need to construct the knowledge they will need to be able to successfully understand subtraction with renaming? The package I would like to suggest for subtraction with renaming would include:

Composing and decomposing numbers

Place Value

Addition and subtraction within 10

Addition and subtraction as inverse operations

Addition and subtraction within 20

Addition without regrouping

Subtraction without regrouping

Addition with regrouping

Subtraction with regrouping

What is being suggested is that if a child is having trouble with subtraction there is probably some deficiency in another area that is the cause. If he doesn't see a relationship between addition and subtraction or he cannot subtract numbers to 20 successfully he will probably have trouble with subtraction with renaming. While the path I have chosen to follow with my students is one way to reach the level of understanding for my students, -- the literature consistently acknowledges that there may be different paths and children may not need all of what is in the package to successfully reach an understanding of subtraction with regrouping..

First it is important to discuss the terminology of addition and subtraction which is often confusing and misleading. For many years the term “borrowing” was used for what is now called “subtraction with renaming”. The word “borrowing” suggests that something is being given for a short term use and will later be returned. Mathematically speaking this is not so in subtraction. Ma mentions the terms “composing and decomposing of units” used by the Chinese math teachers to indicate how numbers are constructed and can be broken down by the processes of addition and subtraction. Currently in the United States “renaming” is the most popular term used. It is used in Saxon Math which New Haven uses for its K-4 curriculum. Throughout the paper I will use all three terms. I have become a fan of the composing and decomposing terms because it seems to express what is happening to the numbers in addition and subtraction.

Place Value

Many of the problems that children have in subtraction especially in subtraction with renaming were attributable to a lack of complete understanding of place value. As Ma suggests, place value concepts must be taught over a lengthy period of time. When students begin to count they need to learn the difference between one and two digit numbers and two and three digit numbers, and so forth. They should understand what is meant by place value in math -- the name of the place and the relationship between the place like 1 ten equals 10 ones. The most important idea they form at the second and third grade level is that digits in different places have different values. In later addition of two or more digits they learn that they must line up the digits by place before they add - in order that they are adding like quantities - 10s to 10s and 1s to 1s..

The numerical system we use expresses all whole numbers as combinations of the symbols 0 through 9. Instead of continuing to make new symbols for each number it was probably inevitable that rules would be made that would allow for other numbers to be made with this finite number of symbols. Indeed if students look at a number line they can easily see the pattern of 0 -- 9 and then the returning back to have 1 combined with each number 0 -- 9, then 2 with each number and so on. Something unique happened after the number nine. The number 1 had already been used and so the 0 was put in as a way to fill in the place value. All numbers became combinations or ones, tens, hundreds, etc. As students learn addition with composing and subtraction with decomposing they learn the relationship between these procedures and their place value.e.

Addition and Subtraction within 10 and Inverse Operations

Addition and subtraction within 10 while an area we would presume students encountered in first and second grades is part of the structure of skills that students need to know if they are going to progress and be successful in subtraction with renaming. Students need to know that 7 can be made by adding 4+3 or 3+4, 6+1, and 5+2. The commutative property of addition, and associative property of addition should be learned and students should be allowed to work out these findings with manipulatives. I know that saying these rules should be learned might better be couched in words like “be exposed to”, or “be allowed to construct the rule themselves.” In Saxon Math, which I use in my classroom, the rule is presented to the students as well as the terminology. My students have become quite adept to recognizing the commutative property of addition. I find it gives them the vocabulary to name something they do recognize.

In the commutative property of addition children recognize that since addition requires putting things together it doesn’t matter in which order two numbers are added. If we add 6+2 we get 8 and if we add 2+6 we get 8. The associative property comes in when we add three or more numbers. Since we can only add two numbers at a time it is up to the person doing the calculations to choose which numbers to add first. We usually teach adding three or more single digits first. Within that lesson we try to have students realize that looking for tens will make adding these numbers easier. For example in adding

6+ 3 +4 = ____, if we can see that 6+4=10 then we can add 3 and get 13. Many students would begin with the 6+3 and then go on to add 4, which is more difficult for them. Another rule that students usually begin to pick up on is that zero added or subtracted to any number does not change that number.

Subtraction with renaming also predicates itself on the idea that students have become familiar with the relationship between addition and subtraction. Students would be working to build what are termed “fact families”: 5+3=8, 3+5=8, and also 8-3=5 and 8-5=3. We would also stress those combinations of numbers that make 10.

Addition and Subtraction within 20

Next students need to expand their understanding of numbers to addition and subtraction facts to 20. For most of the Chinese teachers interviewed by Ma this was a critical area. Now students begin to tackle two digit numbers. Students should have increased experience with number families so that they can see that 17 +3 =20, 3+17 = 20 and also that 20-17 = 3 and 20 -- 3 = 17. The most important examples are the ones that cross 10: 7+8=15, 8+7=15, 7=15-8, and 8=15-7. Students should see that 8+7=8+ (2+5) = (8+2) +5=10+5=15.The Chinese teachers viewed working with the number to 20 as critical. If students didn't understand doing subtraction numbers within 20 how could they tackle higher numbers? In their view early exploration with lower numbers would form the basis of expanding students' abilities in a way that would guarantee success. 6

Here we might use two-sided counters. Each child would be given a different amount under 20 and they would be asked to toss their counters on their desks and record the number of each color. They would continue this trying to find as many combinations as possible. Students would then be asked to discuss what they learned. The class would also prepare a chart showing the different possible combination for each number of coins.

Many of my students have also benefited from using a number line on which they plot the composing and decomposing of numbers under twenty.

Addition and Subtraction without Regrouping

One of the primary things the readings have reminded me of is how important it is to keep the connection between addition and subtraction within the forefront of whatever lesson we are doing. Addition is putting things together and subtraction is pulling things apart. We begin with a number and if we use the operation of addition we end up with a larger number. If we use the operation of subtraction on that number our answer will be smaller. Addition and subtraction both involve part-whole relationships. Addition is reconstructing a whole from parts; subtraction is reconstructing a part from the whole and the complementary part. If we subtract 5-2 together and get 3, it is only natural that we consider the reverse: 3+2 = 5.

In dealing with addition and subtraction without renaming, I would suggest that we are setting students up to work with larger numbers, and also helping students begin to understand the language of these two operations. How do we know when to add or subtract? If we give the example of

32

+ 43

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we want the student to understand that he is adding the ones and the 10s, and arranging the numbers in vertical columns is a way to do this automatically. We would probably have available base ten blocks which the student could use. At this point they would see that all of their calculations go very easily across because they have a value of 9 or under in each column. Also to encourage algebraic number sentences the problem could also be written horizontally as 32 +43 =___. Either way we would hope that from their previous work in decomposing numbers students might easily see this as (30+2) +(40+3) =70 +5 = 75. Another possibility would be (30+40) + (2+3) =75.

Addition is straightforward and pretty easy to understand. Carpenter et. al. in *Children's Mathematics* describes addition problem as joining together. Joe has 324 baseball cards. His mother gave him 15 more. How many baseball cards does Joe have now? In their book they continue exploring the problem types. They finally categorize eleven different problem types which I would prefer to simplify.

In *Guiding Children's Learning of Mathematics* , Kennedy and Tipps simply state that all addition problems are basically joining of two or more groups. They split subtraction type problems into 4 categories. The first and most recognized is what most students would term “take- away” problems. I had 12 stickers. I gave 5 to my best friend. How Many stickers do I have left? Unfortunately by labeling subtraction as “take-away” students get a limited view of these types of problems. There are also comparison problems, completion problems and part/whole problems. Most of the literature advises not allowing children to think of all subtraction as only “take-away.”

In comparison problems we are comparing the size of two sets. For example: There were 345 people at the movie on Friday night. On Saturday night 289 people came. How many more people were at the Friday night show? Or we could ask: How many fewer people came on Saturday than came on Friday?

In the completion or unknown problem we are use subtraction to find a missing set which when put with a second set helps to make a third set. For example: The book Mary is reading has 123 pages. She has 97 read pages. How many pages does she have left? We subtract 123 -- 97 to find the 26 pages. Mary needs to complete the book..

My problem with the Carpenter book is that the authors describe the following as a joining problem: Tony had 5 toy trucks. His dad gave him some more for his birthday. He now has 9 trucks. Because the numbers in this problem are single digit the authors describe the children solving this with counting on from 5 to 9. However, if the numbers were higher --say, Tony starts with 47 trucks and that after his birthday he has 65 trucks- it is my opinion that this would not be a joining problem but a completion problem in which the students would simply subtract 63-47 = 16. Carpenter treats it as part of addition because in truth the action that happens in the problem ends with Tony having an increased number of toy trucks. To me it is still a subtraction problem..

Addition and Subtraction with Regrouping

Once again the composing and decomposing of numbers in addition and subtraction are directly related to each other and students need to see the similarities. If I add

64

+ 28

I can immediately see that now I have more than 10 in the ones space. If I demonstrate with base ten blocks students should be able to decompose the 12 tens into 1 ten and 2 ones. Having done this they can now add the ones column and figure the answer to be 96. Now when subtraction with renaming is introduced I need to fall back on previously learned ideas about addition and subtraction. If I now use the same example but structure it as

92

- 28

I begin to decompose the 9 tens into 8 tens and 10 ones; I can add those ones to the 2 already in the ones column and have 12. When I now subtract I have 8 in the ones column and 6 in the tens column, or an answer of 64. Looking at the two problems students can see that these two examples are like part/whole problems. I put two numbers or parts together and now I separated one part from the other. Usually when we begin subtraction with renaming students are also taught how to check their subtraction with addition. The two go hand in hand and the stronger the relationship students have with proving and verifying these principles the better understanding they will have of subtraction with renaming.

Addition with regrouping plays an important part in students seeing what is happening in subtraction with renaming. If we add 14 +8 = 22 students have an easier time seeing that there is more than 9 in the ones column so that ten must be put over into the tens column making it now 2 tens or 20. If I now show the students 22 - 8 they can see that where I added 4+8 =12 in the addition problem I am now reversing that by taking a ten back and making the 2 ones 12. In the addition problem my first calculation was 4+8 =12, while in the subtraction problem the first calculation becomes 12 -- 4 =8. It becomes obvious that working with an eye toward having students see these relationships can lead to a deeper understanding of the subtraction algorithm and saves it from being a mere set of procedures.

A student might also expand 14 to be 10+4 and so the problem becomes 10+4+8. In this case some children if they understand the zero might find it easier to combine 10+8=18, and then add on the 4 which equals 22. In the subtraction problem 22 -- 8, they could expand 22 to (10 +10 +2) -- 8. They then could subtract 10-8 =2 and add that to the other 10+2 and reach the same conclusion of 14.4.

Throughout the literature the research stresses the need for students to constantly talk and write down their thoughts about what they are doing and why. Students need to work together and talk out what they are doing. Students should have math journals or notebooks where they write about what they are learning in math, or describe how they did a certain problem, or how they know that 5324 is larger than 3259.It is part of the teacher’s role to provide the atmosphere where children can explore and share their ideas- right or wrong. The teacher needs to question the child but most of all listen to their explanations and discoveries.

Following are a group of problems that support the different levels I have identified in my package for learning subtraction with renaming.

Problems

Place Value

These problems offer a variety of ways to review and expand the notion of place value. The problems utilize words and symbols. They ask children to be able to recognize place values into the thousands. They also check to see if students have made the connection of the power and ease of the base ten system. Can they raise a number by 10 or 100. Can they lower a number by 1000? Finally do they know the terminology of expanded notation, value, digit, more than, less than.

1. 1 ten + 3 ones =
2. 11 = __ tens + ___ ones
3. In the number 345 what does each digit stand for?
4. 472 = 400 + ____ + 2
5. In 3547, the 5 stands for ____.
6. In 3547 the digit ____ is in the hundreds place.
7. In 3547 the value of the digit 3 is _______.
8. _____is 10 more than 467.
9. _____ is 10 less than 500.
10. _____ is 100 more than 2467.
11. _____ is 1000 less than 6308.
12. Write the number 5642 in expanded notation.

Addition and Subtraction within 10

In these problem there are a variety of questions all centering on a common theme and all with numbers within 10. These are some of the basic types of questions from simple joining problems to part/ whole, comparison, and completion problems.

1. Mary had 3 fish. Her grandmother bought her 4 new fish. How many fish does Mary have now?

2. Mary had 5 fish. Two of her fish died. How many fish does Mary have now?
3. Mary had some fish. Two of them died and she now has 4 fish left. How many fish did Mary start with?
4. Mary's fishbowl can hold 8 fish. Mary has 3 fish in the bowl now. How many more fish does Mary need to fill the fishbowl?
5. Mary has 9 fish and her friend Crystal has 4 fish. How many more fish does Mary have than Crystal?

Addition and subtraction within 20

In these problems the type of problem does not really change but the numbers are becoming larger. There is also the use of the term "dozen" in question 4. In problem 5 there are 3 addends.

1. John has 14 toy trucks. He got 4 more for his birthday. How many toy trucks does John have altogether?
2. John has 15 toy trucks. Nine of the trucks are red. How many of the trucks are not red?
3. John has 16 toy cars. He has 7 more than his friend Tommy. How many toy cars does Tommy have?
4. Tommy had some toy trucks. He gave 4 to Mike. Now Tommy has a dozen left. How many toy trucks did Tommy start out with?
5. Tommy has 4 cars, Joe has 7 and Mike has 5. How many cars do they have altogether?

Addition and Subtraction without Regrouping

In these problems the type of problems are similar to previous ones except that there is not regrouping. The numbers are increasing and there is also extraneous information in #4.

1. Mary has 86 stickers. She gives 43 stickers to her sister. How many stickers does she have left?
2. Mary's stamp album holds 96 stickers. There are 42 stickers already in her album. How many stickers can she still fit in her album?
3. There are 132 stickers in a small package and 256 in a large package. Mary bought a large and a small package of stickers. How many stickers did she buy?

4. Mary had 34 animal stickers, 25 American flag stickers, and 58 food stickers.
How many more food stickers does Mary have than animal stickers?
5. Mary has 34 more stickers than Kate. Kate has 143 stickers. How many stickers does Mary have?

Addition and Subtraction with Regrouping

In this group of problems we now have larger numbers and we have begun to use money. The problems also include a multi- step problem in #4 and a problem which relies on the child being able to read number words and recognize what is meant by the term “weekend.”

1. Billy had 275 marbles. He lost 87 marbles. How many marbles does he have left?
2. Billy had 166 marbles and his mom gave him a bag of 125 marbles. How many marbles does Billy have now?
3. Billy has $2.34. He needs $1.75 to buy another bag of marbles. How much does the bag of marbles cost.
4. Malcolm needs a new tire for his bike. A tire costs $13.00. Malcolm has $7. 48.
His brother will lend him $3.45. How much more money does Malcolm need to buy the new tire?
5. Five hundred twenty-one people went to the baseball game on Tuesday. Seven hundred eighty-four went on Friday, and nine hundred fifty-four went on Saturday night. How many people went to the baseball games on the weekend?

Lesson Plans

I would like to offer three lesson plans that I feel carrying out the goals of this unit. I thought it would be foolish to try and recreate what Saxon Math already covers and does quite well. What I decided to do was to share a few lessons that would be supplemental or projects done by the whole class. In some cases they would extend over a few days or could become favorite activities students could do on their own or in the math center. The three lessons used are tried and true kinds of activities that students will enjoy. The New

Haven Public Schools have been using the Saxon program for over five years. Teachers have now been encouraged to move beyond the basic program and make it our own by incorporating activities and projects that create excitement and motivation in our students.

I see the first lesson plan: *How much is your name worth?* as a lesson that could be easily done in the first few days of school as a kind of ice-breaker. In the lesson each letter has a corresponding dollar value from $1 to $26. Students have to add up the total worth of the letters in their names. This lesson is an interesting one because at the beginning of the year it allows the teacher to see how students tackle a problem? Do they list the value of all the letters in their names and add them at once. Do they add a letter at a time? Do they put one and two digit numbers in the correct place?

Once students do this activity with their names they can at a later time look for the most valuable 3 letter word they can think of or the 4 letter word with the greatest value. Students might also figure out the values of specific words. By doing this teachers can manipulate the type of addition problems students will be doing. As the process goes along this lesson can also lead to students writing about math. What do they notice about the words that have higher values or lower values? Given two words like *act* and *wet* which one would they guess to be the most valuable? Why?

In the second lesson: *How many seeds does this pumpkin have?* Students are going to review their understanding of place value, which the research says is an important concept for students to understand. Again this lesson is linked to a holiday related theme that is going to get students actively involved and working in groups. Another reason I believe these are valuable lessons is because students of all abilities can do them.

By counting up the pumpkin seeds students are again dealing with counting, grouping into tens, having more than ten and less than ten. They need to be able to add to 10, figure out how many more they need to make ten and they need to be able to go from ones to tens to hundreds to possibly thousands as they count the number. By gluing the seeds onto paper strips and building a place value chart to illustrate the number of seeds students are once again reviewing their knowledge of place value. This activity is also informative for the teacher as a way to see what problems still exist in the understanding of this basic concept.

The third and final suggested activity is called: *How many calories in your favorite MacDonald's meal?* This is a more generic type of lesson that can be used in a variety of ways depending on your objective. Students are given copies of MacDonald menus with the caloric values listed by each food item. The menu is available on the web. The lesson has three tasks. First students are asked to calculate the number of calories in their choice of a drink and sandwich. Once again students are dealing with setting up the problem, adding with regrouping and place value. Second, they will be required to find the meals with the highest and lowest caloric value. In this task students must understand the construction and deconstruction of numbers to ten as they try to with their knowledge of the addends to ten look for reasonable pairs to add. What numbers will most likely add to a higher number? What numbers will likely add to a lower total. Again, this is valuable information for the teacher to see what aspects of the "knowledge package" are missing. In the third task, students are told that a student has a meal worth a given number of calories. They are given two of the items chosen and must figure out the third. Here students must be able to successfully add and possibly regroup large numbers but they must also find a missing part.

These lessons are merely templates and can be used and expanded in various ways. There are numerous menus available and so this activity could be expanded to compare the results from MacDonald's compared to Wendy's or Burger King. Prices could also be used to expand this activity into money amounts. The whole concept of shopping; whether for clothes, or toys, or a class party can be used to initiate math activities that students will relate to and enjoy. Activities like these will be fun and still allow the teacher to review a variety of math topics relevant to subtraction with renaming.

Lesson Plan #1

How much is your name worth?

Objective: Students will calculate the value of their names given a chart assigning a dollar value to each letter.

Materials:

A list of dollar values for each letter: A=$1, B=$2, c=$3, etc.

paper

pencils

Procedure:

1. Students are asked to write down their first names across the top of the paper.
2. Under each letter the student is told to put down the value of each letter in their name.
3. The students then are asked to write a number sentence that will help them figure out the dollar value of their name.
4. Students would be asked to predict who they would expect to have the highest valued name, and why, the lowest, etc.

Extension Activities:

The teacher should collect the papers and hang them up, or put them into a book form to be kept for the class to look at. Students might also do their last names. There is a similar activity that is often used called Dollar Words in which students are challenged to find a word whose letters add up to $1.00.

Lesson Plan #2

How many seeds does this pumpkin have?

Objective: Students will estimate, count and order pumpkin seeds into groups of ones, tens, hundreds, and thousands if necessary in order to visualize how place value works with large numbers.

Materials:

1 large pumpkin

4 different colors of construction paper glue

Procedure:

1. Display the pumpkin so that students can guess the number of seeds it contains. Record their guesses on a chart.
2. Cut the pumpkin and remove the seeds. Separate the seeds from the pulp and let them dry.
3. Have small groups of children count out the seeds into groups of ten and glue them to a piece of the same colored paper cut into strips similar to the rods in the base ten blocks.
4. Those tens should be bundled together and stuck onto a square of another color that represents the flat or 100 block.
5. As groups work together any left over seeds that cannot be made into a ten should be glued to individual square that would represent the ones.
6. After the seeds are all counted they should be displayed on a bulletin board showing the hundreds (or thousands if necessary) from left to right.
7. Check the actual number of seeds with the children’s guesses. Discuss the reasonableness of the guesses.
8. Write the number of seeds with expanded notation with words and numerals and as a compact number. For example:
5 hundreds + 3 tens + 6 ones
500 + 30 +6
536

Lesson Plan #3

How many calories in your favorite MacDonald’s meal (Based on a lesson described in *Making Sense: Teaching and Learning Mathematics with Understanding* by James Hiebert et al.)

Objective: To give students a chance to practice addition and subtraction with renaming in a real world situation.

Materials:

Nutritional Menus from a fast food restaurant -- MacDonaldd’s, Burger

King and others are available on the web. Since many of the items are rounded to the nearest ten the teacher may have to adjust a few numbers in order to necessitate regrouping when adding or subtracting.

paper

pencils

Procedure:

1. The teacher asks the students to suggest what they would choose for lunch if they could have a sandwich and a drink.
2. In the first task, after taking down a couple of suggestions the teacher asks the students to find the number of calories in the meal.
3. In the second task, students then are asked to ask which sandwich has the highest calorie count and what the difference is between the highest and lowest caloried sandwiches on the menu.
4. The third task in this lesson would be to turn it into a multi-step problem, for example: Tommy has chosen a meal worth 824 calories. He had a Big Mac, milk and one other item. What is the other item?

Appendix A

This unit touches on the following standards from the *New Haven Mathematics Framework* which is adapted from the *Standards of the National Council of Teachers of Mathematics:*

Content Standard 1.0

Number Concepts, Arithmetic, and Operation Concepts:

Performance Standard 1.1

Students will add, subtract, multiply and divide whole numbers, with and without calculators.

Performance Standard 1.2

Students will demonstrate understanding of the vase ten place value system and use this knowledge to solve arithmetic tasks.

Content Standard 5.0

Problem Solving and Mathematical Reasoning:

Performance Standard 5.1

Students will solve problems that make significant demands in one or more of the solution process: problem formulation, problem implementation, and problem conclusion.

Performance Standard 5.2 Students will participate in the formulation of problems.

Performance Standard 5.3 Students will make the basic choices in planning and carrying out a solution.

Performance standard 5.4 Students will move beyond a particular problem by making connections, extensions, and/or generalizations.

Mathematical Skills and Tools:

Content Standard 6.0

Performance Standard 6.1

Students will add, subtract, multiply, and divide whole numbers correctly.

Performance Standard 6.8

Students will use recall, mental computations, pencil and paper, measuring devices, mathematical texts,

manipulatives, calculators, computers, and advice from peers, as appropriate, to achieve solutions.

Content Standard 7.0

Mathematical Communication:

Performance Standard 7.1

Students will use appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

Performance Standard 7.3

Students will explain clearly and logically solutions to problems, and support solutions with evidence, in both oral and written form.

Bibliography

Below is a list of books primarily for teachers. Those books that are * are math books and workbooks that can also be used by students.

*Bosse, Nancy. *Math Connection- Grade 3.*

This book provides students with focused practice to help reinforce and develop Math skills in all areas defined by the NCTM as appropriate for 3rd graders.

Burns, Marilyn. *About Teaching Mathematics: AK-8 Resource.* White Plains, New York: Math Solutions Publications, 1992.

This is a book that presents the case for teaching math through problem solving and includes more than 240 classroom-tested activities.

___ , *A Collection of Math Lessons From Grades 3 Through 6.* New Rochelle,

New York: Math Solutions Publications, 1987.

In the book a number of problem solving activities are offered and the author gives some ideas about the responsibility of the students, and teacher in these situations as well as how to organize the classroom.

*___, *Math for Smarty Pants.* Boston: Little, Brown and Company, 1982.

Text, illustrations, and suggested activities offer a common-sense approach to mathematic fundamentals for those who are slightly terrified of numbers.

Carpenter, Thomas P. et al. *Children's Mathematics: Cognitively Guided Instruction.*

Portsmouth, N.H. : Hinemann, 1999.

This is an important book that chronicles children's natural strategies in dealing with math concepts. A CD is also included showing examples mentioned in the book.

_____, Megan Loef Franke, and Linda Levi. *Thinking Mathematically: Integrating*

Arithmetic and Algebra in Elementary School, 2003.

This book explores teaching and study of elementary math and algebraic

principles. It also includes a CD for viewing portions of classroom lessons.

*Chapman, Carolyn. *Real-Life Math Word Problems: Third Grade.* Salt Lake City, Utah: Rainbow Bridge Publishing, 2002.

This book offers a group of word problems that are based on the NCTM standards and uses third grade math skills including addition, subtraction, multiplication, division, graphing, fractions, measurement, money values, and time.

Coffland, Jack A., and Gilbert J, Cuevas. *Primary Problem Solving in Math* . Parsippany: Goodyear Books, 1992.

This book offers 101 math problems for grades K-- 3, which emphasize the idea that the math curriculum should emphasize building conceptual understanding and problem-solving abilities rather than rote memorization.

*Curriculum Planning & Development Division Ministry of Education, Singapore

Singapore Primary Mathematics. Singapore: Times Media Private Limited, 2003.

United States edition of math book, workbook and teacher's guide that were adapted from those used in Singapore. Singapore boasted some of the highest math scores and achievement when compared to other countries world-wide.

Hiebert, James, et al. *Making Sense: Teaching and Learning Mathematics with Understanding.*

This is another excellent book that describes research based ideas on how to

design a classroom that helps students learn math with understanding.

Lampert, Magdalene. *Teaching Problems and the Problems of Teaching.* New Haven: Yale Press, 2001.

The book chronicles a fifth grade teacher and her math class.

Kennedy, Leonard M. and Steve Tipps. *Guiding Children's Learning of Mathematics.*

Wadswoth Publishing Company, 1991.

This book covers different aspects of teaching elementary mathematics. It presents ideas on Mathematics education that are consistent with the curriculum and evaluation adopted by National Council of Teachers of Mathematics (NCTM).

Ma, Liping. *Knowing and Teaching Elementary Mathematics.* Mahwah, New Jersey:

Lawrence Erlbaum Publishers,1999.

The book is about the differences in fundamental understanding about math and how it should be taught as found in a group of American teachers and their Chinese counterparts.

Ritchhart, Ron. *Making Numbers Make Sense: A Sourcebook for Developing Numeracy.*

New York: Addison-Wesley Publishing Company, 1994.

This book provides instructional guidance and a collection of lesson plans and black line masters for the elementary and middle school classroom.

Notes

1. Liping Ma, Knowing and Teaching Elementary Mathematics, (Mahwah, New Jersey: Lawrence Erlbaum Associates, Publishers, 1999)p. 22-23

2. Marilyn Burns, *About Teaching Mathematics: A K-8 Resource Publications,* (White Plains, New York: Math Solutions, 1992) p. 18

3. Leonard M. Kennedy, and Steve Tipps, *Guiding Children's Learning of Mathematics,* (Belmont, California: Wadsworth Publishing Company, 1991) p.123

4. Burns, p. 19

5. Ma, p. 17-19

6. Ma, p. 15-18