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Word Problems Dealing with Ratio and Proportion

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Objective

Ratio and proportion are central ideas in middle school mathematics that can present considerable challenges to many students, but like many mathematical concepts, they can not be understood in isolation. Consequently, there are other aspects of middle school mathematics that can best be served with a clear and concise understanding of ratio and proportion. Some examples from the middle school curricula are as follows: fractions, percents, statistical analysis, investigating patterns in numbers and shape in geometry, divisibility, factoring, and even algebra, especially questions involving rates in travel, interest, taxes, and cooking.

Word problems provide an excellent opportunity to exercise many necessary skills while simultaneously learning ratio and proportion. In particular, there is a specific grammar and form to the word problem that must be mastered in order to do well on standardized tests. These tests seem to require more than a passing familiarity with word problems. It therefore behooves the middle school math teacher to give as many opportunities as possible for the students to not only get familiar with word problems, but to become as comfortable as possible with them. Consequently, what follows are a number of word problems that deal with ratio and proportion, a list of strategies that have been employed by students to reach the answer, necessary tools that I require of my students, a sample classroom procedure, and a list of resources that can provide further access to word problems.

Introduction

“On his mother’s birthday, Juan has cooked dinner for his mother and some guests. He made a huge pot of rice with 4 cups of rice and 8 cups of water. He made two *pernil* (pork shoulders), and baked 3 cakes for dessert . . .”

What follows the word problem above (and all word problems) is that one **final** sentence that causes a great deal of agitation in my students. That final sentence causes a major amount of fear, dread, and even loathing, but it is only until students resolve within themselves the sheer inevitability of it that they can then begin to attempt to understand what the word problem has in store. The final sentence is the sentence by which students are informed of their objective. It communicates what students will be required to find, the eventual prize, that ultimate solution to the jumble of words, symbols, letters, and numbers. Despite the fact that the dynamics of word problems are simple, it is within the language of the words that complexity is found.

The students know that they are reading a problem, and that this problem will require a solution. However, there is a real barrier that forms between the words, the numbers, and the requisite symbols in a word problem. The reality of the situation that I as a bilingual teacher try to convey is that it matters not whether one understands completely the words, but the substance of what is required. The words are mere tools, apparatus, vehicles, if you will. These vehicles of meaning are meant to deliver ideas, and sometimes the vehicles can be sleek, awesome crafts that convey information succinctly, while other times the vehicles can be ancient, rundown jalopies that convey very little. What is the ultimate objective of these problems is to help students acquire the ability to parse meaning from the problem in such a way as to configure correctly the comparisons being made. That is what a ratio does.

Ratios are comparisons between numbers by way of division. They do what math in a more general way aspires to accomplish. Ratios have as their ultimate objective the discovery of relationships between numbers, the asking of “How does this number relate to that number?” or “How does this procedure relate to that procedure?” or “If I follow a certain list of strictures in one type of problem will I need to follow consistently the same list of strictures for these other types of problems.” What makes math fascinating is that sometimes the connections between numbers are concrete and overt, such as the comparison of cups of rice with cups of water in the problem above, and sometimes the connections are more tenuous, like the cooking of two *pernil* compared to the cooking of one pot of rice, and three cakes. Whatever the case, these comparisons whether subtle or obvious can be tremendously fruitful in discussions about establishing relationships between numbers.

Again using the example above, students are informed that there are various parts of a meal prepared by someone for his mother and her guests. These various parts are all related because they are all prepared by one person for several people as one meal. If we were to illustrate the possible comparisons through mathematical means, a few examples would include, but not be limited by:

1:1 - to signify 1 pot of rice for 1 meal,,

1:2 - to signify 1 pot of rice or 1 meal compared to 2 *pernil,*

1:3 - to signify 1 pot of rice compared to 3 cakes,,

1:2:3 - to signify 1 pot of rice and 2 *pernil* compared to 3 cakes,

3:2:1:1 to compare every facet of the meal.

Although there are numerous ratios that can be explored by far the most significant ratio is 1:2 for cups of rice to cups of water, because this is the way you almost always cook rice. The major point of comparison is to illustrate that if there are more or less people attending the dinner, then in order to feed the people attending there will have to be more or less food cooked in similar proportions. Which brings us to another word that is exceedingly important when dealing with ratio word problems.

A proportion is an equality of two ratios. Proportions as ratios hinge on an adequate understanding of division and equivalent fractions. They depend on students understanding that 1/2 = 2/4 = 3/6.

This is a major key. It is a major key because once students acknowledge an understanding of this concept; they **always** will know it and can refer back to it, even if sometimes they need reminding. As a consequence, whenever dealing with ratio word problems that compare one thing to another (proportions), students know that they have to approach the problem as an equation where one ratio equals another ratio. This equals that.

For example, if students were to further consider the word problem above, they can clearly note the ratio that has been purposely set up by the wording of how the one pot of rice is made by four cups of rice and eight cups of water (4/8).Now, if Juan's mother decides to double the amount of people that were invited to the dinner, then Juan knows that he has to cook twice as much food. This sets up a proportion in which Juan needs to realize that in order to cook enough rice in that one bowl he will need to create an equation 4/8 = 8/16. If to cook for x number of people, Juan needs 4 cups of rice and 8 cups of water then, for 2x number of people Juan needs to multiply both the numerator and the denominator by 2. If the students can realize the veracity of the equation then they will have no difficulty when it comes to algebraic equations that propose relationships between specific sets of data.

Ratio and Proportion Problems with Strategies

The following problems are organized into four distinct types of problems dealing with ratio and proportion calculations. The first type of problems will be standard problems that deal with understanding ratios and proportions as equivalent fractions. The second type of problems will be compound ratios which will involve calculating several ratios in succession. The third type will involve doing addition or subtraction in order to find a number for computing a ratio which will be called computational ratios. Finally, there will be a few problems that are complex ratios, which will involve developing some complex relationship between proportions. One reason for organizing the problems in this way is to illustrate an orderly progression in complexity, while underscoring similarity of structure. Of course, there are more problems in the first category than there are in subsequent categories. These will serve as a basis to help the students become familiar with and later recognize structure and format.

Standard Ratio Problems

1. "Bobby has a bag full of marbles that he keeps in his room. He has 35 red marbles and 25 green marbles. Find the ratio of red marbles to green marbles, and put it in its simplest form."

This is a relatively simple ratio word problem that includes the slight wrinkle of including the aspect of fractions that students seem to universally dislike - simplest form. It begins with the relatively harmless ratio of 35/25. The difficulty in this problem is such that there are fraction processes that will be involved like greatest common factor, factoring, and/or the rules of divisibility. Students should have a familiarity with these processes before tackling this type of problem, but even if they do not there is much that they can do with this problem.

Not all students will immediately note that the number five is a common factor in both the numerator and the denominator. In fact most of my students would insist on factoring both the numerator and the denominator to make certain that five is the greatest common factor.

Factors of 35 = 5 x 7
Factors of 25 = 5 x 5

However the students choose to get the result does not really matter, what does matter is that students should have various means for finding simplest form, eventually, dividing the denominator and the numerator by five in order to get: simplest form. Here's another.

2. "To make 20 biscuits, Juanita uses 5 cups of flour to 1 cup of milk. If she uses 3 cups of milk, how many cups of flour will she use?

This is another pretty straightforward problem which sets up a proportion pretty nicely. There is an added bonus that some extraneous information is given which lends itself to the asking of a series of follow-up questions.

5/1 = x/3
15 = x

Some follow-up questions might include, "If we know that 5 cups of flour make 20 biscuits, can we figure out how many biscuits are made with 15 cups of flour?" Most middle school students can figure pretty quickly that 15 cups are 3 times as much as 5 cups which means that they will have to multiply 20 by 3 to get 60. By playing around with word problems in this way, students begin to realize that once you understand the idea of equivalency, they have won half the battle when it comes to proportions.

3. "When 2,000 pounds of paper are recycled or reused, 17 trees are saved. How many trees are saved if 5,000 pounds of paper is recycled? How many trees are saved if 10,000 pounds of paper is recycled?"

This is another standard ratio question that lends itself to being a series of questions, and in fact, I would have my students strategize as to which of the two questions is easier to answer. Some of them might recognize that the second question is an easier question to answer, and that when that question is answered, it will help to resolve the first question.

2000/17 = 10000/x

10,000 is 5 times more than 2,000; therefore x must be 5 times more than 17 or 85.

5000/x = 10000/85

5,000 is half of 10,000; therefore x must be half of 85 or 42.5.

4. “If it costs $90 to feed a family of 3 for one week, how much will it cost to feed a family of 5 for one week? How much will it cost to feed a family of six, seven, and eight?

The ratio again is pretty easy to set up. $90/3 = $x/5. However, I think that I would have my students approach this problem a little different. I would want them to see that 90 is divisible by three, and that once I know how much it costs to feed each person in the family ($30), then whenever I’m asked how much would it cost to feed y number of people in a particular family, I would only need to multiply y by $30.

More Standard Ratio and Proportion Problems

5. “In a school there are four boy scouts to every three girl scouts. If there are forty-two girl scouts, how many boy scouts are there? If there are 81 girl scouts, how many boy scouts are there?”

6. “To make green paint, a painter mixes yellow paint and blue paint in the ratio of three to two. If he used twelve gallons of yellow paint, how much blue paint did he use?”

7. “A rectangle measures 40 cm at its length and 20 cm at its width. Find the ratio of the length to the width in lowest form.”

8. “When a robin flies, it beats its wings an average of 23 times in ten seconds. How many times will it beat its wings in two minutes?”

9. “Inez is 5’4” tall. At a certain time of day, she measures her shadow, and finds it is 8’ long. She also measures the shadow of a tree which is 40’. How tall is the tree?”

10. “The average human heart beats at 72 beats per minute. How many times does it beat in 15 seconds? How many in an hour? How many in a day? How many in a year? Take your pulse and record the number of beats in 30 seconds. How many times does it beat in an hour? How many times does it beat in a day, a month, a year?”

Compound Ratios

11. “On a triangle, each side measures 5 cm, 10 cm, and 30 cm, respectively. In lowest terms, find the ratios of the lengths of the sides.”

In this word problem, there are three numbers that must be placed in a ratio. That will look like this:

5:10:30

Immediately, it should be noted that the numbers are all divisible by five and that the ratio can better be expressed in its lowest form of 1:2:6.

12. “The church is going on a trip to Niagara Falls via several buses. The ratio of men to women to children is 1:2:3. If there are 120 people going on the trip, how many men are going? How many women are going? How many children are going?”

There are a variety of strategies that can be implemented. One can show that by adding up the initial ratio one can decipher that out of every 6 people, three will be children, two will be women, and one will be a man. Then one can divide 120 by 6 which will show that there are twenty groups of 6 people. Lastly, we multiply by 20 each part of the original ratio. Consequently, 20 men, 40 women, and 60 children went on the trip to Niagara Falls.

13. “Kim mixed seltzer, fruit punch concentrate, and ginger ale in the ratio of 2:2:4 to make a special drink for her friend’s birthday party. To make three gallons, how much of each ingredient should Kim use?”

The complexity of this problem, I believe solely revolves around converting gallons to cups. Generally, I give the students a helpful hint in which I might let them know that they will have to convert gallons to cups and that there are 16 cups in a gallon. Once I have given the students that information, it usually puts them on the right track.

1 gallon = 16 cups
3 gallons = 48 cups

2:2:4 means that out of every 8 cups, two of them are seltzer, two of them are fruit punch concentrate, and four of them are ginger ale. 48/8 = 6. That means that I now multiply each part of the ratio with 6 to ascertain how many cups of each ingredient Kim will need in order to make three gallons of punch. Kim will need 12 cups of seltzer, 12 cups of fruit punch concentrate, and 24 cups of ginger ale.

Computational Ratios and Proportions

Computational ratio and proportion problems are problems that require more than just dividing, applying rules of divisibility, or seeking out equivalent fractions. They require addition, subtraction, multiplication, and/or division. As a consequence, these will require a little more thought and strategizing on the part of students to correctly identify the methods that they will need to figure out these word problems.

14. “A rectangle measures 20cm at its length and 5 cm at its width. Find the ratio of the length to the width to the perimeter of the rectangle in lowest form.”

The uniqueness of this word problem revolves around the fact that in this ratio there will be three numbers in the comparison as well as figuring out perimeter, and the fraction processes of lowest terms. The formula of perimeter requires addition of all sides.

P=2 (length) + 2 (width)
P= 2(20) + 2(5)
P=50 cm2
20:5:50

The number five appears to be the common factor. In order to make the ratio in its lowest terms we need to divide all of the numbers by five. The answer will be: 4:1:10.

15. “David received $50 dollars for working at his father’s store. He spent $20 dollars at the movies, and $10 dollars buying comic books. After buying $5 worth of candy, he saved the rest. What is the ratio of the amount of money David spent buying comic books and candy, to the amount that David saved?”

Again we have a ratio problem that looks pretty simple on the face of it, but one might be surprised to find how many students fall into the subtle snare that this word problem involves. There are several amounts of money communicated and some that are not made explicit. For instance, students are told how much money David earned, but they are not told how much money he saved. The problem therefore requires several steps on the part of students. Most students will try to cut as many steps out of the equation in the attempt to make it easier for themselves and as a consequence they will fall easily into the trap.

The key to this word problem is found in the fact that the ratio that is required does not involve all of the money spent. The final sentence only asks for a ratio that involves only the money that David spent buying comic books and candy, not the money he spent at the movies. Many students in order to make the problem easier for them will make the incorrect assumption that they will have to add up all the money David spent. What is all the more interesting is that they do not need to add up all the numbers for the first part of this ratio, but they will need to do it to figure out how much David saved. The mathematical procedures should look something like this

> Comic books + Candy = 1st Ratio
> Movies + Comic books + Candy = Amount Spent
> Amount Earned - Amount Spent = Amount Saved (2nd Ratio)
> Answer = 15:15

This type of problem will serve to underscore that students will need to really pay attention to all aspects of the problem. On standardized tests word problems tend to have distractors. These multi-step problems will help students develop strategies to weather the multi-step problems found on standardized tests.

16. “78 children attended a trip to Six Flags. 60 of them were boys. Find the ratio of the number of boys to girls, and express the number of boys as a fraction in lowest terms to the number of girls.”

Many of these types of word problems involve a tremendous amount of verbiage that serve to distract and confuse students. As a consequence many students tend to either look for the numbers and then to blindly do some calculation that they hope will net them the answer or they just skip over those problems that have a lot of words. This problem is of the type that most children will want to skip but it is really not a difficult problem that requires much calculation.

78 children on a trip, 60 of them are boys, 18 obviously are girls. The ratio of boys to girls is 60/18. Both numbers are even, therefore a common factor is 2. Divide both numerator and denominator by two. 60/18 =

30/9. I again see that both numbers have a common factor, but this time that common factor is three. 30/9 = 10/3. The answer is that for every ten boys that went on the trip, there are three girls that went on the trip.

More Computational Ratio and Proportion Problems

17. “In the United States there is one car for every 1.7 people. How many car tires (on cars) per person are there?”

18. “One out of three students in the school owns a dog. Of these students one out of two owns a cat. What fraction of the students owns both a cat and a dog?”

Complex Problems Involving Ratios and Proportions

In these problems, there are some complex relationships that need to be discovered. There are few students that I find have the patience to really grasp these problems, however, if the students can get through one or two of these problems, I believe that they will be amply prepared for any standardized test.

19. “There are two sisters named Mary and Sue who need to buy a present for their mother’s birthday. The perfume there mother likes is $50 dollars a bottle. Mary is the older sister and she gets more allowance money than her little sister, so they have decided that Mary will give $3 for every $2 dollars her sister gives. For every $15 Mary (the older sister) gives, how much does the younger sister give?”

In order to work with this problem an initial ratio must be the first thing ascertained. This is clearly elucidated by the statement declaring that Mary gives $3 for every $2 given by Sue. Mathematically it would look something like this:

Mary 3 $15

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Sue 2 $x

My students (as well as most middle school students) tend to be creatures of habit and because this is the way they learned and understood ratio, proportion, and percent, upon seeing the problem formatted this way, their minds instantly recall what they learned about proportions and cross multiplication. They become so focused on finding an answer that they hardly recognize the variable as an algebraic entity. After a very short time my students can figure this out easily.

Because my students are bilingual students, I tend to do quite a few word problems as a way of preparing them for the language of standardized tests. My sense is that because students do not do enough word problems in class, when they are confronted by word problems on standardized exams like the CMT, they get “brain lock”, the equivalent of stage fright to an actor. If, somehow we teachers could find creative ways to include word problems in every unit, as well as the usual strictly numerical problems, students would be better prepared for these exams. The answer to this problem is

x = $10.

An added benefit to these word problems is that these problems as stated above can be further complicated by adding a series of follow-up questions to the first question. For instance, how much of the $50 dollars did

Sue pay? How much of the $50 dollars did Mary pay? If they had a little brother named Harry who gave them $5 dollars for their mother's present, how much would each have to put in? These questions make each word problem unique and can be a subject of profound mathematical discussion.

20. "Guillermo drives between Boston to Pittsburgh, a distance of 600 miles. From Boston to Pittsburgh, he averages 50 miles per hour. On the return trip, he averages 60 miles per hour. How long does the trip take?"

Guillermo must drive each way 600 miles. 600/50 = x hours (the trip to Pittsburgh). 600/60 = y hours (the return trip to Boston). Add the number of hours it took to go in both directions (x + y = ? hours). Once the equation is set up, the rest is rather pedantic. 12 hours one way, 10 hours the other way, 22 hours all told. The same problem can be told a couple of ways.

21. "Guillermo drives between Boston to Pittsburgh, a distance of 600 miles. From Boston to Pittsburgh, he averages 50 miles per hour. He is able to go faster on the return trip, and the total trip takes only 22 hours. What was his average speed on the way back?"

22. "Guillermo drives between Boston to Pittsburgh. From Boston to Pittsburgh, he averages 50 miles per hour. On the return trip, he averages 60 miles per hour. His driving time was 22 hours. How far is it from Boston to Pittsburgh?"

Narrative

Many of my Middle School students have a limited understanding as to how vastly important Mathematics is to their lives. They have yet to recognize the importance and the necessity of learning math for the "long run," and how it can help them for the "future." They have yet to comprehend that math is not so much a subject one learns in school as it is a life skill that they will call upon every day of their lives. Oh, they can **repeat** by rote what previous teachers might have told them about computation and other forms of mathematics, and even how they know that math is beneficial because teachers have communicated those ideas, but they generally have not seen the benefit for themselves.

They do not see that throughout their lives they will find themselves in situations where they will be called upon to solve a problem and that they will make the attempt at solving it without even considering that they are "doing math." They do not recognize that every time they go shopping there are some really practical applications of mathematics that are occurring. Obviously, addition, subtraction, multiplication, and division are happening, but there are ratios, percents, proportions, integers, real numbers, and fractions that are also part of the practice, especially when the individual is accessing whether or not to buy an item that is on sale for 25% off. Each of these aspects of math is part of the Middle School curricula.

The problem is clear. There needs to be a conscious effort made at making math not only more accessible as a life skill, but more practical to their thinking. I am not saying that educators need to make math more fun by utilizing and implementing newer strategies for teaching math by using multi-colored manipulatives and other *nouveau* devices, although that is a custom that should be a part of any math teachers bag of tricks, however **something** radical must be done. We need to illustrate the fact that math is a vital part of their lives that is utilized on a consistent basis, not only during the most obvious times, but also in the innocuous times when math is not quite so overt, such as when we weigh ourselves, when we calculate calories, and when we watch television.

Popular shows like *American Idol, TRL* (*Total Request Live*) *,* and *Who Wants to Be a Millionaire* all require some knowledge of Mathematics. Every sport: baseball, football, basketball, and soccer require math as a vital aspect of function. Each of these television programs can be used as a form of enrichment. What student is going to balk at watching television for homework! The math educator must ask him/herself how to keep math away from the impractical, while at the same time conveying theoretical concepts that gear more toward teaching method, less toward busy work, and the conveying of ideas that do not work because they make math complicated, boring, and tedious. Television as math is just one of the methods that can be used to make math less boring.

I freely admit that math **can** be tedious, but it is tedious because by its very nature, it is a process of building a skill. One can not just do something once well, and then make the assumption that somehow in the doing one has acquired the full range of skills that one will need to employ in order to duplicate the feat. Repetition is required. Alternative values and situations need to be postulated. Variations need to be addressed. Distinctions in situation need to be made. Accordingly, one requires repetition. That is how process is typically taught, and hence learned.

The more repetition one employs the quicker the skill is learned, but it is learned with the unfortunate side effect that eviscerates the enthusiasm that students have for learning the skill. With every similar problem tedium sets in, and the avid desire our students have for learning is sapped. Where can be found the happy medium between building mathematical life skills, while simultaneously keeping the meaning and theoretical aspect in such a way as to promote their importance to the everyday vicissitudes of life? What tool can be utilized that can set math squarely in the practical, while striving to educate in the realm of the theoretical? More importantly, how do we “do math,” and still remain “real?”

Problem Solving seems to be a catchphrase that has been proposed as a method of resolving the seeming dichotomy that exists between trying to make math relevant, and the teaching of theoretical concepts that will provide the apparatus that allows a person to figure out the hows and whys of math. Although I agree with the proposition that Problem Solving can bridge the chasm between the real and the theoretical aspects of math, I think we need to go a step further. Before one can promote problem solving as the end all and be all for bridging the dichotomy between theory and praxis one must teach a common language that will help the individual student recognize the process that must be followed to ascertain the result of a given problem. In providing students with a common math-speak, the teacher is providing the **language** necessary for students to identify **what** they are doing, and **why** they are doing it.

I believe that the gap between theory and praxis can only be united by the use of a common terminology that is exercised, and then verbalized repeatedly. Although the proposition seems pedantic, it is a far more complicated prospect. The reason for this is that middle school students are just beginning to become comfortable with the grammar of language in itself, let alone with the academic aspects of mathematical vocabulary. Some other challenges are obvious. Some students have difficulty reading. Other students lack the confidence necessary for tackling math problems directly. Still others see little value in learning algebra, geometry, and trigonometry. Not to mention the fact that many of our students in New Haven have the added burden of being students that speak English in school, while speaking an alternative language at home. In my particular situation, as a teacher of Bilingual Latino students, I am often called upon to spend such a great deal of time teaching a sufficient amount of English to get to the basics of a problem that math instruction suffers. Yet with all these seemingly unassailable difficulties, I still believe that word problems are the most valuable asset to the instruction of a middle school student. Here is why.

Word problems are not just a contrivance that can reinforce the language of mathematics. It is by the oral working through of word problems that students fortify their language skills, verbalize process and theory, and only then put into practice the necessary computations required to achieve a specified result. Word problems train the student to be aware of the specific grammar of word problems. There is not only a specific word usage but also number usage. However, there is one further requirement. In order to do math in the precise way I will suggest, one must employ specific procedures that will assist the student in this practice of math. Further, the student will need certain materials that will be found useful both by the teacher and the student.

Necessary Tools

Notebook - Every student is required to have a notebook just for math. It can be of any type, although I personally prefer a spiral bound quadrille notebook. (It helps students with a sloppy handwriting, especially if they will be copying and working on word problems.) Much of the math done in middle school will require graphing and it is more helpful to have a quadrille notebook as opposed to having to handout pieces of graph paper. Another important aspect of middle school math is that the student will be required to solve multi-step equations, fractions, and ratios. All look much neater when compartmentalized by a quadrille notebook. Also, the notebook can be usable as a log book which can be used to record strategies employed, reflections and the like. Lastly, a quadrille notebook helps students separate distinct parts of multi-step problems which will facilitate clarity of thinking by drawing lines to separate each step of an equation or problem.

Pencils - Many times, I find that my students vest themselves so much in having found an answer that they are reluctant to change it, even when it is wrong. I believe that every student needs to write with pencils in math, because it affords the student the opportunity to erase mistakes without feeling so tied to the answer on the page. When a student can erase his mistakes, it allows the student sufficient flexibility to change errors, and experiment with alternatives.

Erasers - Generally, the erasers on pencils are crumbly and of a very poor quality, it therefore behooves the teacher to either give the students a good quality eraser or ask the students to purchase an eraser. (White gum erasers work best because they limit pencil and pink eraser streaks.) Also, a big eraser affords the student the occasion to erase large segments of work.

Highlighters or Markers - I, along with many math teachers, like my students to show their work. Therefore, on a page there is usually a whole clutter of numbers and symbols, stray pencil marks, erasures, and ghost images where students have erased. Often, while attempting to look at a student’s work, it is extremely difficult to follow a student’s chain of thought because of the haphazard use of the page. In such cases it is always helpful to both the student and the teacher for the student to highlight the problem in one color and the answer in another.

Procedure

Students should be heterogeneously grouped in clusters of 4 to 5 students. Desks should be set up in such a way as to afford every student an unimpeded view of the board or overhead. I prefer the desks to be set up where the students all face each other and need to turn their heads to see the board, because it is far more important for me to witness **their** interactions, than for them to watch **me** every second of the math period. Because I like for the students to work collectively on math concepts, there are often heated discussions and disagreements on which method is best employed in specific situations, it is helpful for the students to know that there are rules and procedures that must be followed even in those times of discord.

Math classes in my school run about 42 minutes and it is important that we rigidly adhere to the schedule in order for the time to be used as productively as possible. The 42 minutes are broken down in the following manner. (Teachers should feel free to alter the schedule according to their own objectives and their students' abilities.)

Tentative Schedule:

2 minutes - students gather and prepare themselves
5 minutes - students copy and discuss what they feel they are being asked to accomplish, strategies that they will need to utilize to fulfill their responsibilities
5 minutes - students are given an opportunity to write their findings, in their notebooks and answers
10 minutes - students share their responses and strategies with the teacher and the class at large, with the teacher acting as facilitator and clarifier of information
10 minutes - an alternative problem is posed, copied, and discussed, building on the information given by the class
5 minutes - the homework assignment is given and explained
5 minutes - wrap-up and last minute questions

When the students walk into the class there is always a Problem of the Day on the board. I have a timer that I use, and when it goes off the students realize that it is time to copy the problem into their notebooks and to begin the process of solving the problem. The students are given 2 minutes when they walk in to get themselves together, to finish the conversations they were having on the way to class and to sit themselves in their seats. Once they have seated themselves, and the timer goes off, the students finish their conversations and begin copying the Problem of the Day.

The timer is reset for 5 minutes and soon I begin to hear conversations as to what strategies will need to be employed in the process of solving the Word Problem. When the timer goes off again, I give my students a verbal cue to let them know that their group sharing should conclude and that they should now decide on the methodology that they will utilize to determine the answer. They then are required to take 5 minutes to write the method chosen and the result acquired. 5 minutes later, the timer goes off again, and we now begin the process of sharing the methods utilized and the answers obtained with the class at large.

For 10 minutes, the students are encouraged to participate. Because I try to make the answers open-ended with a variety of distinct correct responses, disagreement is encouraged, and consistently validated. I want the students to try to identify the word clues in the word problem and to verbalize the methods they used. I try to be as encouraging as possible, while at the same time I try to hear from as many different students as possible in order to get a wider view of who understood the problem and how did they go about attempting to find a resolution. I try to get at least one member of each group to say something that will contribute to the lesson, hoping to authenticate, endorse, and validate the work of each group. At the end of the ten minutes, a further problem is posed. This assists me in understanding how many of my students not only understood the last problem, but can repeat the process. For me, it is far more important that the student can duplicate the process of the first problem, than for the student to give me a correct response. This final problem is generally designed to reveal any glitches in the thinking of the students. If I find that many of the students did not quite comprehend, then I utilize this information. Because it indicates that the majority of the students did not understand, then I realize that I must reteach the lesson.

After further discussion, I give a homework assignment, and explain my expectations. Sometimes clarifications still need to be made. During the final five minutes of the class we do some wrap-up exercises, pose some final questions, and look forward to the following day's lesson.

Sample Lesson - Problem Solving Vocabulary

Problem of the Day: "Juanita is your best friend. You have known her for years. Like all good friends you sometimes have fights. However, school has just begun, and you have made a new friend named Luz. Luz tells you that Juanita is talking about you behind your back. What do you do?"

On the first day that we begin math, it is important that we continue the procedures that had been outlined in the previous few days of the year. The Problem of the Day (or the POD as it will be referred to from here on in) is up on the board to reinforce the procedures. Verbal cues (as well as manual cues like me setting the timer) will be used during the first few lessons to continuously give the lesson an ebb and flow that will hopefully allay boredom, promote excitement, and above all stimulate thinking.

Further, the POD gets the students working and prepared for math. It is especially important that on the very first few days that I let the students recognize that we will be working on vocabulary as well as procedures so that I reinforce the idea that every time they use math terms they will make me happy.

For this particular lesson, there are many graphic organizers that can be used to help the student understand that problem solving is a process that people do instinctively. However, what needs to be made explicit is that even though problem solving is something that all people do instinctively, it is a process that requires some thought. We do not just simply go and solve a problem without pursuing several avenues of inquiry. There is a progression that must be followed. If the sequence is followed adequately and reasonably, then problem solving becomes easier because thought was involved, alternatives were pondered, and the best response was chosen.

The graphic organizer that I generally use contains the four basic shapes: square, circle, triangle, and rectangle. Each shape has an arrow leading from one shape to the next to show that one step progresses to another. Each shape also has a word that communicates the process of problem solving: explore, plan, resolve, examine. The children intuitively realize which space requires what response, but there are subtle variations that need to be explained. A significant note to remember is that despite teacher initiated practice, the lesson is really student directed. I consciously have to continuously remind myself that I want **them** to figure out the correct vocabulary to use, I want **them** to provide the effort, and ultimately **they** should find the result. Although I always try to allow my students the opportunity to consider and achieve for themselves the results, I always have a list of some important ideas that I want my students to ponder, as a consequence I sometimes ask some leading questions in order to guide the students to a specific frame of reference. (Understand that if the students arrive at a specific important concept, these leading questions do not have to be asked).

Some important questions that I might ask about problem solving include:

1. Explore - What is it that I am looking for? What information do I have? What information do I need? What possible strategies might I employ to arrive at a solution??

2. Plan - Should I estimate a solution? Which one strategy would help me to find a response? Are there any alternatives that might be easier or better? Are there any difficulties that I might anticipate?

3. Resolve - Which is the best way to find a solution? How do I do it?

4. Examine - Can I check my answer? How do I check my answer? Why is it important to check my work?

After my students have taken their first ten minutes to discuss what it is that is expected of them, they decide, and begin to fill-in the graphic organizer that I have provided for them. Many of the students try to skip the initial process of discussion, because they have the misconception that schoolwork is all about writing the responses to problems as opposed to the actual working through of the problem. In order to make sure that they do not write before they discuss, I do not hand out the graphic organizer until the second 5 minutes is about to begin.

In this exercise, as in most of the word problems we do, there is no single answer but a number of possible solutions. By giving this word problem as the initial exercise, the students can see math in a situation that most of them have had some experience with. In fact, one of the questions that will invariably arise (generally at the end of the lesson) is, “What does this problem have to do with math?” That question is the opening that I look for to express the fact that math is not so much about solution as it is about process.

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