Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
2004 Volume V: The Craft of Word Problems

Using Basic Properties to Solve Problems in Math

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by Carolyn N. Kinder

Introduction

During the middle school years, some students find the mathematics curriculum dull, uninteresting and some students lack the skills needed to solve word problems. As a result, they perform poorly in math and need help on how to go about solving word problems. Teaching basic properties in mathematics helps to lay the foundation for solving word problems.

This unit is designed to teach to students in grades five to eight. The purpose of this unit is to focus on the Commutative and Associative Properties of Addition, Inverse Properties of Addition and Subtraction, Commutative and Associative Properties of Multiplication; Distributive Property of Multiplication over Addition, Zero Property and the Identity Property for Addition, and the Identity Property for Multiplication. In the content section of the unit under each sub-topic will be: explanations about each topic and a set of word problems that illustrates what is meant by each property.

Lesson plans are supported by proper goals and objectives, assessment tools and resources. Students will be engaged in activities using individual and cooperative learning groups. Students will be exposed to developing skills of thinking, analyzing and problem solving through classroom discussions, and creating their own word problems based on realistic every day situations. In this unit, strategies are given to help students use a variety of approaches for tackling word problem, organizing data based on information given in a word problem, and to translating word problems into equations using algebraic expressions. This unit on “Using Basic Properties To Solve Problems In Math” is framed around New Haven Standards for teaching word problems to students in grades 5-8.

As a result of using this unit, it is hoped that teachers gain more insight to a logical, and consistent approach to teaching students how to think logically to solve problems in math and everyday life; develop understanding of, and interest in math problem solving skills that promotes the best practice; help students interpret different problems presented in words; encourage students to persist when there is more than one step to a problem and encourage students to explain how they can be sure they have a complete solution to a problem when all possibilities are not given. Also, included in this unit are teacher resources, student reading list and a bibliography.

Definitions of Math Properties

Mathematical properties are avenues to higher-level thinking, because they illustrate general cases and lead to mathematical generalizations. The four usual rules of arithmetic for addition are:

1) Commutative Property states that a + b = b + a

2) Associative Property, (a + b) + c = a + (b + c)

3) Identity Property of 0, 0 + a = a (= a + 0)

4) Inverse Property, for every member a, there is - a, such that a + (- a) = 0.

Similar to the addition, the four rules of arithmetic for multiplication rules can be stated as:

1) Commutative Property: ab = ba

2) Associative: (ab)c = a(bc)

3) Identity: 1.a = a (= a.1)

4) Inverse Property, for every a =/= 0, there is (1/a) (or a to the power of -1), such that a (1/a) = 1. It is important to mention that connecting addition and multiplication is the Distributive Rule: a (b t c) = ab + ac. Often rules are consequences of these: for example, a x 0 = 0, because a = a x 1 = a x (1+ 0) x a x1 + a x 0. Now subtract a from both sides to get 0 = 0 + a x 0 = a x 0.

Commutative Properties of Addition and Multiplication

The Commutative Properties of Addition and Multiplication state that the order of two addends (e.g., (4+2 or 2+4) or two factors (e.g., 4 x 2 or 2 x 4) does not affect the sum or product, respectively. The root word of commutative is commute, which means to interchange. Therefore, we can reverse the order of two addends or two factors without changing the result. Sometimes, the commutative property of multiplication can be especially confusing to students. For example, in an addition operation, the addends represent subgroups consisting of the same things: If 3 + 4 is expressed as 4 + 3, only the order of the subgroups changes. In the grouping interpretation of multiplication, 4 x 5 and 5 x 4 represent different groupings- four groups of 5 does not look the same as five groups of fours. Students need to model many multiplication equations in order to see that products are identical. The commutative property of multiplication is also confusing because switching the factors in some multiplication word problems switches the relationships. Sometimes the switched relationships are similar and still make sense; other times they change the problem completely. A good device for justifying the commutative law is the array model for multiplication. The following word problems below can be used to illustrate the commutative property of addition and multiplication.

1. Sally collected aluminum for two days. On Friday morning she collected 20 cans and Friday night she collected 25 cans. On Saturday morning Sally collected 25 cans but on Saturday night only collected 20. Did she collect more on Friday than Saturday?

This problem demonstrates the commutative property of addition. Changing the order in which Sally collected the cans do not change the result.

Friday Morning/Evening 25 cans + 20 cans

Saturday Morning/Evening 20 cans + 25 cans

2. Royale went to a 20% off everything sale at Sports Authority. She brought some running shoes marked at $89.99 and a jersey marked at $39.99.
a) With 6% sales tax, what will be the total cost?
b) Will it be better to pay the sales tax first, and get the discount on the total bill, or to get the discount first, and only pay sales tax on the discounted price?

In order to find the total cost, first find the price with discount first, then taxes (P is the original price)

= p x (.8) x (1.06)

=$129.98 x .8 x (1.06) = 110.21

The sale cost is $110.21

The price with tax first then discount on everything

= p x (1.06) x (.8).

=129.98 x (1.06) x .8

= 110.21

Answer: No, there is no difference in the total cost. The prices are the same because (p x .8) x 1.06 = (p x 1.06) x .8. This is an excellent instance of the commutativity of multiplication because the order of operation does not affect the product.

The addition and multiplication properties can also be demonstrated by the following example: Consider that 2 + 2 give the same answer as 2 x 2. Although 2 is the only number with this property, there are many pairs of different numbers a and b which can be substituted in the equations above. They may be fractions, but they must have a product which is exactly equal to their sum. For example, there are infinitely many pairs of numbers which have the same sum and product. If one number is a, the other number can always be found simply by dividing a by a-1 because a + b = ab then a = ab-b = (a-1)b. Dividing both side by a-1 assuming this is not zero gives a/a-1 =b. For example: 3 times 1 ½ equals 3 + 1 1/2 (Inverse Properties of Addition and Subtraction).

Source: Sam Loyd, *Mathematical Puzzles of Sam Loyd Selected and Edited by Martin Gardner* , Dover Publications, Inc., New York, 1959, Puzzles 51, p. 53.

Inverse Properties of Addition and Subtraction

Students are told that the order of numbers does not matter in addition, but it does matter for subtraction except when both numbers are identical, for example, 10-10 = 10-10 or 0.

When the two numbers are not the same, such as, 3 and 6, the pair of differences is opposites: one difference is a positive number (e.g., 4) and the other is its negative number (e.g., -4). This can be represented as a-b = c and b-a = -c. If a= b, then a-b = 0 and b- a = 0 = - 0. The pairs of differences sum to zero, such as, +4 + (-4) = 0.

Zero Property of Addition

The zero property of addition states that the sum of any number and 0 is that number: For example: 2 + 0 = 2, 99 + 0 = 99. The additive Inverse a + (-a) = 0, a-a = 0.

Zero Property of Multiplication

The zero property of multiplication states that the product of any number and 0 is 0. For Example: 2 x 0 = 0. This can be shown to be a consequence of the zero property of addition and the distributive rule.

Multiplicative Identity Property

The multiplicative identity property states that the product of any number and one is that number, for example 5 x 1 = 5.

Associative Properties

Addition and multiplication are often referred to as binary operations. We can operate on only two numbers at a time. For example, (2+ 3, 9 x 8). If a computation involves three addends, we first add two of the numbers and then add the third to the previous sum. For example, (2+3) +4 = 2+ (3+4) or (a + b) + c = a + (b + c) for all numbers a, b, and c.

The associative property states that the way in which three or more addends or factors are grouped before being added or multiplied does not affect the sum or product. The property is usually used to simplify calculations when adding numbers. The associative property is also used to regroup compatible numbers in order to simplify the calculations. Compatible numbers refer to friendly numbers whose sums or products are easily to calculate mentally. For example 25 x 4 = 100, and 35 and 65 are compatible because 35 + 65 = 100. In general, numbers that can be combined to form multiples of 10, 30, 100, 200, and 1000 are compatible.

The associative and commutative rules can be combined to show that, if you have a collection of numbers to be added the ways you pair up the numbers to do the addition, and the order in which you do the addition have no effect on the outcome. This is sometimes called the Any Which Way Rule. It is the practical outcome of the Associative and Commutative Rules, and gets used in almost every calculation. Because the Associative and Commutative Rules hold also for multiplication, so does the Any Which Way Rule.

The following word problem demonstrates the associative property of addition:

1a.There is a prize for selling the most tickets to the school play. Alphonz, Bela and Chalfont are the leaders. Alphonz sold 42 the first week, 59 the second week and 78 the third week. Bela sold 59 the first week, and 78 the second week but has a disappointing third week, with only 42 sold. The first week, Chalfont sold 78, the second week he sold 59, and the third week he also sold 42. Who wins the prize?

All three won the prize because all three sold 179 tickets during the three weeks.

Alphonz sold 42 + 59 + 78 = 179

Bela sold 59 + 78 + 42 = 179

Chalfont 78 + 59 + 42 = 179

1b. There is a prize for selling the most tickets to the school play. Alphonz, Bela and Chalfont are the leaders. Alphonz sold 42 the first week, 59 the second week and 78 the third week. Bela sold 60 the first week, and 79 the second week but had a disappointing third week, with only 40 sold. The first week, Chalfont sold 77, the second week he sold 58, and the third week he sold 44. Who wins the prize?

Alphonz sold 42 + 59 + 78 = 179

Bela sold 60 + 79 + 40 = 179

Chalfont 77 + 58 + 44 = 179

This is a more subtle use of the Any Which Way Rule. One does not have to do each calculation separately and compare. One can just see that 60 + 79 + 40 = (59 + 1) + (78 + 1) + (42-2) and recombine.

2. Darcy, Egmont and Finian each have a box. Darcy's box is 12 inches long, 8 inches wide and 5 inches high. Egmont's box is 8 inches wide, but it is a foot deep, and it is also 5 inches high. Finian's box is only 5 inches wide, but it is eight inches wide, and it is the tallest box-a full 12 inches high (these are all inside measurements.) They are arguing about which box will hold the most. Which box has the largest volume?

This problem represents both the commutative and associative properties of multiplication. In this problem we have three boxes. First, we know that Darcy's box is 12 inches long times 8 inches wide x 5 inches high. We can represent this by multiplying the length times the width times the height. We can represent this by V= lwh and substitute numbers for literal expressions to solve. For example V= 12 x 8 x 5 =480 square inches. In Egmont's box we need to change a foot to 12 inches and then solve V= wlh and substitute numbers for literal expressions, V= 8 x 12 x 5 = 480 square inches and Finian's box V = 5 x 8 x12 = 480 square inches. Each box has the same volume because the way in which three or more factors are grouped before being multiplied does not affect the product.

3. On a business trip in Upstate New York, Mr. Floyd stopped several times to buy gas. His car held 12.4 liters when he filled up the first time. At his next gas stop, his car held 22.8 liters. The last time he stopped for gas, the car held 18.6 liters. How many liters of gas did his car use on the trip? Answer: 12.4 + 22.8 + 18.6 = 53.81liters of gas

This can be done mentally if you use the Any Which Way Rule and compute it as (12, 4 + 18.6) + 22.8 = 31 + 27.8 = 53.8

4. The senior class was selling tickets to their play. They had two prices, $5 and $8. The tickets were on sale during the week before the play. On Monday, they sold 23 tickets at $5 and 14 at $8. Tuesday, they sold 31 tickets at $5 and 22 at $8. Wednesday sales were 46 at $5 and 28 at $ 8. Thursday sales were 39 at $5 and 32 $8. Friday, including sales at the door, they sold 22 at $5 and 56 at $8. How much money did they take in ticket sales? You can solve this problem by multiplying the number of tickets sold each day using the two different prices and adding up the total amount of tickets sold for the week.

Ticket Sold Ticket Sold Price $5.00 Ticket Sold Price $8.00

Monday 23 $115 14 $112

Tuesday 31 $155 22 $176

Wednesday 46 $230 28 $224

Thursday 39 $195 32 $256

Friday 22 $110 56 $448

Total Amount 161 $805 152 $1216

This problem is about the Distributive Rule (as well as the Any Which Way Rule). Total sales

= (23 + 31 + 46 + 39 + 22) x 5(70 + 45 + 46) x 5 + (14 +22 + 28 + 32 + 56) x 8 + (50 + 70 + 32) x 8

= 161 x 5 + 152 x 8

= 805 + 1216

= 2021

Distributive Property

The distributive property of multiplication allows you to distribute a factor, a, to two different addends, b and c: a(b +c) = ab + ac. The distributive property is used a great deal when computing mentally. For example, how would you mentally multiply 3 times 58? One approach is to think of 58 as 50 + 8 and use the distributive property in the following manner:3 x 58 -> 3 x (50 + 8) -> (3 x 50) + (3 x 8) -> 150+ 24 -> 174.

The distributive property of multiplication can also be applied to subtraction when b -> c. In this case, since 60 -> than 2, it works like this: 3 x 58 -> 3 x (60-2) -> (3 x 60) - (3 x 2) -> 180- 6 -> 174.

The distributive property is often used in connection with coming up with an easier calculation and then make adjustments. For example to compute 3 x 58, 58 can be rounded to 60 = (3 x 60), but 2 groups of 3 must be subtracted from the total 180 to get back to the actual value. The distributive property may be also applied to division expressions. For example, 132/12, If 132 is written is rewritten as 120 + 12, both can be divided by 12:

132/12 -> (120 + 12) /12 -> (120/12) + (12/12) -> 10 + 1 -> 11

The point about the Distributive Rule is, that it lets you compute the sum of products of a given number with a collection of numbers by first adding the collection, then doing one multiplication rather than having to compute each product and then add. This is used all the time in computing sale tax or discounts.

Properties of whole numbers are used extensively when computing. The particular numbers involved in a calculation determine when it makes sense to use the commutative, associative, or distributive properties or some combination of them. The situation is more complex for division and the distributive properties. (b + c) /a = (b/a) + (c/a) for all numbers except a = 0, but it is not true that a/(b +c) = (a/b) + (a/c). 24/ (4 + 2) = 24/6 = 4, but (24/4) + (24/2) = 6 + 12 = 18. Neither subtraction nor division is commutative or associative.

Any selection of numbers will provide a counter example. For example (11- 6)-3 = 2 but 11- (6-3) = 8 or (48/6) /2 = 4 but 48/ (6/2) =16. Multiplication does distribute over subtraction: a x (b-c) = (a x b) - (a x c) for all a, b, and c or (b-c) x a = (b x a) - (c x a) for all a, b, c.

The following are questions for exhibiting the rules of arithmetic; see if you can identify the properties of each problem.

1. Choose a number between 1 and 10. Add 4 and double the result. Subtract 3, then, multiply by 3. Subtract 5 times one more than the original number. Tell me the answer and I will tell you your original number. (It is ten less than the answer.) How does this work? This problem can be represented by this equation (3 (2 (x + 4) -3) - 5 (x + 1). This problem demonstrates the distributive property, as well as the other Rules, because the distributive property of multiplication allows you to distribute a factor, a, to two different addends, b and c: a(b +c) = ab + ac, such as the problem above.

The calculation described in this problem is 3(2 (4+ x) -3) - 5(x + 1), we can use the Rules to rewrite the expression as follows:

= 3 (2 (4 + x) - 3) - 5(x + 1)

= 3 (8 + 2 x - 3) -5 x - 5

= 3 (2 x + 5) - 5x -5

= 6x + 15 - 5x - 5

= (6 - 5)x + 15 -5

= x + 10

2. Choose a number. Add 2, double the result. Subtract 2, double again. Divide by 4. Subject your origin number. I will tell you the answer. (It is 1.)

This can be justified by manipulations similar to those of the previous problem. The instructions say to do the calculation (2 (2 (x + 2) - 2)) x ¼ - x = E. We can manipulate this using the Rules as follows:

E = 2 (2x + 4 - 2)(¼) - x

= 2 (2x + 2) (1/4) - x

= (4x +4) (1/4) - x

= (x + 1) - x

= 1

3. Ames sells flashlights for $7.95 and batteries for $ 3.95. It offers 10 % discount if you buy them together. One Saturday they sold 17 flashlight/ battery combinations, and on Sunday they sold 19. What was their total value of sales of the flashlight/ battery combinations for the weekend? 17(($3.95 + $7.95) - $1.19) + 19 (($3.95 + $7.95) -$1.19) = 36 x $10.61= $385.56. The total value of sales of the flashlight/battery combination for the weekend is $385.56 and can be solved by using the distributive property in the form

(a+ b) (c + d - e) = ac + ad - ae + bc + bd - be. This calculation uses the Distributive Rule on both factors (the total price with discount, and the number of units sold).

4. Suppose you have an uncle who gives you $500 on each birthday for three years in a row, and that you put it in a savings account. Suppose that the first year, the bank pays 4 % interest, the second year it pays 2% interest, and the third year it pays 6% interest. What is the total amount in your savings account at the end of three years? (Assume you opened the account the first time you got the money and that the birthday presents were the only deposits you made.) You can solve this problem in the following manner:

First Year:

500

x .04

20.00

+ 500.00

$520.00

Second Year:

520 First Year

+500 second year

1020.00

x .02

$20.40

$1020.00

+ $20.40

$1040.40

Third Year:

$1040.40

+$500.00

$1540.40

x .06

$93.4240

+$1540.40

$ 1632.82

The value after 3 years can be computed in stages. It may also be (and should be) represented as a single compound expression:

= ((500. 1.04 + 500). 1.02 + 500) 1.06

= 500.((1.04 + 1) . 1.02 + 1 (1 .06)

= 500.(2.04.1.02 + 1) . 1.06

= 500. (2.0808 + 1) . 1.06

= 500. (3.265648) = 1000 .1.632824

= 1,632.824

= 1,632.82

This problem involves use of all the properties, especially the associative property of multiplication and the distributive rule. The associative property states that the way in which three or more addends or factors are grouped before being added or multiplied does not affect the sum or product.

5. Janelle likes to bake. She has 3 aluminum muffin pans, each of which holds 8 muffins, and 2 cast iron pans which also hold 8 muffins each. She also has two stainless steel muffin pans which hold 12 muffins each. If Janelle fills all her muffins pans at once, how many muffins would that be?

Let: aluminum pans: 3 x 8, cast iron: 2 x 8, stainless steel: 2 x 12- This problem can be solved by using the associative property.

= (3 x 8) + (2 x 8) + (2 x 12) = 24 + 16 + 24 = 64 or

= (5 x 8) + 2 x 12

= 40 + 24

= 64

There would be 64 muffins.

6. Prunella is shopping for party supplies. Plastic tableware costs $2.50 per package. Plastic cups are $3.00 per package, plates are also $3.00, and plastic tablecloths are $3.50 each. Prunella gets two packages of spoons, two packages of forks, one package of knives, three packs of cups, one of plates, and two tablecloths. Everything is subject to 6% tax. How much will the party cost Prunella?

((2 (2 x $2.50) + 2.50 + (3 x 3.00) + 3.00 + (2 x 3.50)

10.00 + 2.50 + 9.00 + 3.00 + 7.00 = $31.50

$31.50 x .06 = $1.89

$31.50 + $1.89 = $33.39

The cost of the party is $ 33.39. (Commutative, Associative and

Distributive Properties) In particular, you use the Distributive Property to compute tax only on the total, rather than on each item separately.

Reviewing Properties

- Commutative Properties states when you add or multiply two numbers or variables, you can change the order without changing the result.
- Associative Properties states when you multiply or add numbers or variables, you can group them in different ways without changing the result.
- Zero Property states when you multiply a number or variable by 0, the result is always 0.
- The Identity Properties for Addition states when you add 0 to a number, the result is the same number or variable.
- The Identity Property for Multiplication states when you multiply a number or variable by 1, the result is the same number or variable.
- Inverse Property for Multiplication. The product of reciprocals is 1
- Distributive Property states when you multiply two addends by a factor, the answer is the same as if you multiply each addend by the factor and then add the products.

Rules for Properties of the Real Numbers

Commutative Property for Addition a + b = b + a 100 + 8 = 8 + 100

Commutative Property of Multiplication a x b = b x a 100 x 8 = 8 x 100

Associative Property of Addition a + (b + c) = (a + b) + c (2 + 10) + 6 = 2 +(10= 6 = 18

Associative Property of Multiplication a x (b x c) = (a x b) x c 2 x (10 x 6) = (2 x 10) x 6 = 120

Identity Property for Addition a + 0 = a 5 + 0 =5

Additive Inverse a + (-a) = 0 6 + - 6 = 0

Identity Property for Multiplication a x 1 = 1 x a = a 5 x 1 = 1 x 5 = 5

Inverse Property of Multiplication 1/a, a x 1/a =1/a x a =1 [a does not = 0] 1 x 1/5 = 1/5 x 5

Distributive Property a (b + c) = a x b + a x c 2 (3 + 4) = 2 x 3 + 2 x 4

Lesson Plan I

Content Standard 1.0: Number Concepts, Arithmetic, and Operation Concepts: Students will work comfortably and confidently within the real number system and its operations. They will make comparisons, recognize patterns, and use multiple representations of the same number.

Performance Standard 1.3

Student will be familiar with characteristic of operation and numbers and properties of rational numbers. Students will recognize applications of commutativity and associativity.

Objective 1: To have students rewrite the following problems by using the commutative or associative properties

Use the Any Which Way Rule for addition or multiplication to do these calculations mentally.

For example, 2 + 57 + 28 = 57 + 30 = 87.

2 + 57 + 28, 23 + 9 + 7 + 41

422 + 36 + 24 = 38(50 (47) (2)

97 x 125 x 8 (25) (41) (4) (2)

Strategies:

1. Have students use an example to explain why the order of adding or multiplying two numbers does not change the result.

2. Use an example to explain why grouping numbers in different ways does not change the result of addition or multiplication.

3. Give justification of the commutative Rule of Multiplication using the array model

4. Have students explain why the Commutative Property does not apply to subtraction or division.

Objective 2: Explain and define which property is illustrated.

6+5 = 5+ 6

9 + (4 + x) = (9 + 4) + x

(7 x 2) x 6 = 7 x (2 x 6)

Lesson Plan II

Content Standard 1.0 Number Concepts, Arithmetic, and Operation Concepts: Students will work comfortably and confidently within the real number system and its operations. They will make comparisons, recognize patterns, and use multiple representations of the same number.

Learning Multiplication Facts

Content Standard 5.0: Problem Solving and Mathematics Reasoning: Problem solving concepts and strategies lie at their heart of mathematics. Students will use them in the formulation of problems. They will appropriately test problem conclusions against conditions. They will reason mathematically.

Performance Standard 5.3: Problem Implementation

Students will invoke problem solving strategies, such as illustrating with sense-making sketches to clarify situations or organizing information in a table. Students will integrate concepts and techniques from different areas of mathematics and will justify logical statements.

Understanding the relation between addition and multiplication makes it possible for students to relate the learning of multiplication facts to their knowledge of addition.

Objective 1: To engage students in a discussion of the relation between 3 and 4. The following examples of true or false number sentences can be used to engage students in a discussion about their relation.

3 x 7 = 7 + 7 + 7

3 x 7 = 14 + 7

4 x 6 = 12 + 12

Is it true that 4,287 + 3, 156 = 4, 285 + 3, 158?

Objective 2: To engage students in learning number facts and to learn patterns that makes the number facts easier. Have students to solve the following problems by carrying out the calculations on each side of the equal sign.

3 x 8 = 2 x 8 + 8

6 x 7 = 5 x 7 + 7

8 x 6 = 8 x 5 + 6

7 x 6 = 7 x 5 + 7

9 x 7 = 10 x 7 -7

- Denotes a false number sentence. The above source of information is taken from: Thomas Carpenter, Megan Loef Franke and Linda Levi, Thinking Mathematically Integrating Arithmetic & Algebra in Elementary School, Heinemann, Portmouth, NH, 2003, p 39.

Lesson Plan III

Distributive Property

Content Standard 7.0 Mathematical Communication: Students will demonstrate their ability to communicate, both orally and in writing, mathematical ideas and mathematical applications to ordinary situations.

Performance Standard 7.1: Students will use mathematical language and representations with appropriate accuracy, including numerical tables and equations, simple algebraic equations and formulas, charts, graphs and diagrams to make complex situations easier to understand.

Strategies: Have students explain whether you can apply the Distributive Property to division and subtraction. You can draw a rectangular array to show how to find a product. A rectangular is 5 unit wide and 16 units long. You can use simple multiplication facts to find the area of the rectangle.

(image available in print form)

What multiplication expression describes the area of the rectangle? 6 x 16

Would it be easier to find the area of the rectangle if you divided it into two pairs? Explain! The diagram shows one way to divide the rectangle. Divide your rectangle and shade each part 6 x (10 + 6)

(image available in print form)

How does the diagram show 6 x 16?

How does it show (6 x 10) + (6 x 6)?

How does it show 6 x (10 + 6)?

Find the area of the rectangle. Use the Distributive Property.

Area = 6 x 16 =56 + 36

= (6 x 10) + (6 x 6) = 96

New Haven Math Standards for Grades 5-8

Related Math Problem Solving

Appendix A: Using Basic Properties To Solve Problems In Math

This Unit addresses the following Content Standards and Performance Standards:

Content Standard 1.0 Number Concepts, Arithmetic, and Operation Concepts: Students will work comfortably and confidently within the real number system and its operations. They will make comparisons, recognize patterns, and use multiple representations of the same number.

Performance Standard 1.1

Students will use the inverse operation to determine unknown quantities in equations.

Performance Standard 1.3

Student will be familiar with characteristic of operation and numbers and properties of rational numbers. Students will recognize applications of commutativity and associativity.

Content Standard 5.0: Problem Solving and Mathematics Reasoning: Problem solving concepts and strategies lie at their heart of mathematics. Students will use them in the formulation of problems. They will appropriately test problem conclusions against conditions. They will reason mathematically.

Performance Standard 5.3: Problem Implementation

Students will invoke problem solving strategies, such as illustrating with sense-making sketches to clarify situations or organizing information in a table. Students will integrate concepts and techniques from different areas of mathematics and will justify logical statements.

Content Standard 7.0 Mathematical Communication: Students will demonstrate their ability to communicate, both orally and in writing, mathematical ideas and mathematical applications to ordinary situations.

Performance Standard 7.1: Students will use mathematical language and representations with appropriate accuracy, including numerical tables and equations, simple algebraic equations and formulas, charts, graphs and diagrams to make complex situations easier to understand.

Appendix B Glossary of Math Terms for Basic Properties

1. Additive Inverse-For a given number, the number that can be added to give a sum of 0. Example: -4 is the additive inverse of + because -4 + 4 = 0, also, known as the negative of the number.

2. Associative Property for Addition and Multiplication-Rule stating that the grouping of addends or factors does not affect the sum or product. Examples:

(3 + 6) + 9 = 3 + (6 + 9)

(2 x 4) x 7 = 2 x (4 x 7)

3. Commutative Property for Addition and Multiplication- Rule stating that the order of addends or factors has no effect on the sum or product

Examples: 3 + 9 = 9 + 3 and 4 x 7 = 7 x 4

4. Distributive Property for Multiplication over Addition- Rule stating that when the sum of two or more addends is multiplied by another number, each addend must be multiplied separately, and the products added together.

5. Identity Property for Addition- Zero is the identity property for addition, because any number plus zero is equal that number. Example: 3 + 0 = 3

6. Identity Property for Multiplication- One is the identity property for multiplication, because any number multiplied by 1 equals that number. Example: 17 x 1 = 17

7. Inverse-Opposite. Addition and subtraction are operations. Multiplication is the inverse of division.

8. Property of 0- A property which states that for any integer plus zero, the sum is that integer.

9. Property of 1- A property which states that any number multiplied by 1 will equal the number.

Reading List

Mathematical Puzzles of Sam Loyd (Selected and edited by M. Gardner) Dover Publications, New York, 1959. Martin Gardner offers a selection of the best mathematical puzzles of Sam Loyd, taken from the long out-of-print “Cyclopedia.” Many of Loyd’s most famous puzzles are included in this book such as “Horse of a Different Color which sold millions.

Singapore Primary Mathematics Text 6A, 6B, U.S. Edition, Curriculum Planning and Development Division, Ministry of Education, Singapore Federal Publications. The main features of this book are the use of the Concrete -Pictorial-Abstract approach. The book encourages active thinking processes, communication of mathematical ideas and problem solving. It consists of six units which are divided in parts. Each part starts with a meaningful task, practice and review designed to help students.

Knowing and Teaching Elementary Mathematics , by Liping MA Lawrence Erlbaum Associates, Mahwah, NJ, 1999. This book describes the nature and development of the "profound understanding of fundamental mathematics" that elementary teachers need to become accomplished mathematics teachers, and suggests why such teaching knowledge is much more common in China than in the United States, despite the fact that Chinese teachers have less formal education than U.S. teachers.

Teaching Problems and the Problems of Teaching , by M. Lampert, Yale University Press, New Haven, CT, 2001. This book is about teaching with problems and the problem in teaching. It identifies problems that must be addressed in teaching.

Thinking Mathematically, by T. Carpenter , M.L. Franke, and L.Levi Heinemann, Portsmouth, NH, 2003. In this book, the authors revealed how children's developing knowledge of the powerful unifying ideas of mathematics can deepen their understanding of arithmetic and provide a solid foundation for learning algebra. It provides numerous examples of classroom dialogues that indicate how algebraic ideas emerge in children's thinking and what problems and questions help to elicit them.

Houghton Mifflin Mathematics , Houghton Mifflin Co. Boston MA. 2002. This is a traditional textbook in math that consists of a problem solving plan that can help students to become better problem solvers. There are many strategies that can be used to solve problems.

Math 4 today: 10-Minute Daily Skills Book , McGrow-Hill Children's Publishing, Good Apple 1998. *Math 4 Today* is a comprehensive easy to use supplement to any fourth through fifth-grade math curriculum. In only ten minutes each day, several essential skills are reviewed during a four-day period and evaluated on the fifth day. The book is designed on a continuous spiral so that concepts are repeated weekly. Complete answer keys are provided for 54 weeks worth of reproducible worksheets. Great practice for standardized tests!

Teacher Resources

Mildred Johnson, et al: McGraw Hill, 1999. This popular study guide shows students easy ways to solve what they struggle with most in algebra: word problems. *How to Solve Word Problems in Algebra* is ideal for anyone who wants to master these skills. With this easy-to-use pocket guide, solving word problems in algebra becomes almost fun. The anxiety-quelling guide helps you get ready for the most difficult word problems, one step at a time. With fully explained examples, it shows you how easy it can be to translate word problems into solvable algebraic formulas.

Oswego City School District Regents Exams Prep Center, 2003. Lists and explains all the major properties of real numbers, including the identity, inverse, and closure properties.

Elizabeth Stapel: Purplemath.com, 2003. Discusses the simplification of expressions via the distributive, commutative, and associates properties.

Bibliography

Carpenter, Thomas P., Franke, Megan L. and Levi, Linda, *Mathematically Integrating Arithmetic & Algebra in Elementary School,* Heinemann, Portmouth, NH, 2003, p 39.

Chapin, Suzanne H. and Johnson, Art, *Math Matters: Understanding the Math You Teach* . Math Solutions Publications, Sausalito, Calif., 2000.

Curriculum Planning and Development Division, Ministry of Education *Primary Mathematics Text 6A,* U.S. Edition, Singapore Federal Publications, 1984.

Forte, Imogene, Frank, Marjorie, Breeden, Terri and Sukow, Andrea, *The Basic Not Boring Wole Numbers & Integers Grades 6-8 Math Skills Middle School* , Incentive Publications, Nashville, TN, 1997.

Forte, Imogene and Frank, *The Basic Not Boring Problem Solving Grades 6-8 Math Skills Middle School* , Incentive Publications, Nashville, TN, 1997.

Gardner, Martin, *Mathematical Puzzles of Sam Loyd* , Dover Publications, Inc., New York, 1959.

Houghton Mifflin Mathematics , Houghton Mifflin Co. Boston MA. 2002

Lampert, Magdalene, *Teaching Problems and the Problems of Teaching* ,Yale University Press, New Haven, CT, 2001.

Ma, Liping, *Knowing and Teaching Elementary Mathematics* , Lawrence Erlbaum Associates, Mahwah, NJ, 1999.

Pearson, Donna, *Math 4 today:10-Minute Daily Skills Book* , McGraw-Hill Children's Publishing, Good Apple 1998.