



Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
2004 Volume V: The Craft of Word Problems

Introduction

The premise of this seminar was that word problems play a key role in learning mathematics. More specifically, they provide occasions to give concrete interpretations to the compressed symbolic language of mathematics, and thereby help the student access the power of this language. In this view, the act of translation, from common language into mathematical symbols in order to solve a problem, and then back again in order to interpret the solution, is a key aspect of doing the problem, as important in the learning process as the solution itself.

The seminar used a variety of sources to examine the role of problems in the mathematics curriculum. Magdalene Lampert's account of a year of teaching a fifth grade class (*Teaching Problems and the Problems of Teaching*; Yale University Press, 2001) was read as an example of a way to approach mathematics instruction exclusively through problems. Sixth grade mathematics textbooks from Singapore were studied both for their well-constructed problem sets and for a specific highly effective diagrammatic method of solving a wide variety of problems.

We also considered broader curricular issues related to word problems. The kind of knowledge needed to teach mathematics well, and in particular, to deal with a wide variety of problems, both mathematical and pedagogical, is explored in the book *Knowing and Teaching Elementary Mathematics* (Erlbaum Associates, 1999) by Liping Ma. The data for this study was gathered through interviews in which teachers were posed problems situated in the classroom. The interaction between arithmetic and algebra, and types of arithmetic problems which can be used to promote algebraic understanding, were discussed in the book *Thinking Mathematically* (Heinemann, 2003) by T. Carpenter, M. L. Franke and L. Levi. The seminar Fellows who teach at the primary level also found that the earlier book, *Children's Mathematics* (Heinemann, 1999) provides an interesting discussion of how very young children approach simple arithmetic problems.

Much of the seminar time was taken with analyzing specific problems, both for their own interest, and as part of larger systems. We especially focused on understanding how a given topic can be explored through groups of related problems. This involves analyzing how the given problems are related to each other, their similarities and differences, and asking how the problems together articulate a given topic, or set of topics. In the seminar, the process of analyzing connected configurations of problems was referred to as *exploring the problem space*. This was the organizing principle for the units created for the seminar. Each unit assembles a collection of problems, and discusses how these problems together explore a given topic. The topics addressed frequently key parts of the mathematics curriculum.

Michele Murzak and Roberta Mazzucco have created sets of problems to explore subtraction with renaming. Jennifer Ulatowski has constructed a set of problems with the goal of enabling students themselves to create

word problems. She has provided a detailed analysis of the structure of each problem.

Diane Powers provides a sequence of lessons which introduces students to aspects of per cent. Sheila Wade investigates the use of the number line in dealing with fractions and percents. Joyce Bryant has collected a variety of problems constructed by her students, as well as some she constructed herself. These are accompanied by an essay on the importance of problem solving. Carolyn Kinder has devised a set of problems with the ambitious goal of advancing student understanding of the key properties of the basic operations, also known as the Rules of Arithmetic.

Luis Matos constructs problems which deal with ratio and proportion. Susan Gudas's substantial collection of problems again takes up the issue of percent, now at a more comprehensive level appropriate for 8th grade.

Finally, Anthony Wight presents us with a wide-ranging collection of problems which touch on a number of topics, from arrangements of blocks and counting problems, through various problems of "Singapore type" and several kinds of rate problems.

Each author has accompanied his/her collection of problems with a discussion of the subject area which the problems explore, and sometimes comments on how the problems fit together or how they address the subject.

Every collection of problems is a work in progress, always open to adaptation and refinement. The selection and refinement of a set of problems to address a given topic with effectiveness and insight may be a gradual process, carried out over a period of years. Problems may be added, deleted, or modified according to the needs of a given lesson or a given class. Problems may be selected from a larger group according to specific needs. It is hoped that teachers will find valuable material here for various purposes. One may borrow a set wholesale, another might select an individual problem. Some may modify some of these problems to their own needs, or be inspired to create a new set of problems. All these uses would further the basic goal of the seminar: to encourage use of word problems in mathematics instruction.

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