Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
2004 Volume V: The Craft of Word Problems

Do the Math 100%

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Introduction

Do the Math 100% is a problem solving unit involving percentages. Although written for use with eighth grade students, due to the various ability levels within all classrooms the material may be appropriate for other grades as well. The real world word problems will explore a variety of uses of percent. The design and analysis of the word problems are to enhance Mathematical-thinking skills and develop Mathematical problem solving skills. Both of these skills are important to the students' capability in taking the high school Connecticut Academic Performance Test (CAPT).

Rationale

Why teach with word problems? When teaching eighth grade Mathematics in the New Haven Public Schools or any school system, some of the issues facing teachers are the daily struggle of getting and keeping the students' attention and maintaining their interest level. Students' level of understanding drops significantly when their focus on their schoolwork is lost. By using word problems, a real world or more meaningful purpose can engage the student and make the Math more interesting to them. When designing the problems to use the teacher should try to relate them to current issues in students' lives. The percentage word problem can present students with a "picture" or real world concept involving mathematics, which can then lead to improving their critical reading skills. The story of the word problem and the student's prior knowledge often times can be supportive to figuring out the solution.

Current testing has shown that students generally do well in computation, but do poorly on items involving reading a word problem and solving it. The student must learn to translate the verbal into mathematical language using often times a formula or equation. A guided plan for students in order to solve problems is of utmost importance in the teaching of word problems. With repeated practice, students improve when they learn and use the terminology needed to decipher what operation(s) to use when solving. Students need to discover relationships between problems to gain familiarity in solving them. There is a need for this practice to begin more formally in the middle school grades in order to provide the skills necessary for him/her to perform

confidently on the CAPT in their tenth grade in high school. Percent problems are widely assessed on the CAPT in performances tasks, circle graphs and statistical analysis. Students also need these skills to provide them with success and therefore, the self-assurance needed to take advanced level Mathematics courses in high school. It is important for our students to take calculus and statistics in high school in order to be able to compete for admission to colleges and in the job market. Research has shown that many of our students do not take advanced placement courses and this puts them behind their peers in other school systems that are doing so. Often, our student population may feel they do not have the ability to succeed in advanced mathematics. Raising their success level may lead to self-confidence in their abilities. Real world problems and multi-tasks involving finding and writing your solution, will allow students to become more comfortable and familiar with the terminology and strategies needed to succeed.

Do the Math 100% provides these opportunities for achievement that encourages students to complete the lessons and increase their capability to solve the percent problems. The activities require students to use critical reading to analyze real world word problems and strategize to find solutions. They also will need to use writing skills in order to justify their answer. This unit provides the children with a strategic plan necessary for them to master in order to become better problem solvers.

Mathematics Standards

The New Haven Mathematics Framework and Grade-Level Standards are based on the Standards of the National Council of Teachers of Mathematics (NCTM). These are the qualifications required on the Connecticut Mastery Test (CMT) and the Connecticut Academic Performance Test (CAPT). This unit is designed in accordance with the mathematics standards listed in the appendix.

Strategies/ Role of the Teacher

Strategies for solving all types of word problems are multifaceted. It is not so much the strategy that determines success but rather that a student follows a plan for solving. If students can see relationships between problems that they are successful in solving to new problems they are presented with, then they have a basis for their plan. Generally, solving problems should be broken down into three areas:

- understanding what the problem says;
- thinking and doing the math
- responding and giving the answer

After reading the problem, understanding what to do can be a major difficulty for some students. They often are weak with the vocabulary necessary to translate the problem into mathematical language. Repeatedly using and charting math terms and their meanings are helpful. Student writing in journals where they use the clue words that are needed reinforces their meanings. When they write their own story problems, they gain further familiarity with the terms they need for understanding. For example, the use of the word “of” when combined with “percent of” in mathematical language means indicates multiplication. When using percent, to understand that percent often means out of 100, is as important, as how to represent the percent out of 100 by using a decimal or fraction. Both are vital to solving the problem. Encourage students to reread the problem for understanding and try to relate this problem to ones that have seen and solved before. It is seeing the relationship between problems that often will guide a student towards understanding what to do. With percentage problems, continuous practice will allow the student to identify what information is needed to be solved, the percent, the part of the quantity, or the whole quantity.

As students reread the problem and look for the information that is given to them in the word problem, the plan for thinking how to solve can begin. First uncover the clue words needed to indicate what operation must be done in order to correctly solve the problem. The teacher should encourage the student to circle the clue words and numerical information that they see in the problem. When students see the percent sign they will be reminded that they can use a formula to solve. By continually presenting students with practice problems that require them to see relationships, students learn to relate the clues to new problems that they are trying to work out. When teaching percent for instance, the teacher will introduce methods for solving the problems that students can practice and gain a level comfort. By using formulas students can make sense of the procedures they need to do to solve the problems they are given. Once comfortable with using formulas, they can plug in the numbers from the problem that go into the formula. This unit explores the use of formulas to solve percentage problems and the analysis of this. Use cooperative learning groups and classroom discussion about how to solve to find the answer when appropriate. Students often benefit from small group discussion on how to set up a mathematical sentence and then find the solution.

The response of the student will occur in several ways. Obviously, you want the students to calculate and give the answer. You also want students to be able to write a written response which explains and justifies their method of how they found the solution. This may be asked of them on certain tests. They need to be sure that theirs is a reasonable answer for the question that is being asked. Again proper understanding of terminology helps this process. Students must also be prepared to answer using a grid format as this is frequently asked of them.

As teachers we need to set up successful situations, as well as, challenging situations for all of the students. We need to have the students achieve a measure of success and to challenge without frustration, if possible. Solving words problems and writing a response to the solution are the driving force to mastery of the CAPT goals that students are required to achieve. “… basic skills and conceptual understanding are intertwined, and both are necessary before students can successfully apply mathematics to the solution of problems.”1 CAPT activities in mathematics will require the students to discover the ways to solve real world problems in a performance task. Students will attack the tasks with more confidence when they are able to draw on what they already know and have repeatedly practiced in previous grades.

Frequently when beginning a new type of problem to set up a successful situation, we can choose to model our own strategy for solving a problem and let the students “see” our thinking. Students often want to “help” us, as well. We cannot expect all students in the class to jump successfully into problem solving in today’s diverse ability level classrooms. Therefore, teachers should devote instructional time to discussion and

modeling how to solve word problems. It is helpful for the students to see you use an illustration, chart or table to solve the problem if appropriate. When the students can observe your plan as you go through the steps of understanding, thinking and coming up with an answer, it often times helps them to begin to use a plan on their own. Our class instruction must involve effective communication, questions, and discussion in order to engage students in learning and providing feedback.

It can be helpful for the students when attacking the percent problem to draw on individual familiarity, as can be the case with story problems. The teacher can point out examples that students may have had prior experience with, such as going to the mall and purchasing sneakers or music. Students are familiar with paying tax or looking for a discount. Using catalogs for tabulating items can involve percent both with tax and shipping costs. As students get to 8th grade, the level of difficulty of the word problems increases, however, their skill levels may still be weak and can still be a deterrent to their getting the correct answer. Use of the calculator can help to bridge the arithmetic gap. Even though this weakness of skills may occur, they still need to experience the type of word problem that will be challenging them. "… to best serve Connecticut's students, we encourage educators to adopt the following measures: overall, set higher expectations for all students…in curriculum, provide a more rigorous study of mathematical skills and concepts and their applications in today's world…"2 They need to be ready to face the more daunting job of preparing for more advanced mathematics courses in high school and taking the CAPT. The exploration in this unit of the differing types of percent problems are typical of what will be asked of students to solve on the CAPT.

Percentage Review

In beginning the study of problem solving using percentages, the idea of percent is foremost. Students have been using percents for the past few years in school and have done extensive exercises with them. They have changed the percent to decimals and fractions, done calculations with these and rounded the answer, and used pictorial representation involving percents. Spend time reviewing these because what a percent truly represents quite often may have escaped some students. In your review use hundreds grids and number lines as models for percents, fractions and decimals. Remind students that when you have a denominator of 100, you can use the term percent to mean "for each hundred". When working with a grid, you can ask them to shade in 3 squares, 14 squares, or 40 squares. To reinforce out of 100 ask: How many squares are in the grid? What percent have you shaded? Consider these questions to reinforce 100:

Shade 72 squares. What percent are shaded? What percent are unshaded? Check the total: 72% shaded + 28% unshaded = 100%. What fraction of squares is shaded?

These questions ask the students to compare and verify. Students can make needed connections with fractions and decimals using follow up questions about the grid.It is imperative in questioning that you include a distracter (such as unshaded) occasionally. This is frequently seen on standardized tests and to practice it in class helps the students to read more critically.

It is important for the students to see what percents such as 100, 50, 25, 0% can be related to in real world. Consider, for example:

The entire class is present in school today. What percent of the students are present? What percent are

absent?

With this type of question, you want the students to realize that the whole amount is 100%. That 100% is equal to 1 whole class. No one is absent therefore it is 0% absent. Next, try this for a quick review:

Half of the students in this class walk to school each day. What percent of the class are walkers?

This question gets the students thinking about what 1/2 represents in terms of percent. Students will begin to see a relationship between 1/2 and 50%, if they do not see it already. Many students also will know a variety of equal fractions and percents. They should be comfortable with changing familiar fractions to percents easily. A review of changing percent to fractions and vice versa will be needed at this point.

The class will be at various skill levels with percentages and seeing relationships between problems will not be easy for all of them. But some students can go beyond. If you add to the problem above that there are 24 students in the class and ask: How many of the 24 students are walkers?

With this added question several students will calculate using mental math to get 12. Again you can point out the relationship between half and 50%, and 12. Continue with practice such as this:

One fourth of the class is left handed. What percent of students in this class are left handed?

Conversely use the complement: What percent of the students are right handed in this class *?*

These questions create benchmarks for students as to what common percents and fractions are related to. Some will throw out there that 6 students are left handed. Let them explain their reasoning to the class. As they see this 1/4 and 25% connection, a question that can be proposed to the class is “Why do we work with percents?” The response that truly answers it is that it allows us to work with fractions yet, give the appearance that we are dealing with whole numbers. Many students are more at ease with whole numbers, and for the most part two digit numbers, which are used to express most percents, than they may be with fractions. Using percent allows the student to avoid the difficulties they incur with fractions, a skill that many have not yet truly mastered. Also using percent is a skill that is real world and is frequently associated with purchasing items at the store or from a catalog involving tax and discount. Budgeting and circle graphs are a usual tool involving percents This real world skill lends itself to creating word problems that provide meaning to the child and hopefully peaking their interest. It also is a necessary skill for the child to develop because it is so vital to adulthood. A set of review problems to work with percent review follows the unit.

The Percentage Formula(s)

The true essence of percent is that it means “parts out of 100”. Thus the fraction, p/100 (p out of 100), which shows its representation, has been taught to the student from early on. Then p % of a whole quantity, w, can be described as q, the part of the quantity. This is the basis for the our main formula (1): q = (p /100) x w = (p x w) / 100

This formula (1) is all the students need to perform all the basic calculations involving percent. Other forms of this formula will be studied but they can all be brought back to our standard percentage formula. This formula

allows us to solve for the quantity. By moving letters and numbers around we can solve for the percent, p, and the whole amount, w.

Students often have difficulty working with formulas therefore addressing how to use formulas themselves is a priority to be delved into before the teaching of percent problems. Beginning in seventh grade exploration of the use of formulas is more widely addressed. (For example the use of area, perimeter and volume formulas.) By eighth grade, the use of formulas needs to be a comfortable task for the students in mathematical solving. Using formulas is an objective that is tested on by the Connecticut Mastery Test in September of the eighth grade year. Problem solving and using formulas serve as a precursor for Algebra.

Percents can also involve fractional percent, which can complicate the significance for using them. Not every problem can be solved as easily using the same method. Although they are accurately correct, it can then lose its simplicity. Address these difficulties by calculator use, rounding or teaching other methods for solving. There are enough routine examples to provide for the student in order for them to gain confidence with solving percents.

Regarding the use of the calculator: Consider our first equation:

15% of 60 = q. By grade 8 students should to be comfortable solving this equation both with and without a calculator. However, in many circumstance the ability levels within our classrooms are diverse. It is up to the good judgment of the teacher to know who does and does not have the skills necessary to do these types of problems without use of the calculator. This is where differentiation needs to be taking place within our classroom space. The teacher is trying to maximize student development. "… when an entire class moves forward to study new skills and concepts without any individual adjustments in time or support, some students are doomed to fail."3 Developing understanding of what needs to be done to calculate should take precedence over the actual ability to calculate. We should be trying to increase students' success and personal growth. However, for those students who do have the ability to multiply and divide accurately, using these types of problems can reinforce those skills as well. Teachers need to monitor both computational and conceptual understanding in the classroom because both are important to student success in Mathematics. Each student needs to be challenged at an appropriate level. In the eighth grade mathematics classroom, each teacher must carefully decide when student success of understanding a concept is handicapped by a lack of good computational skills and act appropriately.

The first objective in working with percent would be to understand the relationship between the p (%) and the q, (quantity) of the whole, w. Extensive discussion of this percent, part, and whole is important to student understanding of using percent. Students need to know which of the three they will be solving for. Given a great deal of practice, students can see this relation. When beginning to develop this relation, begin with sets of problems such as our first equation:

15% of 60 = q (q being the part of the quantity, as taught to the students)

$q = (p /100) \times w = (p \times w) / 100$

$q = (15/ 100) \times 60 = (15 \times 60 /100) = 900/100 = 9$

Next, consider the following problems which students must see are asking essentially the same question: Find 15% of 60. 15% of 60 is what number? What is 15% of 60?

The solving of these three examples above uses the same equation (1). The answer 9 is the same. The terminology used however is different. Use all of these ways to express 15% of 60 because students must understand they will see different ways . Then they can take that familiarity and transfer it to word problems using the same numbers or involving estimation. Consider asking:

The teacher announced to the class that only 15% of the 60 students in the eighth grade had made honors that quarter. How many of the students made honors? Or

The teacher announced to the class that approximately 15% of the 58 students in the eighth grade had made honors that quarter. About how many of the students made honors? Or

If nine out of 58 students made honors is it better to say 15% or 16% of the 58 students made honors that quarter?

You can assign these problems independently, sequentially, or simultaneously. These three problems can elicit a variety of responses leading to a variety of considerations. For example, students will interpret the issue of rounding the quantity in varying ways. The students can write their thinking in their journals about the similarities and differences between the problems. A question for their reflection might be to decide which problem is more realistic to this type of situation.

Another consideration for the teacher is being cognizant that students may bring up differing ways to solve instead of using the formula. For example, changing the percent to a decimal or fraction and multiplying by the whole, or using a percent proportion (%/100 = part/ whole). When solving percent problems these may be methods with which students are comfortable. When this occurs, it is a good time to explore formula (1) and address how the different methods they have been taught is related to the formula. This helps students see how this formula works. The students, in groups, can figure out how to justify the relationships of the formula to other methods of solving. “Justification is central to mathematics, and even young children cannot learn mathematics with understanding without engaging in justification. …as they share their ideas and are asked to convince others that a procedure they have used to solve problems is valid, they have to use arguments that are convincing to other people.”4

An added theme with finding part or quantity, q, which is important for investigation, is regarding the complementary part or quantity, q?. When we have a percent of a whole, we also have the complementary part, the part left over. (If 10% made honors, 90% did not.) If the whole is w and the part is q, then the complementary part is q?.

(2) q = w - q = w - p/100 * w = (1 - p/100) * w = (100 - p)/ 100 * w

This equation (2) gives alternative ways to compute the q?. First use formula (1) to find q; then, subtract to find q?. Or compute the p?, by 100 - p, and then use formula (1). Some problems involving complementary part and complementary percent:

Troy and Kevin shot 60 baskets each at the gym. Troy made 75% of his shots. How many shots did Troy miss? Or

There is a 25% discount on the $80 basketball jersey that I want to buy. How much will the jersey cost?

The percent decrease is 25% and the amount of decrease is $20. This is the savings. The complementary percent is the percent left after the decrease. The jersey will actually cost 75% of $80. Students are ready for

you to introduce the new terms, percent decrease and increase.

Let us also consider percent increase: 6% of 50 = q; 6% of 50 + 50 = q + w, or, 100% of 50 = w; 106% of 50 = w + q. Either way the answer is the same.

A large exchange regarding the relations of these problems should be discussed. This can lead to the lesson of computing percent increase or adding tax onto a purchase with word problems. Begin with a quick review discussion of 100%.

Try , 100% of the 710 students at Troup have a lunch period , for discussion . Reminding students that 100% would mean everyone. Then:

The price of a $60 baseball glove is increased by 10%. Find the new price.

Students should be prepared to use formula (3) to solve for 10%. Remind them that the original price of the glove, $60, is 100% of the cost. Suppressing the $ relate:

(3) q? = w + q = w + p/100 * w = (1 + p/100) * w = (100 + p)/ 100 * w

60 + 6 = 66 is the same as: 100% + 10% = 110% 110/100 * 60 = 66 this represents the original price (100%) and the additional percent increase (10%). (100/100 + 10/100)*60 = (100 + 10)60. Set up a way to display plenty of practice using a table if you choose. See table 1 at end of unit.

Additional Ways to use the Formula (1)

When we need to find the percent of the part and the quantity, a new format with which to write our formula (1) needs to be introduced.

From (1) q = (p /100) x w. We move variables to get: (4) p = (q / w) x 100 =

(q x100)/ w

This formula now allows students to solve for percent, p. Students will need to be shown how the letters or variables have been moved around, yet remain consistent with the meaning of the percent formula (1). Consider our practice type of example: What percent of 25 is 20?

Students now familiar with using formulas (1-3), should not have difficulty using our new formula (4): p = (20/25) x 100= (20 x100) /25 p = 80%

Consider this word problem using the formula (4): On the last quiz, Sheila answered 18 out of 20 questions correctly. What was her percentage of correct answers on the quiz? Or

What is Kevin’s free throw percentage if he made 30 baskets out of 40 attempts?

These problems require the student to divide the quantity by the whole and then multiply by 100. Students before trying to solve, need to examine the problem to see what they already know and what they need to

find out.

The last formula we will introduce to the students that is developed from formula (1) is finding the whole, *w* , when we know the quantity, q, and the percentage, *p* : (5) w = (q / p) x 100 = (q x 100) / p

Consider the word problem using formula (5): James scored 85% on his quiz. He answered 17 questions correctly. How many total questions were on the quiz?

Students will need to put in the correct quantity, q, 17 and multiply by 100; then divide the result by the percent, p, 85. 17 x 100/ 85 = 20 questions on the quiz

Another-Katie's free throw percentage was 90%. She successfully made 45 shots. How many free throws did she attempt? 45 x 100/90 = 50 free throws

Again careful examination of the problem must be performed. Are you looking for the QUANTITY, PERCENT OR WHOLE? Students can work in pairs or groups to brainstorm the problem and try to agree on the formula to find the solution. They should be able to support their decision and if their answer is reasonable. Sets of word problems involving finding quantity, percent and whole follow the unit. Activities follow the unit.

Extensions of percent

Consider word problems of this type: House of Pizza has increased their prices by 15%. What will the new cost of my favorite $10 small pizza be?

They could use their formula (1) to find the quantity of increase and then add it to $10. They may choose to add 100% + 15% =115% and then use formula (1) to get $11.50. After calculating the answer, you can ask the students what the new costs of all the items at House of Pizza would be giving them a menu format or list of the costs.

To challenge a more advanced learner, this is a good time to work with matrices and percent increase. A matrix is a rectangular arrangement of numbers (data) in rows and columns. They are written within brackets [ ], not large parentheses. When you multiply a matrix by a number it is called scalar multiplication. Every number (element) within the matrix is multiplied. Consider the pizza costs:

small medium large

Cheese and tomato [10 12 16]

Pepperoni [12 14 18]

Supreme [14 16 20]

The manager at House of Pizza wants to increase all of his prices by 15%. Use scalar multiplication to do this for him.

[10 12 16] [11.50 13.80 18.40]

1.15 [12 14 18] = [13.80 16.10 20.70]

[14 16 20] [16.10 18.40 23.00]

The application of matrices to business is very important. By combining percent increase (or decrease) with scalar multiplication you can put this all together for the student. You can also have discussion with students about if rounding the prices would be suitable here. Graphing calculators will also assist in this teaching.

Consider this performance task. Mom went to Party City for supplies for my graduation party. She spent $5.99 on paper plates, $3.49 each on 2 packages of cups, and $4.99 on forks, spoons and knives. She also bought a graduation balloon that cost $1.50. Tax is 6% on these items.

From this story a variety of questions can be asked, each of which should require a skill that has been previously learned by the students. Depending on the level or levels of the students within your classroom at that time, you can ask a sequence of questions that will guide individuals of various ability ranges to answer. Alternatively, you can ask a question that will require the student sequentially to find answers leading up to the bigger question without guiding them. Prior to giving this real world problem as a task, several skills have been previously learned and mastered to varying degrees. The task may ask:

- Estimate what Mom has spent at Party City without tax.
- What was Mom's actual total cost at Party City without tax? Write one or two sentences comparing your estimation to the actual total.
- What is the amount of tax on Mom's items purchased with a rate of 6%?
- What was Mom's final cost including 6% tax?
- A new skill-How much change did Mom receive from $25?

Or go directly to the bigger questions:

- What was Mom's final cost including 6% tax?
- How much change did Mom receive from $20?

Asking students to estimate is a question that is important because it focuses on what consumers really need to do at the store. The students often have difficulty with rounding and estimating, but not as frequently when money is involved. There may be different strategies of estimating used by the class. Students need to be prepared to defend their answers.

Students will most often meet success solving addition problems with and without a calculator. The terminology 'how much spent' is familiar due to both real world exposure and previous practice. Often in

cooperative learning groups, they will discuss their strategy for solving. You can encourage this discussion by requiring all the students in the group to do their own work but then come up with an answer agreed upon by all members. They will then have one answer they can all defend as a group when called upon. If, after group discussion they cannot come up with one common answer, they may ask the teacher for assistance. Using group activity with problem solving, correction of minor addition errors are fixed, the order of operations can be addressed, and entering the numbers correctly into a calculator can be checked. Now you want students to compare the exact cost to the estimation.

For finding tax, students should be able to calculate using formula (1). For final cost, students should be familiar with adding the tax to the exact amount. To find the change, this should be one small subtraction step that some students will have difficulty remembering. Groups or pairs will help those students. Some students are ready to skip the sequential questions and go directly to what Mom spent or what her change is. That is a decision best left to the teacher of each particular class.

By adding in some additional information you can create a new set of challenges for the student: Party City is having a special graduation sale: 10% discount on all items!

Students may not be quite sure where to put in the 10% portion of the problem. Items for discussion within groups will include where does the new information fit in to the solution in finding the final cost, and how do we work with discount. You may give this problem on its own without the previous one and let them work to discover the final cost or guide them to the answer with a group of questions. Again, your determination of which way to question the students will depend on your classes' ability to complete the word problem. This is a good example of differential instruction in adding a challenge to the story for those in the class who are up to the task. Further questions for individual groups to consider is to ask if there is a 10% discount and a 6% tax, is it better to take off the 10% first, then pay the tax? Or decide to add the tax and get the discount on the total? Also, if a store raises its prices by 10%, then offers a 10% sale, what is the relation between the original price and the now sale price?

Follow up with this task next to assign the students. You and your friend go to Electronic Boutique. Estimate the cost of buying a video game for $49.99 and 2 controllers for your system at $19.99 each. Then find the exact cost, including 6% tax. (Optional: EB is having a 20% off sale on all video games)

After doing the performance task problem involving Party City, you would like the students to see a similar relationship in attacking this new problem. They should be familiar with the terms and plan for solving. Using cooperative learning groups let the students strategize ways of solving this cost and question each other. They will explain their methods to each other. A sample homework problem may include:

I went to Sports Authority and bought a beginner's tennis racket for $39.99 and two packs of tennis balls for $1.99 each. Sports Authority is having a 20% off sale. Tax on all purchases was 6%.

You can make up a question(s) about the story or you can ask the students to make up one or more questions about the story and answer their own questions at home. If some of your students do not have calculators at home, you may opt to let them just write their questions at home and estimate a solution. They can then take a few minutes at the beginning of class to solve or check their solution. Students can share their questions with the class or their group the next day or you can collect them and go over selected questions for the class to solve. They enjoy having input in a lesson and as the teacher; you can access their level of understanding of the mathematical computational skill as well as the mathematical understanding of the concepts. Another sample homework assignment to follow up is to have students write their own story giving them the numbers

to use. This is an extension of types of assignments asked on the CMT. Consider as a homework assignment.

Write a story about the following equation: 25% of 80 = q Solve your equation. Or:

You are going to the mall shopping. You have $100 to spend. Make up a story about what you purchased and the costs. Do not forget to include any sales (discounts) you came upon while shopping and you cannot forget some items may include 6% tax. Show your total purchases and if you have any change left. Have fun!

Conclusion

When working with percent problems, the real world situations that can be presented are abundant. Students can relate to the story of the problem due to the prior experience and knowledge they have with so many of the occurrences. They can enjoy creating their own problems based on the world in which they, as teenage consumers, are surrounded by. Their mathematical skill level with the many aspects of working with percentages will be tested and challenged. The key to effective teaching in today's world is to help children learn to think, reason, and solve problems. Students must be able to demonstrate and communicate understanding. Critical thinking skills are the hardest and most important skills to teach. The learners' ability to interpret and communicate mathematics is the most important for them to learn. The design of the word problems will stimulate the students' mathematical thinking and enhance their capability to meet success with the future performance tasks that lie ahead for them.

Activities in the Classroom

All activities will follow the mathematics standards located in the index of this unit. The materials needed include the chalkboard or overhead projector; handouts; grid paper; and calculators. At the discrimination of the teacher, students may do activities in pairs or small groups.

Activity one:

Review of conversion of percents to fractions-numerical and word problem practice. One class period is needed followed by homework and continual checking for understanding in subsequent classes. Handouts will be used for class and homework.

Performance Task :

Students will convert a percent to a fraction in lowest terms.

Objectives:

Students will review changing a percent to a fraction. Students will review simplifying a fraction to lowest terms. Students will recognize the magnitude of a fraction and compare that fraction to one closest to it. Students will read and solve a word problem changing a percent to a fraction.

Procedure :

Following a short demonstration that when you have a percent it converts to a fraction by putting the whole number percent over 100, have students shade a hundreds grid paper with the whole number of the percent shaded. Also ask students what percent is unshaded. Students can then find the lowest terms of the fraction through the use of the grid paper, a calculator or manually depending on the students' abilities. Model a sample of each for parts 1, 2, and 3 on the chalkboard or overhead projector.

Handout:

Conversion of percent to fraction.

1. For the following percents, shade the hundreds grid to represent the percent. Then change the percent to fraction. Simplify the fraction to lowest terms. Change the percent that is unshaded to a fraction in lowest terms.

a) 10% b) 25% c) 50% d) 75% e) 30%

2. For the following percents, find the fraction in lowest terms which is equal to the percent.

a) 15% b) 20% c) 35% d) 40% e) 55% f) 60% g) 70% h) 80%

i) 90% j) 98%

3. For the following percents, find the fraction with denominator 10 or less which is closest to it.

a) 10% b) 12% c) 15% d) 16% e) 20% f) 25% g) 42% h) 45%

4. For the following word problems, find the fraction in lowest terms which is equal to the percent.

a) The entire class is present in school today. What percent of the class is

present? What percent are absent?

b) Twenty percent of the class collects baseball cards. What fraction of the class is

baseball card collectors?

c) 75% of the students at Troup eat lunch. What fraction of the students do not eat

lunch?

d) If 50% of the students at Troup are walkers, what fraction of the students is

walkers? What fraction is bus students or gets a ride to school?

e) 40% of the class is wearing tee shirts. What fraction of the class is wearing a tee

shirt?

f) The eighth grade consists of 30% of the students at Troup. What fraction of the

students at Troup are eighth graders?

g) 10% of the class participates in football after school. What fraction of the class is

involved in football?

Handout : This student homework sheet provides similar examples for the students.

1. For the following percents, find the fraction in lowest terms which is equal to the percent. a) 85% b) 12% c) 31%

2. For the following percents, find the fraction with denominator 10 or less which is closest to it. a) 30% b) 50% c) 70%

3. For the following word problems, find the fraction in lowest terms which is equal to the percent.

a) Deanna spends 80% of her money. What fraction of her money does she not spend?

b) 10% of the students are absent. What fraction is not absent?

c) If 5% of the students were tardy and 10% of the students were absent, what fraction of the students was in school on time?

d) John ate 50% of the pizza and Jeremy ate 25% of the pizza. What fraction of the pizza did I eat?

e) Mom spends 30% of her budget on food, 40% on the bills, and 5% for miscellaneous expenses. What fraction is left for rent?

Activity two:

Review of conversion of fractions to percents-Numerical and word problem practice. One class period for initial review is needed with subsequent follow-up.

Performance Task :

Students will convert a fraction to a percent.

Objectives :

Students will review how to change a fraction to a percent. Students will compare different fractions that equal the same percent. Students will round the percent to the nearest whole number.

Procedure :

Model for the class the numerical calculation of dividing the numerator by the denominator (a/b), and multiplying the result by 100 (a/b x 100) to get the equivalent percent. Also, review with the students that the expression 5 out of 6 is written as the fraction 5/6.

Handout :

Conversion of fractions to percents with numerical and problem practice.

1. For the following fractions, find the whole number percent, p. Indicate which if any of the fractions are equal to the same percent.

a) 1/10 b) 1/9 c) 1/8 d) 1/7 e) 1/6 f) 1/4 g) 1/3 h) 1/2 i) 2/3 j) ¾

k) 5/6 l) 6/7 m) 2/10 n) 1/5 o) 10/100 p) 5/10 q) 6/8 r) 12/16

2. For the following words problems, change the fraction into a percent.

a) One fourth of the class is left handed. What percent of students in this class are left handed? What percent of the students are right handed in this class *?*

b) Alexis made three fifths of her shots from the free throw line. What is her field goal percentage?

c) John ate one third of the pizza. What percent of the pizza is remaining?

3. Write a fraction and convert the fraction into a percent.

a) 90 of the 500 seats at the movies are empty. What percent of the seats are empty?

b) 14 out of 20 students brought their books to class. What percent of the students brought their books to class?

c) 10 out of 25 students wear glasses in the class. What percent of the students wear glasses?

d) 45 is what percent of 135? e) 17 out of 20 is what percent?

f) 6 is what percent of 24? g) 10 out of 10 is what percent?

Handouts :

This student homework sheet includes similar examples for the students.

1. For the following fractions a/b, find the whole number percent, p. Indicate which if any of the fractions are equal to the same percent.

a) 19/20 b) 3/10 c) 4/8 d) 7/14 e) 5/20 f) 3/5 g) 2/8 h) 18/30

2. Express the fraction as a percent. About 3/20 of the total land surface of Earth is covered by deserts. What percent is this?

3. Write a fraction and convert it to a percent.

a) Four out of five students brought their pencil to class. What percent is that?

b) Out of 25 students in the class, 24 will be going on our school trip. What percent of the class will be going?

c) Five out of 25 students made high honors this marking period. What percent is that?

Activity three:

The three main types of percent equations-numerical practice. One or two class periods for practice and reinforcement is suggested.

Performance Task :

Students will solve an equation for the part or quantity, the whole number or the percent.

Objectives:

Students will interpret an equation and write it into formula format. Students will identify which of the items-quantity, whole or percent is needed to be found. Students will solve the formula for the correct missing item.

Procedure :

The teacher will introduce the formula (1) as explained in the unit. Students will use a calculator to solve the formula to find the quantity. Once the teacher is comfortable that the students can work the formula to satisfaction, introduction of solving for percent and whole amount is modeled. Be sure the students understand the vocabulary which identifies the part that is needed to be solved for. Then mixed numerical problems such as in the handout can be worked on in small groups. A review of rounding to nearest percent, tenth, and hundredth (penny) should be included.

Handout :

Solving for the quantity, q; solving for percent, p and whole, w.

1. Find the quantity, q, using the formula: q = (p x w) /100.

a) 15% of 60 b) 25% of 40 c) 60% of 90 d) 65% of 300

e) 50% of 322 f) 5% of 40 g) 6% of 100 h) 2% of 50

2. Find the quantity, q, using the formula: q = (p x w) /100. Round your answer to nearest hundredth, if necessary.

a) 10% of 39 b) 2% of 43 c) 75% of 102 d) 15% of 30

e) 50% of 116 f) 65% of 93 g) 12% of 80 h) 33 1/3 of 240

i) 4% of $10.40 j) 5% of $20 k) 6% of $42 l) 5.5% of $30

3. Solve for the quantity using the formula q = (p x w) /100.

a) 10% of 12. b) 10% of 12 is what number? c) What is 10% of 12?

d) Find 6% of 50. e) 6% of 50 is what number? f) What is 6% of 50?

g) Find 50% of 91. h) 50% of 91 is what number? i) What is 50% of 91?

4. Solve for the quantity, percent or whole amount. If necessary round your answer to the nearest whole number.

a) 19 out of 20 is what percent? b) 20% of 140 is what number?

c) 66 is 10% of what number? d) 45 is what percent of 60?

e) 38 out of 40 is what percent? f) What number is 34% of 50?

g) 35% of 70 is what amount? h) 80% of what number is 48?

i) What is 6% tax of $60? j) 30 is 75% of what number?

Handout:

The student homework sheet includes similar examples for the students.

1. Find the quantity, q, using the formula: q = (p x w) /100.

a) 15% of 90 b) 25% of 60 c) 50% of 322 d) 5% of 40

2. Solve for the quantity using the formula q = (p x w) /100.

a) What is 10% of 32? b) Find 6% of 50. c) 80% of 40 is what number?

d) What is 50% of 50? e) 50% of 91 is what number? f) What is 25% of 50% of 320?

3. Solve for the quantity, percent or whole amount. If necessary round your answer to the nearest whole number.

a) 10% of 120 is what number? b) 30 is 75% of what number?

c) 5 is what percent of 10? d) 38 out of 40 is what percent?

e) 80 is what percent of 40? f) 30% of 75 is what amount?

h) 75 is 30% of what? i) What is 20% of 15/22? j) 1006 is what % of 1001?

4. Try these. Solve each problem for the part. Round to nearest penny, if necessary.

a) There are 240 students in the eighth grade. About 20% of the students made the honor roll. Approximately how many students made the honor roll?

b) Of the 240 students in the eighth grade, 10% made high honors. How many students made high honors?

c) During a recent fundraiser in the 8th grade, students raised $2000. 70% of the $2000 will be used for school trips. How much money will be used for school trips?

d) The other 30% of the $2000 raised will be used for a special graduation breakfast for the parents and the graduates. How much money will be used for this breakfast?

Activity four:

Mixed Percentage Problems. Two class periods for word problems.

Performance Task :

Students will solve mixed word problems for quantity (part), whole and percent.

Objectives:

Students will determine what part of the word problems is missing. Students will solve word problems involving mixed percentages.

Procedure :

Review type of problem using the previous day's homework. Ask students to discuss what type of problem (part, whole or percent) it is before attempting to solve.

After a certain time allowance, go over the problems with the whole class.

Handout :

Classwork day 1. Solve these mixed percentage problems. First determine what you need to solve for - quantity, percent or whole.

1. John deposited $850 in the bank. The bank paid 2% interest per year. How much money did he receive as interest after one year? 1b) How much money did John have in the bank after the one year?

2. There are 12 rooms at my school use by eighth graders. 20% of the rooms are for the eighth grade students. How many total rooms are in the school? 2b) How many rooms are not used by eighth graders?

3. I bought my CD player for $20 less than the original price. It was on sale at 15% off. What was the original price?

4. Sally went shopping with $100. She spent 25% of the money on new shoes and 20% on a new jacket. She spent the remainder of the money on new sneakers. How much did she spend on sneakers?

5. Troy and Kevin shot 60 baskets each at the gym. Troy made 45 of his shots. What percent of the shots did Troy make?

Handouts :

Homework day 1. This includes similar word problems for the students.

Read the word problem and identify the part that needs to be solved for. Then solve the problem.

1. Mike has saved $90. He spent 30% of his savings at the mall. How much did he spend?

2. James saved $120 for his shopping trip to the mall. He spent 30% of his savings. Predict who spent more, Mike or James. How much did James spend?

3. Last month Mrs. Jones spent $150 on utilities. Her electric bill was $75. What percent of the total was her electric bill? Mrs. Jones's phone bill was 10% of her monthly utility cost. How much was her phone bill?

4. Miguel has 250 baseball cards in his collection. He has traded 20% of his cards. How many cards has Miguel

NOT traded?

5. Of the cards that Miguel has not traded, he has 80% of them in a card collecting book. How many cards does he have in the book?

Handout :

Classwork day 2. Solve these mixed percentage problems. First determine what you need to solve for - quantity, percent or whole.

1. In the class of 25 students, 20% are left handed. How many students are right handed?

2. In another class of students, 20% are left handed. Six students are left handed. How many students are in the class?

3. Shelia's savings account earns interest per year. She has $460 in her account now. If she leaves it there for a year, she earns $23. What is her interest rate (percent)?

4. The football team has 55 eighth graders on it. The volleyball team has 24 eighth grade players. No one plays on both teams. If there are a total of 169 eighth graders what percent of the class does football or volleyball? Round your answer to the nearest whole percent.

5. 48 students will attend the Philadelphia trip. Out of the 169 students in the 8th grade, approximately what percent will attend the trip?

Handouts :

Homework day 2. This includes similar word problems for the students.

1. A new fitness center has opened up in town. There is room for 500 members. Find the number of members if 75% of the 500 openings are full.

2. The number of apartments rented in a building is 40. If 80% of the apartments have been rented, how many apartments are there in total?

3. 5 of the 50 apartments need painting. That is what percent?

4. Find the percent of students who take a bus to school if about 200 out of 800 students take the bus to school.

5. What is the amount of students who get a ride to school if 10% of the 800 get rides?

Activity five:

Finding sales tax, discount, tipping, interest, and other forms of percent change. Two class periods recommended with homework.

Performance Task :

Students will solve problems involving real world situations with percent change. Be sure students justify and

show work to support their response.

Objectives:

Students will solve consumer percentage problems.

Procedure :

Introduce to the class this type of problem: I have saved $20 to buy a new DVD. The one I want to buy just came out for $18.99. Do I have enough money to buy the movie including 6% tax? Brainstorm with the students their thoughts on how to solve this problem. Ask how it is different from previous days' problems.

Handout :

Where can I get the better buy? There is a 25% discount on the $80 basketball jersey that I want to buy at Sam's Super Sports Store. In Connecticut I know that I will have to pay a 6% tax also. On the internet, I found an NBA website where the same jersey will cost me $62. I do not have to pay tax but I need to pay a shipping and handling fee of 5% for all items costing $50-$99.99.

Do the math to find out where I can buy that jersey at the least possible price.

Handout:

Ray bought some running shoes for $89.99 and a jersey for $39.99 at Foot Locker. There is a 25% discount today, with 6% sales tax, estimate what will the total cost be?

Handout :

Action Sports is selling the skateboard that I want for $21.80, plus 6% tax.

The same skateboard costs $25 at Sports Authority, but this week they have a 20% discount on all their sports equipment with wheels. Of course I still have to pay the 6% tax. Compare the cost of the skateboard at both stores and help me choose which store has the better buy. Please show and label your work in order to support your decision. Buy the way which sport on wheels do you like the best?

Reading List: Electronic Resources

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http://www.mathleague.com/help/percent/percent.htm good information; practice with many types of percent problems

http://www.aaamath.com/pct.html good teacher resources; homework help; easy to navigate website; lots of practice

http://www.math.com/school/subject1/lessons/S1U1L7GL.html#sm6 contains first glance; in depth; examples; computer will work out problems that you input

http://www.mathgoodies.com/lessons/vol4/meaning_percent.html pictorial and fun website; good puzzles and word search activities; challenge exercises

http://www.exrx.net/Calculators/Percent.html will calculate percent problems

Print Resources: Teacher

Carpener T, Franke M.L., and Levi L., *Thinking Mathematically* , Heinemann, Portsmouth, NH 2003 how to handle what students are thinking about math

Connecticut State Board of Education., *Connecticut Academic Performance Test Second Generation Mathematics Handbook* , 2001 good resource

Gardner, M. ed., *Mathematical Puzzles of Sam Loyd* , Dover Publications, New York, 1959 fun activities

Kordemsky, B., (edited by M. Gardner), *The Moscow Puzzles* , Dover Publications, New York, 1992 challenging puzzles

Lampert, M., *Teaching Problems and the Problems of Teaching* , Yale University Press, New Haven, CT, 2001 good strategies

Larson R., Boswell L., Kanold T., Stiff L., *Algebra One,* McDougal Littell Inc., 2001

Good reference

Liping Ma, *Knowing and Teaching Elementary Mathematics* , Lawrence Erlbaum Associates, Hahwah, NJ, 1999 delves into mathematical thinking

Mathematics: Applications and Conncections , Glencoe McGraw-Hill, 1999 text resource

Primary Mathematics Text 6A, 6B , U.S. Edition, Curriculum Planning and Development Division, Ministry of Education, Singapore, Federal Publications, 2003 great problems; illustrations

Tomlinson, C. and Demirsky Allan S., *Leadership for Differentiating Schools & Classrooms* , Association for Supervision and Curriculum Development (ASCD), Danvers MA, 2000 ideas for differentiating due to varying student abilities within classrooms

Print Resources: Student

Coffland, J. and Coffland, D., *Basketball Math* , Goodyear Books, Glenview, IL, 1995 fun facts and statistics.

Stenmark, J., Thompson, V., and Cossey, R., *Family Math* , Lawrence Hall of Science, Berkeley, CA, 1986. challenging activities; good explanations.

Zaccaro, E., *Challenge Math* , Hickory Grove Press, 2000. real world interests; good illustrations.

Mathematics Standards Appendix

This unit is designed in accordance with the following mathematics standards:

- Content Standard 1.0 Number Concepts, Arithmetic, and Operation Concepts: Students will: understand, represent and use numbers in a variety of forms in real-world and mathematical problem situations; compute accurately and make estimates with whole numbers, fractions, decimals, percents, integers, and rational numbers.
- Content Standard 3.0 Function and Algebra Concepts: Students will understand and use the concept of a variable.
- Content Standard 5.0 Problem Solving and Mathematical Reasoning: Students will: formulate problems from situations and given data; develop and apply a variety of strategies to solve problems-particularly multi-step and non-routine problems; will make and evaluate conjectures and arguments; verify, validate, and interpret results and claims, and generalize solutions.
- Content Standard 6.0 Mathematical Skills and Tools: Students will use estimation to assess the reasonableness of results.
- Content Standard 7.0 Mathematical Communication: Students will: express mathematical ideas and arguments with clarity and coherence; comprehend mathematics from reading assignments and from other resources.

Notes

1. Connecticut State Board of Education-2001, *CAPT*

2. Connecticut State Board of Education-2001, *CAPT*

3. Tomlinson, *Leadership* .

4. Carpenter, *Thinking Mathematically*

Table 1

Calculation	In words
110 % of 60	A 10% increase of 60 is 60 increased by 6 or 66.
115% of 80	A 15% increase of 80 is 80 increased by 12 or 92.
120% of 90	A 20% increase of 90 is 90 increased by 18 or 108.
200% of 30	A 100% increase of 30 is 30 increased by 30 or 60.
106% of 30	A 6% increase (such as tax) of 30 is 30 increased by 1.8 or 31.8