Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
2004 Volume V: The Craft of Word Problems

The Number Line as a Mathematical Tool

Curriculum Unit 04.05.05
by Sheila Wade

Introduction

This unit is designed to be taught to fifth, sixth and seventh grade students. The fifth grade curriculum for mathematics requires students to develop an understanding of fractions, and to interpret percent as part of 100 and as a means of comparing quantities of different sizes or changing sizes (Content Standard 1.4). In sixth grade students continue to develop an understanding of fractions, decimals and percent. Problem solving and mathematical reasoning (Content Standard 5.0) is the central focus and goal of teaching mathematics to children. It is critical that students understand the purpose and reason for the mathematical operations which they learn in the middle school years. To apply the concepts of fractions and percents to word problems is the goal of this unit.

While in a workshop to develop vertical teaming for the mathematics department in the city of New Haven, I had an opportunity to confer with high school teachers. I asked, “What do we need to do in the middle school to prepare students for their math program?” Aside from the obvious of having students memorize their multiplication tables they said, “Students need to place fractions on a number line.” In another conversation with a high school math teacher he shared with me that he felt strongly that we lost students in math because students never quite understand the relationship of fractions to percents. In addition, the professor of this seminar during early discussions, shared with us that the problem faced by many students is that they memorize formulas, and arithmetic operations with little understanding as to why and how to apply in real situations.

The objectives in this unit address the above mentioned concerns; the relationship of fractions and percents by solving word problems with fractions, and placing fractions on a number line. Students will use the number line as a mathematical tool. The intent is for students to see clearly the relationship of fractions to percents, and solve word problems which require answers in fractions and percents. Concepts to be covered will include the alignment of fractions on a number line. A series of number lines will be developed of equal length but divided in different fraction lengths. Lessons will answer why the fraction line separating the numerator form the denominator considered a division line.

Strategy for teaching this unit will be presented in three sections. In the first section students will use a line divided into fourths beginning with from 0 through 2, and another line divided into fifths from 0 through 2, and

a third line will be divided into 20 from 0 through 2. A series of word problems will be presented that can be answered using this series of number lines. The objective of this first section is for students to add and subtract fractions using this number line. Word problems will be presented that involve fractions, improper fractions and mixed numbers.

The second section will require students to solve problems using a number line with percents. The third section will combine percents and fractions in a in word problems. The objective is for students to further their understanding of fractions and the relationship of fractions to percents.

Tools for this unit

I have developed a set of tools for students to use for this unit. I went to the local hardware store and had 2in. by 20 in. pieces of acrylic cut. They look like blank rulers. I would give each student or a pair of students four clear acrylic blank rulers of equal lengths. Students will be instructed to measure and draw a 20in. line down the center of the acrylic piece. On the first they will divide the line into fourths from 0 to 2 labeling each section in red. On the second piece they will do the same using a green marker dividing the line into fifths from 0 to 2. The third will be marked in blue and the line will be divided into 20 equal sections from 0 to 2. The fourth will be marked in black and will identify the corresponding percents of the lines divided into the fourths and fifths. This last ruler would not be made until students are ready to answer problems in the second section of this unit.

Students will answer word problems using their clear acrylic fraction rulers by placing them on top of each other moving them in order to add or subtract fractions. It is clear that I have not included 1/3 but certainly teachers can add this. The objective is for students to see the relationship of adding, subtracting, and comparing fractions and their relationship to a whole and to mixed numbers. Although it is not intended that these lessons necessarily be presented in a sequential series it is intended to be taught after students have developed an understanding of fractions using fraction blocks. The objective is for students to develop an understanding of the relationship of whole numbers, fractions, and percents, and that a number line can be used as a tool to solve word problems.

1. Solving Word problems with fractions.

It is interesting to me that most students when asked, “What is 1/2 of 60?” will answer 30. However, when asked what is 2/5 of 60 they are often confused. Mathematics is a series of relationships which are logical and if they know that 1/2 of 60 is 30 then 2/5 of 60 is 24. In order to further students’ understanding of this relationship of fractions and percents to a whole the word problems in this section require students to add and subtract fractions.

All students will be given three acrylic blank rulers all 20 inches long. See above for directions to create fraction rulers. This series of lessons is intended after students have developed an understanding of equivalent fractions most often having used fraction towers or blocks. When students place 1/4, 1/2, and 3/4 on a number line they can visually see the relationship to the total similarly with 1/5, 2/5, 3/5, 4/5, and 5/5.

1. Word Problems
1. If Ms. Wade made 20 cupcakes and gave 1/2 to the fifth grade how many did she give to the fifth grade? _____
2. If Ms. Wade made 20 cupcakes and gave 1/4 of them to the sixth grade and 1/2 to the fifth grade how many cupcakes did she have left for the teachers? _____
3. If Ms. Wade made 20 cupcakes and she saved 1/5 of them for after school how many did she save for after school? _____
4. Ms. Wade gave 8 cupcakes to the fifth grade class. This represents 1/5 of her total. What was the total number of cupcakes?
5. Ms. Wade gave 12 cupcakes to the teachers. This represents 1/2 of her total. How many did she have originally?

Solutions and instruction:

Each student will mark with an erasable marker on their 1/4 and 1/5 fraction line the total at the 1 mark. It is important to reinforce that the 1 represents the whole. In the first 3 problems 20 is the whole or total. It will be clearly visible that 1/2 of 20 is 10 (**P#1**). 1/4 is 1/2 of 1/2 so the answer to **P#2** would be 5. 5 + 5 + 5 + 5 = 20. Using the number line divided into 1/5 it is clear that 1/5 of 20 would be 4. The number line to get to 1 is divided into 5 sections 4 + 4 + 4 + 4 + 4 = 20. In problems 4 and 5 students should mark on their number line at the 1/5 mark 8 (**P#4**) and at the 1/2 mark 12 (**P#5**). Reading their number line it will be clear that if 8 is 1/5 then the total will be 40 and if 12 is 1/2 than the total will be 24.

6. Students baked cupcakes for a bake sale to raise money to buy calculators. The goal was 200 cupcakes. Grade 5 baked enough cupcakes to reach 1/4 of the goal and grade 6 baked enough cupcakes to reach 1/4 of the goal and grade 7 baked the remaining cupcakes to reach goal.

a. How many cupcakes did grade 5 bake?
b. How many cupcakes did grade 6 bake?
c. How many cupcakes did grade 7 bake?

7. The goal was to raise $500.00. On Monday students raised 3/5 of their goal. On Tuesday they raised 2/4 of their goal. On Wednesday they raised 1/5 of the goal.

a. How much money did they raise on Monday?
b. How much money did they raise on Tuesday?
c. How much money did they raise on Wednesday?
d. Did they reach their goal? On what day did they reach their goal?
e. How much over or under goal were the students by Wednesday?

8. The students sold 200 cupcakes. The teachers bought 3/5 of the cupcakes. The students bought the rest. How many cupcakes did the students buy?

9. The teacher read 30 pages of a book on Monday and 60 pages on Tuesday. She still had 1/4 of the book to read. How many pages are in the book?

10. The students were assigned to read 1/5 of a book Monday and 1/4 of the book the following Monday. What fraction of the book remained for them to read?

Solutions and instruction:

Using the 1/4 ruler students will mark 200 at the total (**P#6**). Students will determine that 50 is 1/4 of 200. Grade 5 therefore baked 50 cupcakes and grade 6 baked 50 cupcakes the remaining 100 was baked by the 7th grade which represents 1/2 of the total **(P#6**).

Similarly students will do the same for **P# 7** . Using the fifths ruler students will mark at the 1 the total $500.00. Monday raised 3/5 of goal which is $300.00. Using the fourths ruler students will determine that on Tuesday students raised 2/4 which is 1/2 of goal it would be $250.00. Wednesday they raised an additional $100.00. By adding Monday and Tuesday's totals students will see that goal was reached on Tuesday. Adding all the totals Students will calculate that students reached $650.00 or $150.00 over goal. I would instruct students to mark on their ruler 1/5 and add 3/5 which will bring them to 4/5 and using that as the beginning point I would ask them to place the 1/4 acrylic ruler on top and determine where 4/5 plus 1/4 ends on the ruler. It us greater than a whole and when placed on the 20th ruler it is 26/20.

Problem **#8** requires the students to calculate 3/5 of 200 or 2/5 of 200 to determine how many the students bought. I would model two ways of calculating this answer; 3/5 of 200 is 120, the amount teachers bought, subtract 120 from 200 to determine how many the students bought which is 80, secondly, determine that if the teachers bought 3/5 of the cupcakes then the students bought 2/5, 3/5 + 2/5 = 5/5 which would represent the total, 2/5 of 200 using the ruler would be 80.

Problems **#9** and **#10** do not give totals. Students must add 30 + 60 = 90 (**P#9**) which represents 3/4 of the total. Placing 90 at the 3/4 mark on the acrylic ruler students can calculate the total. Each 1/4 section is 30

pages therefore the total is 120 pages. Problem **#10** requires students to add 1/4 + 1/5. By placing 1/5 on top of the fourths ruler using the 1/4 mark as the beginning and placing both on the 20ths ruler students will determine that students read 9/20 of the book. The remainder is 11/20. I would discuss this answer in relationship to 1/2. Did the students read more than 1/2 of the book?

Fractions with mixed numbers:

11. A recipe calls for 3/4 cup of flour for cookies and 2/5 cup of flour for the cake. Will 1 1/2 cups of flour be enough?

12. In order to have enough ribbon for a craft project I need 4/5, 1/4 and 2/5 of a yard. Will 1 1/2 yards be enough?

13. The teacher asked the students to read 1/4 of the pages of a 200 page book each night for 5 nights. Did she assign too many nights to read the book?

a. How many pages a night do the students need to read of the assigned book?

b. If the students read 1/4 of 200 pages every night for five nights how many pages would they have read after five nights?

14. Tanya had 2 cups of sugar. She used 1/4 of a cup of sugar for her cake recipe. How much sugar did she have left?

15. Amber had 2 cups of walnuts. She needed 4/5 of a cup for a recipe. How many cups of walnuts did she have left?

16. The cake recipe required 3/4 of a cup of sugar and the cookie recipe required 1 1/4 cups of sugar. How many cups of sugar were required for both recipes?

17. The total dry ingredients for a cake were 1-1/2 cups of flour and 3/4 cup sugar. How much more flour than sugar was used?

18. Jamie needs 2/5 of a cup of flour for each cookie recipe. She used 2 cups of flour. How many batches of cookies did she make?

19. Olivia needed colored pencils for a project. Each box had 20 pencils. What fraction of the box did she need for the following colors?

a. 5 Red b. 16 Blue c. 15 Green d. 10 Yellowe. 30 Black

Solutions and Instructions :

When using acrylic rulers for mixed numbers it is important to point out the features of the rulers that represent more than 1. On the 1/4 ruler, review that 4/4 is 1, 6/4 is 11/2, 7/4 is 1 3/4 and 8/4 is 2. I would have the students write above the line the whole number equivalents to the mixed numbers. Problems **#11** and **#12** requires students to add a series of fractions and determine if it is more or less than 1-1/2. When adding fractions with unlike denominators I would guide the students to add the fractions with the like denominators first. I would then show how to use the fifths ruler to add 2/5 + 4/5 = 6/5. Starting at 6/5 add 1/4 by placing

1/4 onto the acrylic ruler starting at the 6/5 mark. Place on the 20ths ruler. It will add to 29/20. Is it more or less than 1 1/2? Students should see that is less than 1-1/2 which would be 30/20.

Problem **#13** gives a total, 200 pages, and asks the students to determine what 1/4 is? When placing 200 at the 1 mark it is easy to determine that 1/2 or 2/4 is 100 and 1/2 of that is 1/4 or 50. If the students read 50 pages a night for 5 nights they will finish the book after night 4. Problems **#14, #15** and **#16** are subtraction problems using mixed numbers. In P **#15** students should identify that 2 cups would be represented at the 2 number and simply move 1/4 of a distance to the left and the answer would be 7/4 which would be 1 3/4 cups of sugar. P **#16** and **#17** are similar.

Problem **#18** : Students are asked to calculate how many 2/5 are in 2. This of course is a division problem. Students can simply move the length of 2/5 within the fifths ruler and determine that there would be 5. This is a sample question for division and I would develop a set of similar problems for students to develop competence in this series of calculations.

Problem **#19** is pretty straightforward. Students will use both the fourths and fifths ruler. Asking what fraction 30 represents is misleading because it is not a fraction of a box it is a whole plus a fraction. I would expect students to question however; the intent is to understand mixed numbers.

2. Solving word problems with Percents

Students will be given at this point of the unit the fourth blank acrylic ruler. They will be instructed to draw a 20 inch line down the middle of the ruler. At the 10 inch mark identify as 100% at the 20 inch mark 200%. At the 5 in. mark 50% at the 21/2 in. mark 25% at the 15 in. mark 150% at the 12 1/2 mark 125% and at the 17 1/2 inch mark 175%. Likewise I would have students mark at the appropriate intervals: 2in. 20%, 4in. 40%, 6in. 60%, 8in. 80%, 10 in 100%, 12 in 120%, 14in. 140%, 16in. 160%, 18in. 180%, and 20 in. of course is 200%. Once students have created their percent rulers we will discuss why percents are used and why they are easier than fractions. The final section will have problems with both percents and fractions.

2. Word Problems

1. If Ms. Wade made 20 cupcakes and she gave 25% of them to the teachers and 50% to the students how many did she have left?

2. There are 48 students in the fifth grade. 25% received A’s on their report card. How many received A’s?

3. There are 64 students in the sixth grade, 75% received honors. How many received honors?

4. Ms Wade gave 12 cupcakes to the teachers.

a. If this represents 50% of her total how many did she have originally?
b. If this had represented 40% of her total how many did she have originally?

5. If 50% of her 20 cupcakes were vanilla, 25% was strawberry and the remaining was chocolate what percent was chocolate?

6. Ms. Wade baked cupcakes. 20 cupcakes had white frosting this represents 40% of her total. How many cupcakes did she bake?

7. There are 20 students in the fifth grade that went on the field trip. This represents 40% of the class. How many students did not go on the field trip?

8. Of the 80 bottles of soda 75% are orange.

a. How many are orange flavored?
b. The remaining bottles are cola. How many are cola?

9. The fifth grade class had a cookie sale. The goal was to bake 300 cookies. Students brought in 450 cookies. What percentage over goal were they?

10. The goal for the fifth grade class was to collect 200 pennies. On Monday they collected 80.

a. What percent of the total goal did they collect on Monday?
b. On Tuesday they collected 70 more pennies. What percent of the total have they collected altogether?
c. On Wednesday they collected 100 additional pennies. What percent of the total have they collected?

11. Ms. Wade bought a gross of pencils at the beginning of the year. In November she used 36 of them. By March she had used an additional 72. What percent of the total did she have remaining?

Solutions and Instruction :

Problems 1, 2, 3, give the totals. Students would place the totals at the 1 mark. To calculate 50% it is 1/2 or 75% is 1/2 plus 1/4. It is important for students to understand that 1 or 100% represents the total and of course that value of total varies.

Problems **#4** , **#5** , **#6** , and **#7** give a percent of the total. Students would place the value of 25%, 50% or 40% on the ruler to determine the value of 100%.

Problem **#8** gives the total of 80 bottles. Students are required to calculate how many bottles are orange, 75% of 80. The second step requires a calculation for the remaining cola. This could be the total of orange (60) minus the total (80) to give 20 or students could calculate 25% of 80.

Problems **#9** and **#10** illustrate samples of more than 100%. On their acrylic rulers students would mark at 100% the total goal of cookies and pennies and determine the percents of the given values. Problem **#11** requires students to know that a gross is 144 which will represent the total or 100%. 36 + 72 = 108. 108 is

what percent of 144. When 144 is placed on the ruler it is easy to calculate 1/2 or 50%, 72. 36 is 25% or 1/4 therefore 108 is 75%.

3. Solving word Problems using fractions, mixed numbers and percents

This section of word problems combines fractions and percents. Students would be instructed to add the equivalent fractions to their percent rulers. The acrylic rulers are designed to be used with non-permanent markers. The percent values should be placed above the line in black ink. Below the line using the corresponding colors of red for fourths and green for fifths students should mark the fractions at the appropriate lengths. The objective is to see the relationship and to fully understand that fractions and percents can be used interchangeably.

3. Word Problems

1. Ms Wade made 24 cupcakes. She gave 50% of them to the fifth grade winners of the math contest. She gave 1/4 of them to the sixth grade students.

a. How many did she have left over for the teachers?
b. What percent does this represent?

2. Ms Wade gave 8 cupcakes to the students in the fifth grade. This represents 1/5 of her total.

a. What was the total number of cupcakes?
b. What percent did she have remaining?

3. Ms. Wade gave 1/4 of the cupcakes, 6, to the teachers. How many did she have left over for the students?

4. 48 students attended the honors assembly. This represents 2/4 of the class.

a. How many did not attend?
b. What percent of the students did not attend the assembly?
c. 1/4 of the students who attended received medals. How many received medals?

5. Students brought in 80 bottles of soda and stored in the closet of the cafeteria.

a. How many bottles if 2/5 of the soda is for the eight grade party?

b. How many bottles if 20% is for the fifth grade field trip?
c. How many bottles if 25% is for the sixth grade teacher's party.
d. The rest is for the seventh grade. How much soda is for the seventh grade?

6. There are 200 students in the 7th grade. 75% of them stayed after school to watch the basketball game.

a. How many stayed to watch the basketball game?
b. If 1/2 of those who stayed were boys how many were boys?
c. 40% of the boys at the game wore red jerseys. How many did not wear red jerseys?

7. I have 30 pencils in my desk. This represents 2/5 of the total I had at the beginning of the week.

a. How many did I have at the beginning of the week?
b. What percent do I have left to give to the students?

8. 1/5 of the class went to the library, 2/5 of the class remained in the classroom. There are 60 students altogether.

a. How many students are in the library?
b. How many remained in class?
c. How many students are not accounted for?
d. What percent of the total are not accounted for?
e. Is the percentage of students unaccounted for the same as the percentage who remained in the class? Explain

9. 60% of the fifth grade went on a field trip. 120 students went on the field trip. How many students are in the fifth grade?

10. The recipe calls for 3/4 of a cup of chocolate chips to make cookies. There are 3 cups in a bag of chocolate chips.

a. What percent of the total do you need to make cookies?
b. How many cups will remain in the bag?

11. I have 1 1/2 cups of walnuts. I need 3/4 of a cup for the cookies. What percent of the total does this represent?

12. I received $240 to buy a gift for a teacher.

a. 50% came from the 7th grade teachers. How much did they contribute?
b. 25% came from the 5th grade teachers. How much did they contribute?
c. 25% came from the 6th grade teachers. How much did they contribute?

13. I have $240.00 to buy a gift. The gift I wanted to buy cost $360.00. I had a coupon for 25% off any item in the store.

a. Did I have enough money?
b. What percent was the cost of the gift to the total I had collected? (150%)
c. How much more money do I need? _____
d. What would the gift cost if I had a 2/5 off coupon? _____

14. The goal was to raise $500.00. On Monday students raised 3/5 of their goal. On Tuesday they raised 2/4 of their goal. On Wednesday they raised 1/5 of the goal.

a. How much money did they raise on Monday?
b. How much money did they raise on Tuesday?
c. How much money did they raise on Wednesday?
Change fractions to percents to solve:
d. Did they reach their goal? On what day did they reach their goal?
e. How much over or under goal were the students by Wednesday?

15. For one recipe I need 2/5 of a cup of flour and 1/4 of a cup of flour for another.

Solve using percents.

a. Do I need more or less than a cup?
b. If I have 3/4 of a cup of flour will I have enough?

Solutions and Instruction:

Problem **#1** gives a total, 24. Students should place 24 at the 1 or 100% mark on their number line. 1/2 or 50% is clearly identified at 12. 1/4 of the total is 6. 12+6 =18. 24–8=6. Therefore, 6 remain for the teachers, represented as 25%. Problem **#2** gives the value of 1/5 which is 8. Students will place 8 at the 1/5 mark which is also 20%. 5/5 or 100% will be 5x8=40, total number of cupcakes, or 1/5(20%) is 8, 2/5(40%) is 16, 3/5(60%) is 24, 4/5(80%) is 32, 5/5(100%) is 40. Remaining is 5/5 – 1/5 = 4/5 (80%), 32 cupcakes remain. Problem **#3** is similar to **#2** .

Problems **#4** , **#5** , and **#6** give totals and should be placed at the 1 or 100% mark on the ruler. In problem **#5** when using the ruler with all the fractions of fourths and fifths and percents on the same ruler it allows students to determine the different fractions of the same amounts. 2/5(40%) of 80 is 32, 1/5 (20%) of 80 is 16 and 1/4 (25%) of 80 is 20. Problem **#6** requires students to determine 75% of 200 which is 150. Once that is determined students then use 150 as the total to answer b. and c.

Problem #7 gives 30 as representing 2/5 of a whole. Students should be instructed to place 30 on the 2/5 mark of the line and determine that 1/5 is 15 and 5/5 is 75. 2/5 represents 40 %(answer to b). Problem **#8** gives the total as 60 and accounts for 1/5(library) and 2/5(remain) of the students. Placing 60 at the total each 1/5 has a value of 12. 12 students in library, 24 students remain in classroom which accounts four 3/5 of the total which is 60%. 40% or 2/5 which is 12 are unaccounted for. Problem **#9** gives the value of 60% which is 120. Students should place the number 120 at the 60% (3/5) mark. This problem uses a number greater than 100 to represent less than 100%.

Problem **#10** requires an understanding of the value representing 100% which is more than 1 cup. Students should place 3 cups at the total, 1,100% by doing so the calculation of 1/2 would be 1-1/2 cups. 3/4 of a cup is 1/2 of 1-1/2 therefore the answer is 25%. Problem **#11** is similar to **#10** . 1-1/2 represents the total, 3/4 is 1/2 of 1-1/2, and the answer is 50%.

Problem **#12** and **#13** are related. $240.00 represents the total and should be placed at the 1,100% mark. 50% is $120.00, 25% is $60.00. Problem **#13** illustrates the most common use of percents which is a portion of money which is added or subtracted to the total. The cost of an item is $360.00. I have a coupon for 25% off. Students can use their number line to calculate the value of 25% of $360.00, $90.00. To answer **#13b** students should place $360.00 on the number line in relationship to the total they collected, $240.00. This will illustrate that the cost was 150% of the total collected therefore I need $120.00 more which is 50% of the total collected. Problem **#13d** describes the discount using a fraction instead of a percent. This offers an opportunity for discussion of the relationship of fractions and percents. 2/5 is the same as 40%. The gift would cost $236.00.

Problem **#14** has multiple parts and is rewritten here from the selection in section 1 to illustrate the relationship of fractions and percents. The goal was to raise $500.00. On Monday students raised 60% of their goal. On Tuesday they raised 50% of their goal. On Wednesday they raise 20% of their goal. It the goal is represented as 100% it is quite clear that after Tuesday they had already raised 110% of goal. Problem **#15** when phrased in percents, I will need 40% of a cup of flour for one recipe and 25% of a cup for another. How much of a total cup do I need? 65%. It is easy to determine that more than 1/2 is need but less than 1 whole.

Sample Lesson Plans

Lesson: Creating fraction rulers and number lines.

Objective:

Students will place fractions on a number line 0 through 2. Using fourths, fifths, and twentieths will compare, add and subtract fractions.

Materials:

Each student will be given 3 blank acrylic rulers already cut on 20 inch lengths, a set of non-permanent markers, and a standard inch ruler.

Procedure:

Students will draw a 20 inch line in the middle of each blank ruler using a different colored marker for each ruler.

1. With the red marker at the 10 in. mark students will write 1 above the line, 2 at the 20 inch mark, 1/2 at the 5 in. mark, and 0 at the beginning. Students will then proceed to label at 1/4 intervals, 1/4 at the 21/2in., 2/4 (1/2) at the 5in. mark, 3/4 at the 71/2 in mark, 4/4 at 10 in., 5/4 at 121/2 in., 6/4 at 15 in., 7/4 at 171/2 in., 8/4 at the 20 in. mark.

2. Using a green marker label the beginning of the line, 0; at the 10 in. mark label 1 above the line; and at the 20 in. label 2. At 2 in. intervals label below the line,1/5, 2/5, 3/5, 4/5, 5/5, 6/5, 7/5, 8/5, 9/5, and 10/5.

3. Using a blue marker, label ruler at 1/2 inch intervals 1/20 through 40/20.

When rulers are complete students should using one ruler at a time add 1/4 sections to 3/4, or 3/4 + 2/4 = 6/4, and subtract 1/4 segments to develop an understanding that the operation of addition and subtraction for fractions is the same as adding whole numbers. The denominators do not change. Students should do the same with the 1/5 ruler and 1/20 ruler. When using their fraction rulers students can add, subtract or compare using their eye or a separate piece of paper as a measure to move right or left of the number line.

Problems:

1/4 + 3/4 = 4/4 6/4 – 2/4 = 4/4 1/4 + 3/4 + 5/4 = 8/4

1/5 + 1/5 = 2/5 1/5 + 3/5 = 4/5 6/5 – 3/5 = 3/5 2/5 + 3/5 + 1/5 = 6/5

4/20 + 5/20 = 9/20 using three rulers add 1/4 to 1/5 and place on top of 1/20 acrylic ruler students will see that 1/4 +1/5 is the same as 4/20 + 5/20.

Students should make up a series of problems on their own to become more familiar with their new mathematical tool.

Closure:

If I need 3/4 of a cup of four for one recipe and 3/4 of a cup for another will 1 1/2 cups be enough?

Lesson: Solving Word Problems using fraction rulers.

Objective:

Students will solve word problems using fourths, fifths, improper fractions and mixed numbers.

Materials:

Fractions rulers.

Procedure:

Students should be familiar with the properties of their fraction rulers. Point out that 1 represents a whole, 4/4, and 5/5 are the same value as 1. 2/4 is 1/2. The denominator represents the sections to make a whole. On the 1/4 ruler there are 4 sections to make 1 whole and 8 sections are equivalent to 2. On the fifths ruler there are five equivalent lengths to make a whole and 10, 1/5 lengths are equivalent to 2. A good question to ask, “How many 1/5 lengths are equivalent to 1/2?” Students should also continue to add fourths and fifths and place on the twentieths ruler to compare answers. This will develop an understanding of common denominators.

Problems:

See word problems in Section 1 and 2.

Lesson: Solving Word Problems using percent rulers.

Objective:

Students will create a percent ruler to use as a tool to solve word problems. Students will identify equivalent fractions and percents.

Materials:

Blank 20 inch acrylic rulers and a black non-permanent marker.

Procedure:

Students will draw a 20 inch line in the center of their ruler. Label 100% at the 10 inch mark, 200% at the 20 inch mark, 50% at 5 in., 25% at 21/2 in., 75% at 71/2in., 125% at 121/2 in., 150% at 15in. On this ruler

students will also label, 20% at 2in., 40% at 4in., 60% at 6in., 80% at 8 in., 120% at 12in., 140% at 14in., 160% at 16in., and 180% at 18 in. I would then have the students write the equivalent fractions to the percents using the appropriate color, red for fourths, and green for fifths.

Discuss properties of their new tool in relation to the fraction rulers they have already been using. Students should solve simple addition and subtraction problems.

20% + 20% = (40%) 1/5 + 1/5 = (2/5) Are the values equivalent?

75% - 25% = (50%) 3/4 -1/4 = (2/4) Are the values equivalent?

When adding and subtracting percents it is easier than adding and subtracting fractions with unlike denominators:

20% + 25% = 45% 1/5 + 1/4 = 9/20 The values are the same. 9/20 = 45/10.

Word problems are presented in Section 2 and 3.

Models for fractions and percent acrylic rulers:

1 2

1/4 2/4 3/4 4/4 5/4 6/4 7/4 8/4

1 2

1/5 2/5 3/5 4/5 5/5 6/5 7/5 8/5 9/5 10/5

1 2

... * 5/20 ... 10/20 ... 15/20 ... 20/20 ... 25/20 ... 30/20 ... 35/20 ... 40/20

1 2

20% 25% 40% 50% 60% 75% 80% 100% ...* 125% ... 150% ... 175% ... 200%

The 1/4 ruler should be marked in red.

The 1/5 ruler should be marked in green.

The 5/20 ruler should be marked in blue.

The percent ruler should be marked in black.

*When using 20 inch acrylic rulers space will allow including all equivalent fractions and percents.

Mathematical Standards

This unit addresses the following content standards.

Content Standard 1.0 Number Concepts, Arithmetic, and Operation Concepts.

Performance Standard 1.2:

Students will describe and compare quantities and compute using fractions and decimals.

Performance Standard 1.4:

Students will interpret percent as part of 100 and as a means of comparing quantities of different sizes or changing sizes.

a. Students will interpret percent as part of 100.
b. Students will use percents to compare quantities of different or changing sizes.
c. Students will apply percent concepts to solve problems.

Performance Standard 1.6:

Students will order numbers.

b. Students will locate numbers on a number line.

Content Standard 5.0 Problem Solving and Mathematical Reasoning.

Performance Standard 5.2:

Students will formulate and solve a variety of meaningful problems.

Content Standard 6.0 Mathematical Skills and Tools.

Performance Standard 6.7:

Students will use recall, mental computations, pencil and paper, measuring devices, mathematics texts, manipulatives, calculators, computers, advice from peers, as appropriate, to achieve solutions.

Content Standard 7.0 Mathematical Communication.

Performance standard 7.1:

Students will use mathematical language and representations with appropriate accuracy, including numerical tables and equations, simple algebraic equations and formulas, charts, graphs, and diagrams.

Bibliography

Carpenter, Thomas P., Megan Loef Franke, and Linda Levi. *Thinking Mathematically.* (New Hampshire: Heinemann) 2003.

This book is a good resource for elementary mathematics teachers. It provides in depth description on how to create a classroom culture for questioning and reasoning. The equal sign is given focus so students clarify early and avoid misconceptions at a later date.

Lampert, Magdalene. *Teaching Problems and the Problems of Teaching.* (New Haven: Yale University Press) 2001.

This book chronicles a year the author spends with a fifth grade class. Students solve word problems articulate their reasoning and defend their answers. This book provides invaluable resource for teachers who wish to develop this teaching method.

Ma, Liping. *Knowing and Teaching Elementary Mathematics.* (New Jersey: Lawrence Erlbaum Associates) 1999.

The author describes American and Chinese teachers' approaches to solving mathematical problems. The descriptions are insightful and the comparison of teaching styles is very beneficial for school systems or individual classroom teachers looking to enrich their program.

*Hong, Kho Tek. *Primary Mathematics 6A Textbook.* U.S Edition. (Singapore: Curriculum Planning & Development Division Ministry of Education.) 2003.

This is a series of textbooks for primary grades from Singapore. This is a highly recommended series for teachers to supplement their math program with word problems. To order books online: www.singaporemath.com

* Appropriate for children