



Curriculum Units by Fellows of the Yale-New Haven Teachers Institute  
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## **Problem Solving Through Communication**

Curriculum Unit 04.05.06  
by Joyce Bryant

This unit will be developed for middle school students. As I develop this unit and make use of it with my students it will enhance their ability to solve problems while developing math skills in the four basic math operations in problem solving.

Problem solving is the process by which students acquire knowledge, skills, habits, values and attitudes. They must learn to read mathematics in order to become able to use math in everyday life. Like skills, problem solving must be practiced; when it is practiced it becomes far less difficult. Some problem solving techniques at times are so very important, they should be the focus of instruction.

Problem solving is a challenge for the majority of our students. Real life situations, applications and interdisciplinary connections are a part of everyday lessons to be learned. In most math classes, there is a wide range of pupil ability. Pupils do not mature at the same rate, and they vary in both the ability to learn and the level of learning attained. Furthermore, different pupils learn in different ways. Instructors are faced daily with the problem of dealing with the many variables existent in their classes.

Other resources are needed to augment the existing curriculum and my unit on problem solving will aid the curriculum in that area. The most important aspect of problem solving is the quality of thinking it engenders among students. Clarity of thought is aided by precision; the development of precise language is more than a matter of memorizing mathematical words and phrases. Students should be led to think about concepts and then generalize about them. Systematic attention should be given to the development of problem solving skills as they relate to word or story.

There are several ways to present problems other than to refer to textbooks and workbooks. Problems can be written on the chalkboard, thus inviting a discussion. An overhead projector may be used when presenting a problem, it can be presented either in its entirety or line by line to focus students attention and pinpoint parts in case there is difficulty in understanding the problem.

Problem solving is the process by which students acquire knowledge, skills, habits, values and attitudes. Problem solving includes a variety of learning experiences and it takes place outside the classroom as well as inside the classroom. It involves both learning and teaching. Some students learn by teaching themselves and they also learn with the help of other people, such as parents and teachers. Problem solving is as old as humanity. People have always needed problem solving in order to survive. Today, our society must become skilled in government, industry, commerce, the arts, and agriculture in order to survive. A society can not

survive without being skilled in problem solving, especially in math.

Problem solving is more important today than ever before. It helps students acquire skills that are needed in everyday activities. It also gives them training that they may need to prepare for a career or job. It is also important because it helps students get more out of life, increases their knowledge and understanding of the universe, and helps students acquire skills that make their lives more interesting and enjoyable. Problem solving provides skills needed to participate in sports, play a musical instrument, or paint a picture and helps them to adjust to change.

Problem solving in and out of the classroom makes learning a delightful adventure. Problem solving is for improving the lives of others as well as our own and leaving the community and the world a better place for it. Mathematicians identify sources of change, distinguish between patterns, and seek multiple representations through mathematical language, such as numeric, graphic, verbal, and symbolic to express what it transpires.

The art of problem solving is the developed ability over a period of time. The information and activities in this unit will be designated to offer students opportunities to enrich their problem experiences.

It is important to realize that students cannot be expected to use strategies that are unfamiliar to them. Problem solving skills are learned just like other skills are learned. Students need to be exposed to a wide variety of problems so that they can try out new strategies and practice using them.

Learners will become familiar with problem solving as they relate to everyday life. This unit will allow students to relate to real life situations. The activities in this unit fosters quantitative thinking in the learner, which will lead the students to develop interest, objectivity, attitudes and problem solving skills. It is my intent that these skills will grow with adequate use, that is, by having certain thought patterns recur in a wide range of problem solving. The adequate use will enhance the required amount of application of a concept necessary to insure its future availability, and in this way the students will really become mathematically literate in problem solving.

The middle and high school learner is an entity in himself / herself with unpredictable reactions to problems and personal situations. The learner should see that in problem solving the thought processes help to analyze the factors involved when something needs to be done as well as to organize the chosen factors in a problem in order to bring about a satisfactory outcome. The learners are introduced to problems that are common. It is my hope that by solving some of the problems the learners will acquire some of the skills and understandings that will be needed in the future. There are understandings to be mastered. There is insight to be gained. One of the purposes is to guide the learners thinking to help make decisions. It is hoped that the learners will not only develop problem solving skills, but take a critical look at himself / herself in this ever changing and complex society of ours where problems of all kinds do exist.

Problem solving is more than the ability to compute. It is believed that the ability to solve problems develops automatically from the mastery of computational skills. This is not true, problem solving is itself a skill that has to be taught.

When the learner is confronted with a problem situation whose solution is not known, the student must rely on his or her problem solving skills. One problem solving skill is the ability to pick out the important facts that are given and disregard the facts that are not pertinent. Another skill is to recognize what information is missing and how to find it. Another is the ability of the learner to recognize similarities between problems which are being solved and have been solved.

Problems can be solved in a variety of ways, yet not one of these ways is appropriate to the solution of all problems. However it is helpful to have at least a general framework within which students can organize their efforts.

There are several techniques, methods and strategies used in problem solving. Problem solving involves trying several ways to solve the problem before deciding on which to use. It involves putting together the facts that the learner has with mathematics that he or she knows in such a way that the result is a solution that was unknown to begin with. A model of a problem can be a picture worth five hundred words, a sketch, a scale drawing, a chart or graph. It's whatever helps the student. There are times when information is missing and the skill is the ability to recognize what additional information is necessary.

Problem solving situations are likely to be representative of those which the learner will face sometime in the future. Problem solving will give the students experiences to help them in their thinking and decision making processes. The main thrust of problem solving should be toward the development of logical thinking. Taking the numbers out of problems seems to be one way of doing this without the negative reaction that usually accompanies mathematical problems.

Applications of mathematics to problems from the daily lives of pupils give added depth of understanding. In making and applications, the essential problem is to choose the appropriate mathematical structure for the application. As students study and analyze a given situation to determine the most appropriate mathematical structure, they learn to solve real life problems, and also broaden and expand their knowledge of the mathematical structure involved. Problem solving, in its broadest sense, includes the way one approaches any mathematical idea as well as the way he or she approaches a practical problem stated in words.

Learning is an individual matter. However, it often takes place in a group situation. Students learn by doing, thinking, discussing, and it is important that problem solving be the kind that invites students to participate, to do, to think and to respond. Students should be challenged to react, to reason, and come to their own conclusions.

Students should be led to explore, experiment, and analyze unusual problem solving within the four basic mathematical operations, such as addition, subtraction, multiplication, and division. Problem solving, in it's broadest sense, will include the way the learners approach any mathematical idea as well as the way they approach any practical problem stated in words and the writing of story problems. No mathematical problem solving idea is completely developed at any one time. Each problem solving idea is introduced at an early level and is expanded at different levels. Numerous exercises should be provided for fast and slow learners. Some adaptation is desirable. Each instructor should make use of problem solving ideas suggested.

Applications of mathematical problem solving ideas from the daily lives of students can give added depth of understanding. In making applications, the most important problem is to create an appropriate mathematical structure for the application. As they study and analyze a given problem to determine the most appropriate mathematical structure, the students learn to solve life problems, and also broaden and deepen their knowledge of the mathematical structure involved. Emphasis should always be given to the relationship between the situation and the appropriate mathematical structure.

Another important aspect of problem solving is motivation. Motivation for the study of problem solving comes from a variety of resources. One important resource of motivation is the inherent interest of mathematical ideas themselves. Many students study with enthusiasm when they are allowed to explore the why and how of the mathematical situations. Students are also motivated by working with special activities that require the

application of mathematical principles, by exploring such topics as early ways of computing or primitive ways of measuring and by seeing how the mathematics that they study are applied to their lives and others.

Another motivating factor, and one that should not be overlooked, is the success in learning. A learning student is a motivated student. When the student's experiences in math lead to success and achievement, the learner has excellent motivation for more and further study.

Problem solving ability develops over a long period of time and grows with experience in solving a variety of problems in many different ways. Students must learn to be flexible and make use of a variety of methods, techniques and strategies.

If a problem is more complex, the strategy for solving it may not be immediately apparent. Problem solving requires some degree of creativity on the part of the problem solver. The problem solver can be the actor by acting out the problem or by devising a plan to solve the problem.

Problem solving can be accomplished by using a four step or five step problem solving plan.

A Four Step Plan Involves:

1. Understand the problem
  - a. what are the facts?
  - b. what do you need to know?
2. Make a plan
  - a. what do you do to solve the problem?
3. Show the work
  - a. do the arithmetic
4. Interpret the answer
  - a. Is the answer reasonable?
  - b. Does it answer the question?

A Five Step Problem Solving Plan Includes:

1. Reading the problem carefully, understanding what it says, and reading it more than once

2. Ask questions like the following
  - a. what is asked for?
  - b. what facts are given?
3. What facts are not needed to solve the problem?
4. What facts are needed to solve the problem?
  - a. will a sketch or diagram help?
5. Determine which operation or operations can be used to solve the problem, carry out the operations carefully and give the answer.

There are several ways to present problems other than to refer to textbooks and workbooks. Problems can be written on the chalkboard, thus inviting a discussion. An overhead projector may be used when presenting a problem; you may project the entire problem at once on each individual part pinpointing any difficulty in the understanding of the problem.

Duplicated masters is another way of presenting a problem where by each student have its own individual sheet. Some students may find its easier to refer to a problem in this way. It also eliminates possible student errors in copying a problem from a chalkboard or an overhead projector.

Oral presentation is another way of presenting a problem. This can be done by the teacher reading the problem aloud while the students read it softly or by having a student so the same.

Students need help in solving problems. Giving students too much help will leave them with nothing to do. Too little help will cause frustration. The right amount of help will allow students to experience the challenge of a problem and the pleasure of finding its solution.

Students may need to test survival strategies before an appropriate one is found. Once the problem is solved the students think that's the end, but they should be encouraged to take a second look and consider the reasonableness of the of their answers.

### Learning Objectives

1. Help students increase their thinking skills and decision making process.
2. Apply questions to decision making process.
3. Students will understand that there are certain conditions under which they must work in order to bring about a desirable and valued solution.
4. Students will become familiar with and understand the language that is used in problem solving.

5. Evaluate the process and consequences.

## Goal

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To help students familiarize themselves with problem solving dealing with the four basic operations of whole numbers, fractions, decimals, and percentages.

- a. the majority of students will be able to discuss problem solving in relation to careers and real life situations.
- b. evaluative device, teacher observation, quizzes and tests.

Once students understand the problem situation, they must apply some problem solving strategies. The strategies appropriate to solving mathematical problems are many and varied. The most common strategies used in problem solving are:

1. draw a picture or diagram
2. acting out the problem
3. trial and error
4. write an equation
5. experimenting
6. make a table or list
7. finding a pattern
8. estimating

## Problems Written by Students

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1. Stacy had to call her Mom an hour after she came home from school. Her Mom gets off work 5 hours after Stacy gets home from school. If Stacy's Mom gets off at 8:30 p.m., then what time does Stacy get home from school?
2. Mark has 5 books in his backpack. Ruth has two more than Mark but one less than Rosie. How many books do Ruth and Rosie each have? How many books are there all together?
3. Mr. Finney's class was selling lemonade. If the lemonade costs 25¢ (a quarter) and twenty-two people bought lemonade, how much money did they take in?
4. We have twenty-six people in our class. Half of the class went home sick. How many people are in the classroom?
5. Michelle's Mom baked 3 cakes for the bake sale. If Martha's Mom made two times as many cakes, how many cakes did Martha's Mom make? How many cakes are there all together?
6. I went to a sports store with \$90.00 to spend. A jersey costs \$30.00 and a pair of NBA sneakers costs \$55.00. How much change do I have left over?
7. I went shopping in the clothing store, and the sales tax was 10%. I had \$40.00 and bought a shirt for \$5.00 and a pair of jeans for \$10.00. What was the total for all of the clothes?
8. Mr. K has 900 tickets for graduation and there are 180 8th graders and each one has to have 4 tickets. How many tickets will be left over?
9. There is a sale on K-Swiss shoes. The regular price is \$80.00 and they go on sale for 20% off. How much money will you save?
10. Mrs. Bryant has 99 Math books and she gave out  $\frac{1}{3}$  of them. How many books does she have left?
11. In Tonya's drill team, she had 50 people to begin with. 10 people dropped out! How many people are left?
12. Caleb Jr. had 2 apples, 1 orange, and a bottle of grape juice. He gave half of the apple to Mike, half of the juice to Tim, the orange to Melissa, and the other half of the apple to James. What was left for Caleb Jr.?
13. 20 kids are in a club. If they split up into 4 equal groups to do a project, how many kids are in each group?
14. Mark had \$5.00. He lost \$2.00 when he was playing. How much does he have left?
15. LaToya bought 20 pieces of gum and they each cost 5¢. If she gave the cashier a dollar (\$1.00), how much change will she get back?
16. Julie went to the mall with \$20.00 of allowance. She was looking at a prom dress that cost \$115.00. If she gets \$20.00 a week, how many weeks does she have to wait until she has enough?
17. Troy was looking at some sneakers at the mall. The store had a sale on sneakers for 50% off. If the regular price is \$69.99, how much will he pay with the 50% off?
18. Karen had 7 pencils and lost 3. Kiyana had 14 pencils and gave Karen a fair amount so they can have the same number of pencils. How many pencils did Kiyana give Karen? And what amount do they have?
19. Cameron has 2 packs of cookies with 4 cookies in each pack. If Cameron has 4 friends including himself, how many cookies will each friend get?
20. There was a single foul line. If Nate was behind Kevin, and Kevin was behind Kiyana, then

Kiyana was behind Bianca, who was third in line?

21. Nate had \$25.00 and bought \$11.50 worth of candy, how much money does he have left?

22. We are having a pizza party for my sleepover. If there were 9 friends including myself and 15 pieces of pizza, how many slices did each person get?

23. The market around the corner sells apples and tangerines for 49¢ each. If Tanya wanted to get \$10.00 worth of fruit, how many would she purchase?

24. My Mom baked 47 oatmeal raisin cookies. I ate 7, my Dad ate 5, and my 3 brothers ate 10 each. How many oatmeal cookies were left?

25. Homeroom 213 took a math test. 25 students were there. If 13 out of 100 got good test scores, what percent is that?

26. Robert went to Jimmy's Army Navy Store. Robert wanted to get a jersey for \$30.00 and a pair of jeans for \$15.99. Robert gave the clerk a \$50.00 bill. How much change will Robert get back?

27. Toby went to Staples to get two packs of paper. Each pack costs \$2.50. Toby only had \$1.35. How much more money did Toby need?

28. There were 32 students at the pizza party. We ordered 8 large pizzas with 8 slices in each pie. How many slices will each student eat?



## Word Problems by Joyce Bryant

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1. The local news team covered the Democratic political convention for four nights and five days. On Monday they covered 250 minutes, Tuesday 500 minutes, Wednesday 480 minutes, Thursday 350 minutes and Friday 200 minutes. How many hours did they cover? At \$200 per minute, how much money did the news team earn?
2. The six o'clock news has 90 minutes broadcast time. The consumer reports, weather, and sports use 30 minutes. How much time is left for reporting the other news if commercials use 15 minutes?
2. The anchorperson on the evening news works from 4:00 p.m. until midnight, with one hour for dinner. How many hours does the anchorperson work?
2. The local network charged \$500,000 for one-minute commercials during the super bowl football game. How much money did the local network earn for 5  $\frac{1}{2}$  minutes?
2. Invite a representative from the local television station and have them speak to the classes concerning their work at the TV station.

Topics of discussion:

- a. How much time is spent preparing the news to be televised?
- b. How does one determine the number of commercials?
- c. How much money is spent on airing commercials?
- d. Does one who reports the news like their job and why?

Objective

To solve word problems using the five step plan.

Remind students to use a five step plan because more information is given in the problem than before.

Have students work in groups to try and solve the problem. List the steps on the board.

1. Read the problem carefully and make sure that they understand what the problem says.
2. Ask questions like the following:
  - a. what is asked for?
  - b. what facts are given?
  - c. are enough facts given?
  - d. are unnecessary facts given?
  - e. will a sketch or a diagram help?
3. Make a determination as to which operations can and will be used.
4. Carry out the operations.
5. Check solution and give the answer.

**Have students work in groups and solve the following problem:**

George's Restaurant received a shipment of 20 cases of lettuce and 18 cases of tomatoes. Each case of lettuce had 10 heads of lettuce in it and each case of tomatoes had 12 tomatoes in it. How many heads of lettuce and tomatoes did George's Restaurant receive?

After the students have solved the problem, review the steps and strategies that were used in solving the problem by the different groups.

Word problems may be written by students and may be used in the following manner:

1. students may try and solve the problems
2. rewrite the word problem
3. make a determination as to whether the problem should be discarded.

1. Jason went into a store and saw a sign which read 20% off of all men's suits which were \$100.00 each. If Jason bought three suits, what would be the price of the suits with 20% off?

2. Pat went to math class and wanted to know how to change a fraction to a decimal. If she was to change  $5\frac{3}{4}$  to a decimal, what would her answer be?
3. Marie bought an electric mixer for \$80.50. She was charged 6% sales tax on \$80.50. How much tax did she pay?
4. Steve's test scores were 91, 90, 97, 89, and 75. Janet's test scores were 100, 96, 88, 75, and 80. What is the average of Steve's score and the average of Janet's score?
5. Janet's father traveled at a rate of 64 miles per hour for 6 hours. How many miles did Janet's father travel?

## Lesson Plans

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The following are a variety of problems for students to solve dealing with the four basic mathematical operations.

The problems are designed for students so that they will be able to relate to real life situations.

### Problems

1. The local news team covered the Democratic Political Convention for four nights and five days. They covered 250 minutes on Monday, 500 minutes on Tuesday, 480 minutes on Wednesday, 350 minutes on Thursday, and 200 minutes on Friday. How many hours did they cover? At \$200 per minute, how much money did the news team earn?
2. The six o' clock news has 90 minutes broadcast time. The consumer reports, weather, and sports use 30 minutes. How much time is left for reporting the other news if commercials use 15 minutes?
3. The anchorperson on the evening news works from 4:00 p.m. until midnight, with on hours for dinner. How many hours does the anchorperson work?
4. The local network charged \$500,000 for one-minute commercials during the Super Bowl football game. How much money did the local network earn for  $5\frac{1}{2}$  minutes?
5. The news camera specialists brought two thousand feet of film for a television show. If they used sixteen hundred feet of film, how many feet of film were left?

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