



Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
2006 Volume IV: Math in the Beauty and Realization of Architecture

The Math in the Design and Building of Bridges

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Introduction to Unit

What would our world be like without bridges? There are over half a million bridges in our country alone. Do you know how they work? Why are some curved while others are not? Some are delicate, ingenious and innovative while others are sturdy, functional and dull. Each is the result of centuries of creative design, constant technological development and of imagination held in check by the need for safety, reliability and peer approval. Every bridge grows out of people's needs. The story of bridges is a story of materials. Engineers must consider many things before determining the size, shape and overall look of a bridge. The next time you see a bridge after completing this unit, you will see it with a fresh sense of understanding and a greater level of appreciation of all that went into its construction. Engineering is an art as well as a science.

These lessons will enable the students to learn the history of bridges, identify the basic types of bridges, understand bridge vocabulary, determine the most appropriate type bridge for a specific area based on factors like cost, climate, and function, research and apply information on the internet, understand the construction, explain the forces of compression and tension, use model software to discover the physics in bridge building, create scale drawings, measure and compute math problems accurately, build a model bridge to test load bearing and design a community bridge that will inspire hope and friendship.

This unit is written for grades nine and up average math students. The students in New Haven are from a wide variety of backgrounds but mostly inner city. Their mathematical skills are often below grade level and their interest in school and math is low. There is a pressing need to improve academic performance, proficiency on the CAPT test, and completing high school. The development and use of a curriculum that focuses on the application of math and problem solving of the real life problems in the building of bridges should help inspire and motivate learning.

The unit will begin with the history of bridges, discuss some famous bridges, disasters and bridge facts, explain the basic types of bridges, bridge parts, explore the basic forces at work, truss designs, research and experience the multitude of designs, materials and constructions, find and graph information on bridges, and draw a blueprint. It will conclude with the students designing a bridge for their community that will bridge diversity and promote understanding and community.

Unit Objectives

1. Describe the common bridge types, including beam, truss, cantilever, arch and suspension.
2. Understand some of the basic design considerations bridge engineers take into account.
3. Understand basic bridge building vocabulary.
4. Understand the function of bridges in modern society.
5. Use the Internet as a tool in research and design.
6. Identify and explain the forces of tension, compression, torsion, shear, and stress and strain as they relate to bridges.
7. Measure, calculate and follow directions accurately.
8. Draw and label tables and diagrams to display information.
9. Design or choose a bridge for a specific cause
10. Explain the principles behind supporting a beam using the truss, arch and suspension method.
11. Understand the history and development of bridges

The History of Bridges

The history of bridges is fascinating. The oldest and simplest type is the post and beam which the Trojans used over 3,000 years ago. The Roman Empire contributed the arch, cofferdams (encircled watertight walls from which water is removed and bridge foundations dug inside), concrete, and the concept of public works which enables an empire to flourish. The Dark Ages that followed collapse of the Roman Empire was a confusing time of battles, famines, and epidemics. The great bridges of the Romans seemed superhuman to those living in a simple feudal system. Myths sprang up about evil spirits in the streams and bridges. The Roman Catholic Church became the best bridge builders in Europe. The religious orders had the education and travel to research bridges so every bridge had a chapel and an alms (donation) box (for those evil spirits).

In the twelfth century the first London bridge, across the unruly tidal Thames River, was begun with twenty

cofferdams to build the piers. There were nineteen pointed arches for a distance of 937 feet. Piers blocked two-thirds of the river which twice daily rose up to 15 feet with the tide. Water rushed through the remaining one third and a dangerous sport evolved called "shooting the bridge" (riding a boat in the fast current between the piers was extremely dangerous). Dozens of tall buildings with shops and homes perched on the bridge. It was a small village on a bridge and the best address in town. By the late 1700's the bridge had to be rebuilt and two more since then.

In the middle ages, bridges that could be used for invasion were fortified heavily with drawbridges, iron gates and high walls. Through the centuries invasions were always a looming danger. More bridges have been destroyed by war than by wear.

By the late 1400's the Dark Ages awoke to the Renaissance in Europe. Stability, wealth and great artists combined to create great bridges that were not only useful but beautiful. Powerful city states competed for architectural beauty. In Venice, 1587, the best known architect, Palladio designed a monster, heavy, showy and grand bridge. It was not good engineering, however, because it restricted canal traffic with its three arches and tradesmen had to carry their heavy sacks up long stairs to get cross. Da Ponte solved these problems by building the Rialto Bridge, the most famous and beloved covered bridge in Europe. It is a single shallow arch bridge, still strong and beautiful. Because of the soft soil on the sides, he built cofferdams on either side of the Grand Canal with special terraces with a ramp that met the diagonal thrust of the bridge. Engineers were just beginning to understand that directional forces, not just weight, held structures together. Leonardo Da Vinci and Galileo developed theories that helped architects make structures of strong, lightweight material.

The Age of Reason arrived in the 1700's. Complex forces, tension (stretching) and compression (squeezing), inside structures were explored. Triangles were engineered into trusses. Long, strong beams were designed with the least weight. A huge wooden train trestle built in the 1800's spanning a deep canyon was very strong with its layers of trusses. In 1813, Lewis Werwag, a master of wood construction, created a huge and beautiful bridge that was rot proof, light and strong. He knew how to eliminate sources of decay and his bridge could resist hurricane winds. It was a monument to reason and grace and its ingenious engineer.

In the late 1700's came the Industrial Revolution, quickly transforming the rural economy into industrial. The development of metal, first iron, then wrought iron and then steel transformed bridges. Steel could be mass produced in large quantities. The first iron bridge, the Coalbrookdale Bridge, built in England in 1779 used cast iron. In 1826, he built another high suspension bridge of wrought iron spanning 1000 feet across an important harbor. Cast iron is cast from a molten state into molds then cooled and hardened. It has low tensile strength. Wrought iron, three times the strength of cast iron in tension, was made into 3.5 inch bars to form the chains of the suspension bridge, the Menai Straight Bridge. In 1850, rectangular hollow tubes of wrought iron were used to make beams on the Britannia Tubular Bridge which were strong but used less material.

In 1878 in Scotland, the longest train bridge at the time was almost two miles long. Considered one of the seven wonders of the modern world, the Firth of Tay Bridge, an iron girder truss bridge, collapsed in a storm and a train with everyone aboard went down with it. The design had not allowed for wind forces and a gale of almost 80 mph hit the bridge. In addition, parts were uneven, misshapen and loose or weak. This disaster resulted in new solutions and designs. Four years later, the Firth of Forth Bridge, in 1890, was built as an immense, massive cantilever bridge with new material, steel, stronger than iron was used. It was the greatest railroad bridge ever built. In 1990, a commission declared that it would last even another hundred years. Engineers now knew how responsible they had to be about inspection of materials, building methods and

maintenance.

Drawbridges were designed where very high bridges were impractical. In the late 1800's, came the practical availability of steel and huge, watertight iron caissons. Caissons are chambers that create an air lock so men can dig the bridge foundation under the water. Men had to climb down iron tubes and entered the air lock of high pressure atmosphere. The difference in air pressure made them dangerous and cost many lives. These caissons did allow the Brooklyn Bridge, spanning almost 1600 feet, and the Eads Bridge, the first to crossover the Mississippi River at St. Louis, Missouri, to be constructed in 1874. The Brooklyn Bridge has more than 1000 suspended cables hanging from the four main cables. The deck is stiffened by trusses. Diagonal bracing increased the stiffness. P.T. Barnum took twenty-one elephants across the Brooklyn Bridge to declare it stable. These bridges were built during the Gilded Age when ornamentation was considered beautiful. These bridges declared the beauty in engineering that worked.

In 1925, the George Washington Bridge was erected. It was two times the length of any previous suspension bridge. It had 600 foot towers, a record height. In 1937, the Golden Gate Bridge, spanning the almost 4200 feet between towers, crossed the San Francisco Bay. It was the longest suspension bridge for the next 28 years until the Verrazano-Narrows Bridge. Its deck is 220 feet above the water. The towers rose to 736 feet tall. Nothing surpassed this achievement for many years. Joseph Strauss had never built a suspension bridge. Only six miles away is the San Andreas Fault, so because of new technology, it is being reinforced to withstand an 8.9 earthquake. The Verrazano Narrow Bridge, one of the most expensive structures ever made, had cables alone that were worth more than the Golden Gate Bridge.

In the early 1900's, reinforced concrete impressively heavy and durable was used. Stressed concrete created much stiffer beam (girders). This miraculous construction material created two kinds of bridges - brilliant and boring. New materials, strong, rust resistant cables, new connectors and new designs for stressed concrete beams created new astonishing bridges like the cable stayed bridges.

Students can research and imagine what kind of bridge they could cross twenty years from now. Imagine the impossible because the impossible sometimes leads to the possible.

Motivation for Unit

Ask: Which bridge totally disappears at times? The Golden Gate Bridge disappears in the San Francisco area heavy fog.

Students will be the most motivated to learn the math and physics and perform the activities if they can see the purpose of them. Inspire the students. An interesting lesson set of showing the most amazing bridges, model bridges being tested for huge loads and a couple of bridge catastrophes should encourage students to want to learn as much as they can about bridges. Showing the video and relating the story of the 1940 collapse of "Galloping Gertie", the Tacoma Narrows Bridge, are good enticements.

<http://www.wsdot.wa.gov/TNBhistory>

That bridge collapse was the most spectacular failure in bridge engineering history and the bridge was only four months old at the time. From the beginning, the bridge became an attraction for its galloping roadway.

On the morning of the collapse the wind blew at only 42 miles per hour. The only casualty, however, was a dog who refused to leave a car on the deck as its owner crawled to safety.

Math

The 600 foot center deck fell 195 feet into the water. The resulting spray of water was 100 feet high. What was the ratio of the spray to the drop, in lowest terms?

$$100/195 = 20/39$$

What rounded fraction can this be simplified to? $1/2$ (20/40 rounded)

Research more articles and web sites about this collapse.

<http://www.education.guardian.co.uk/higher/newstory>

Bridges are fascinating and amazing because they are designed to encourage, connect and inspire people. Bridges can be works of art. Suspension bridges are magical in their grace and elegance.

Manuel Rondon is project manager for the newest Tacoma Narrows Bridge, budgeted for \$849 million. It will be the longest suspension bridge in the USA in forty years, crossing that channel infamous for high winds and tidal currents. When asked about what makes a successful construction team, he said to choose talented people and entrust them with responsibility. Discuss how this relates to Donald Trump's television show, "The Apprentice". Students will work in teams at times during this unit. Effective team dynamics, support, expectations and skills should be taught and promoted throughout the lessons.

<http://www.eneews.Tufts.edu>

Models of bridges to be built are tested in wind tunnels today. Research and discussion on the latest methods to test bridge safety, as in earthquakes, are also motivating for students. There is a web site where you can build a bridge near San Francisco, between two faults and test it for different tremor forces.

Assure the students that they will be given step by step instructions on the activities, bridge language (terms) will be readily available and they will be fascinated and amazed by their research and study of all that goes into designing and building bridges. We will all never look at bridges the same again.

Introduction Discussion with Students

Local Bridges

All notes, work, calculations and handouts must be clearly dated, organized and filed in each student's portfolio.

Get an overall grasp of the students' knowledge and experience with bridges. After a slide show of some famous bridges, ask them if they knew any of these.

New Haven is a waterfront community where rivers flow into the harbor which becomes part of Long Island Sound. It is also on the New York City to Boston I-95 corridor and a major railway. It is home to many bridges; therefore students will have local relevance and experience to draw from for this unit on bridges. Have the students describe the bridges they have seen in their area, why we need bridges and what qualities a bridge should have. They must be safe, durable, functional, and visually pleasing.

There are three local bridges in particular we will discuss:

1. the new Quinnipiac Bridge section of the I-95 interstate being built downtown over the river
2. the closing of the Ferry Street bridge in town
3. the 1983 collapse of the Mianus River Bridge in Greenwich, CT. Copies of newspaper articles on these bridges can be obtained and used for practicing reading skills. Students would underline vocabulary words, answer questions and make up a question with answer.

Show the students pictures or slides of great bridges. To get the students thinking and talking, ask what type of bridge they would choose to build and where it would be.

Which type would they like to see in their neighborhood? Do they know that a major bridge has just begun to be built in New Haven? It is to be completed in 2014. Which type would be their choice of the new Q-bridge? Discuss, and then show a picture of the one chosen. Have the students explain their opinions. For a photo, go to

www.i95newhaven.com/upload/files/Fact_Sheets/program_overview_092305_hi.pdf

Ask who has taken a train into New York City. If they did then they rode over bridges. What would they do if they were going over a bridge and it collapsed? That happened down the road from here. The Mianus River bridge portion of I-95, in Greenwich collapsed on a June morning in 1983 sending three people to their deaths. What could have caused a section of Interstate 95 to fall out? The collapse was blamed on the failure of the steel pins to hold the horizontal beams together and inadequate inspections. What would you demand of your legislators after knowing the cause of the disaster? (New inspection laws) Ask the students if anyone remembers the 2004 accident on an I-95 bridge in nearby Bridgeport. The bridge was partly melted by a tanker truck explosion. Could this have been prevented? Traffic was going to be diverted for over two weeks but the bridge was reopened within a day. How? Huge crews were called in to fix it in record time, with much publicity. The Mianus River bridge traffic was delayed for 25 days after its collapse on this extremely busy interstate.

What would you do if the bridge you took to work was closed until further notice? That is happening in Fair Haven, next door. Imagine having a business on the other side that depended on the bridge to bring customers and inventory to you. What effect does that closing have on the neighborhoods and businesses on either side of the bridge? Do you know any business or person affected this way? What could you do about it? Discuss.

There are 583,000 bridges in this country. About how many is that per state on the average? $583,000 / 50$ is

over 10,000 per state. There are three billion bridge crossings a day. The number one cause of bridge failure is flooding.

Math

Make a bar graph of the lengths of the following ten structures, labeling completely and clearly.

1. Tacoma Narrows Bridge, 1950 5,979 ft
2. Empire State Building , NYC 1,453 ft
3. Golden Gate Bridge, San Francisco 6,450 ft
4. Brooklyn Bridge, NYC 5,989
5. Space Needle, Seattle 605
6. Titanic 885 ft.
7. Football field 300 ft.
8. Statue of Liberty, NYC 305 ft.
9. Average 9th grader 5.5 ft.
10. Eiffel Tower, Paris 1063 ft.

How many Titanics could have fit on the Tacoma Narrows Bridge?

$5979/885 = \text{over } 6 \text{ Titanics}$

Make another graph of the current longest bridge of each type:

Arch (concrete) - Yugoslavia - 390

Arch (steel) - USA - 518

Box Girder - 300

Cable-stayed - Japan - 890

Cantilever - Canada - 548

Suspension - Japan - 1990m

Make up two math sentences using the inequality math symbols $>$ and $.$

For example, $1990 > 300$.

Bridge Language

Many of the students have a limited vocabulary. Vocabulary will be learned from the readings, discussions, labeling diagrams, and word puzzles, like unscramble, matching, hangman, word search and crossword puzzles. Here are some bridge terms:

Abutment, Anchor, Arch, Beam, Bedrock, Caisson, Cantilever, Causeway

Centering, Cofferdam, Compression, Deck, Falsework, Floor, Force, Foundation, Girder,

Hanger, I-beam, Keystone, Load (dead/live), Pier Pile, Pontoon, Post, Pylon, Reinforced

Shear, Span, Stress, Superstructure, Suspension, Tension, Torsion, Trestle, Truss, Vector

The Types of Bridges

There are seven main types of bridges: the beam, truss, arch, suspension, cantilever, cable-stay and drawbridge. Bridge types differ in length, function and the load they can bear. Each holds weight in a different way. It is the balance between the downward forces (the weight and gravity) and the upward forces (supports) that causes a bridge to stand and carry loads. Many bridges use a combination of designs.

The beam bridge is the simplest and cheapest and the most common. The first one was a tree falling over a stream. The beam type made up of a pair of girders that rest on piers and support a deck. Usually they can span up to 250 ft. It has to withstand compression (pushing down of the weight of the bridge, the load and gravity) on top and tension (stretching) on the bottom. The piers balance the downward forces by pushing up.

If the linear structural elements are arranged in a lattice, such as a triangular pattern, you get a truss bridge and each member shares only a portion of the weight on any part of the structure. Truss bridges get their strength from the rigidity of the triangle. It is commonly used for railroad bridges because it can span great distances and carry heavy loads. If the truss is shaped into an arch, even longer bridges are possible. Most American bridges are girder designs. Straight steel beams, girders, lying on abutments, support concrete slab decks. Longer girder bridges up to 700 ft have steel deck.

Arch bridges were built 6000 years ago. The Romans took arch bridges to grander designs. Nearly all their bridges used the arch design because it could support more weight than a flat bridge. The arch is most successful when its two legs can be set on either bank. The traditional shape is made from a series of blocks cut to fit together perfectly. These "voussoirs" are wedge-shaped and gradually take the curve of the arch. The forces are spread around the arch to the legs that are kept from spreading out (buttressed) by the abutments.

The arch is built curving in from the ends, supported by scaffolding called falsework until the keystone is in place. The Romans often built multi arch bridges and cofferdams to cross rivers for its armies. Arch bridges have evolved with the changing of fashion and materials (from stone to concrete and steel). In 1809, Perronet transformed and perfected the stone arch by finding that pier that are $\frac{1}{5}$ of the arch are ideal for safety and that forces thrust downward and horizontally all the way to the abutments. Arch bridges can span up to 1800 feet.

Activity

Find pictures of two arch bridges. One should be a complete semicircle. The Darby Bridge was built from 1777-1779 near Coalbrookdale, London. It was the world's first iron bridge, 140 foot single arch and is a semicircle. The other picture of an arch bridge should be a shallower arch that is only the top of a semicircle. T. Seyrig designed the Luis I bridge in Oporto, Portugal. Completed in 1885, it has an upper road tangent to the circle at the top and a lower road that forms a secant through the arch above the diameter of the circle. This means the bridge arch is less than a semicircle.

Activity

Two students will form an arch by pressing their palms together high over their heads and walking their feet back. See Ice-Breaker Activities.

The cantilever design, a beam type, is more common for long spans. It consists of two independent beams/levers called cantilevers that extend from opposite banks and are weighted by piers. The downward force in the middle is counteracted by the weights on the side. Pulling out a kitchen drawer and a diving board demonstrate the cantilever concept. The two ends are joined in the middle by a beam, girder or truss. A pier firmly anchors one end and the arm extends freely out. Most cantilever bridges are made of steel or pre-stressed concrete. In 1890 the Forth Bridge in Scotland, the first to be made entirely of steel, broke records with spans of 1,710 feet each.

Activity

Have the students make a human cantilever bridge. To prove to the public that the new Firth of Forth Bridge would be stable after the Firth of Tay disaster in Scotland, the designers had three men act out the principle of a cantilever bridge. A photograph is easily obtained on:

http://www.makingthemodernworld.org.uk/learning_modules/maths/02.TU.03/?section=3

Long ago, suspension bridges were built in Asia and South America of vines, ropes and bamboo. How was the first cable connected to the other side, especially if there was a deep gorge to cross? Originally, it was attached to an arrow and shot over. Kites have been used. Today helicopters fly the first cable to the opposite bank. In a suspension bridge the cables are anchored in one abutment, carried over the top of one tower, draped over the other tower and carried down to the opposite anchor in another abutment. The deck is supported by suspenders attached to the cables and ride on saddles that allow the cables to slip back and forth as the bridge load changes. Steel cable suspension bridges, begun in Great Britain in the early 1800's, are used for the longest spans of 2000 to 7000 feet. They are light, strong, aesthetic but are the most expensive to construct. Two huge extremely strong cables are draped over two high, sturdy towers that may rise hundreds of feet into the air. The ends of the cable are deeply embedded in concrete and the anchorages are usually fastened to solid rock. The deck can be high enough for even the largest ships to pass under. After

the Civil War, the design of suspension bridges advanced when John A. Roebling (Niagara and Brooklyn Bridge creator) developed wire cables and stiffening trusses which minimized sway or twisting. He also learned to add more stays after the Wheeling Bridge disaster. At the same time, St. Louis, Missouri, desperately needed a train bridge to cross the 1500 foot span over the dangerous Mississippi River. They needed to take economic advantage of the cross country rail route soon to be available. Eads, a man who had never before built a bridge but knew the unpredictable river, accomplished the feat. He used innovative materials (steel), foundations and superstructure construction. Most of the weight of the bridge is transferred through the cables to the anchorages. Cables of high tensile wire can support an immense load. The longest main span is in Japan and measures 6,529 feet long.

Math

How much more than a mile is this length?

$$6529 - 5280 = 1249$$

Write this length as a decimal of a mile rounded to hundredths.

$$6529/5280 = 1.24 \text{ mi.}$$

Activity

Research at least three suspension bridge tower designs and draw them on graph paper as accurately and detailed as possible. Label each with the bridge name and the dimensions of the towers. The scale should be the same for all three so their relative sizes can be easily compared. The Golden Gate, Brooklyn Bridge and the 1940 Tacoma Narrows Bridge are a good start. The Narrows Bridge art deco towers are 50 ft wide at the base and 39 feet wide across the top. Research the height and draw a diagram to scale.

<http://www.wsdot.wa.gov/TNBhistory?Art/art1.htm>

There are also good pictures of the massiveness of the anchorages in three great suspension bridges.

A fascinating hands-on web site demonstrates step by step the building of this bridge.

Combination spans are often used to bridge even longer stretches of water. The San Francisco-Oakland Bay Bridge is noted for its three long spans, two of which are suspension spans and the third a cantilever.

The cable-stayed design is the most modern, coming into prominence in the 1950's, and resembling sailboats. It may look similar to the suspension bridge but the two types of bridges support the load of the roadway in different ways. In suspension bridges, the cables ride freely across the top of the towers, sending the load down to the anchorages. In cable-stay bridges the cables are attached to the towers, which bear the weight alone. Computer modeling that constructs a three dimensional virtual bridge model allows engineers to make strong bridges of light, minimal material. The longest is in Japan and spans 2920 feet. The deck is hung from diagonal cables that exert force towards the towers as well as vertically. This makes the tension in the steel cables very high and thus very stiff. The cables stabilize the towers from both sides.

Activity

Tie a 5 foot rope to each of your elbows and place the middle of the rope on your head. A partner can help

you. The rope acts like a cable-stay to hold up your elbows. Next take a longer length of rope and hold it your outstretched hands, place it over your head and experience two cable-stays.

The last category is the moveable bridge. There are several types and many are located in the area. The bascule bridge has a moveable deck that can be lifted quickly, either straight up or as the typical drawbridge. The swing bridge is built on a huge wheel that moves the bridge at a right angle. Pontoon bridges are quick to build and take down. Armies in ancient times used sealed and inflated animal skins or small boats floating end to end. Today metal cylinders or hollow concrete boxes are filled with air and attached to permanent piers or supports. In 1940 a permanent pontoon bridge more than two miles long was built on a lake near Seattle, Washington.

Interesting Famous Bridges

A handout of a world map is given out to each student. The following bridges will be located and labeled on this map.

London Bridge

London Bridge has an almost 2000 year rich history, beginning with a Roman built bridge in 43 AD. In 1176, the first stone bridge stood for 600 years. Its construction and upkeep was paid by rents and tolls. In 1824 a five arch bridge was built and replaced in 1972. The old bridge sank $\frac{1}{4}$ inch each year and so by 1968 this became alarming to the authorities. The old bridge was dismantled and bought by McCullough (chain-saw manufacturing) who moved it 10,000 miles to Lake Havasu, Arizona. How did he do it? Each of the over 10,000 pieces were carefully removed and coded. The bridge was reconstructed according to the original blueprints. Originally it took 800 men and 7 $\frac{1}{2}$ years to build this bridge but only 400 men and 2 years to rebuild it, although they did not have to sink foundations. It is in the Guinness World Records as the world's largest antique. There is an excellent web site: (<http://www.oldlondonbridge.com/timeline.shtm>) which shows an amazing time line, in full color pictures, of London Bridge's eventful history. Another site shows the reconstructed bridge: www.londonbridgeresort.com/history/london_bridge.htm

Math

Find the total depth the London Bridge sank over the 1824 to 1968 at $\frac{1}{4}$ inch per year.

$$1968 - 1824 = 144 \text{ years} / .25 = 38 \text{ inches}$$

Calculate how many feet this is.

$$38 / 12 = 3.17 \text{ ft.}$$

Brooklyn Bridge

The famous Brooklyn Bridge in New York City, over the East River, has been celebrated in stories, poems and paintings. It may be the most photographed bridge. The design of its towers and graceful pattern of the stays create a bridge of beauty. It is close enough to New Haven to visit and walk over. An engineering wonder, it

was the longest suspension bridge in the world at its 1883 opening. It had to have a large clearance of 135 feet above the river. The idea came to its designer, John A. Roebling, when he was crossing that river on a ferry and it became stuck in the ice. Ferries would stay tied up for weeks because of the ice in the river. All new techniques were used. It was the first bridge built of steel. It cost double the original estimate, about \$1 billion in today's dollars, and took 14 years to build. It was supposed to take only 5 years to build.

Math

How many extra years did people have to wait to use it? $14 - 5 = 9$

Discuss if structures are commonly completed past their due date.

The huge opening ceremonies were elaborate with the US President, Chester A. Arthur, attending and 14 tons of fireworks exploding in the sky. For a virtual tour go to:

(www.newyork.com/attraction/panorama/tour4a/htm)

Each cable is made up of 3,515 miles of steel wire, each galvanized with zinc to protect them from the adverse effects of weather.

Each wire was the thickness of a pencil. Each strand is made up of 286 wires. One strand was at the center, surrounded by 6 more in a hexagonal shape. Surrounding those were 12 more.

Math

Calculate the amount of wires in one cable.

$$286 \times (1+6+12)$$

$$286 \times 19 = 5,434 \text{ wires}$$

Math

Each cable has a thickness of $15 \frac{3}{4}$ inches. Students will cut a 2 inch strip of paper the circumference length of a 15.75 diameter circle.

$$C = \pi d = 3.14 \times 15.75 = 49.5 \text{ in. rounded to the nearest tenth.}$$

The cables of the Verrazano-Narrows Bridge are 3 ft. thick (diameter). Find the circumference. $3.14 \times 3 \text{ ft} = 9.42 \text{ ft.}$

Convert these feet to inches. $9.42 \times 12 = 113 \text{ inches}$

Compare the circumferences of the cables of the two bridges.

$$D = 15.75 \text{ in. } C = 49.5 \text{ in}$$

$$D = 3 \text{ ft} = 36 \text{ in. } C = 113 \text{ in. Thus roughly double.}$$

Next students will find the cross sectional area.

$$A = \pi r^2 = (15.75/2) \times 3.14 = 24.7 \text{ in}^2$$

Math

Workers traveled over 10,400 times from each end (anchorage) to loop the wire.

How long did it take to just stretch the wires?

$10,400 \times 10 = 140,000/60 = \text{over } 2,333 \text{ hrs. } 2,333/24 = 97 \text{ days}$

Roebing used pneumatic caissons modeled after the ones Eads used on the St. Louis suspension bridge. The ones for the Brooklyn Bridge were approximately 170 by 100 feet. The walls of the air chamber at the bottom were 9 feet thick on top. The extremely thick roof in one of the caissons the chamber was 22 feet thick to support the masonry of the tower foundations. There were sharp iron triangles on the bottom of the caisson to cut deeper as more river bottom was removed to get to the bedrock.

Activity

Take an empty glass. Turn it over in a pan of water to experience an air lock.

Roebing died from complications in a ferry accident, so his son took over. His son became paralyzed by the bends from the caissons and could not leave his home for years. His wife, Emily, had to quickly learn the business enough to become the on site director.

Activity

Rewrite these parts of a suspension bridge in the order of which they are built:

1. Suspenders
2. Towers
3. Foundations
4. Deck
5. Cables

A fascinating hands-on web site demonstrates step by step the building of this bridge.

Another time line enables you to see the progression of the different types of traffic at six different times in its history. Originally only cable cars and horse and buggies crossed with a center elevated lane for pedestrians. Trucks have been banned. Discuss why.

(http://www.brooklynexpedition.org/structures/buildings/bridge/bl_construct_b.html)

Lake Pontchartrain Bridge

The longest bridge in the world is near New Orleans, LA, and crosses Lake Pontchartrain.

It is 24 miles long. The piles were driven 70 ft. down into the sandy bottom. It has a maximum vertical height of 50 ft. and a minimum of 15 ft.

Math

What is the range of the clearances? $50 - 15 = 35$.

Why is there a difference in clearance? It costs less to make a bridge lower but boats also need to pass under at some point.

Math

The bridge length, 24 miles, is 1/1000th of the earth's circumference.

How long is the earth's circumference? $1000 \times 24 = 24,000$ miles.

The Pearl Bridge

The longest suspension bridge in the world, the Akashi Kaiko Bridge, also called the Pearl Bridge, was built in Japan in 1998 and cost \$4.3 billion. It crosses 110m deep water with a very swift tidal current, heavy winds and lies only 150m from a potential 8.5 earthquake. The towers alone are over 282m. tall. The cables over the towers have a diameter of 1.12m. Each hanger cable/rope has a diameter of 7mm.

Math

Find the circumference of each and cut a 2 inch strip of paper the length of those circumferences and tape them into loops to feel the size of these cables.

$$C = 3.14 \times 1.12 = 3.5\text{m.}$$

Compare these to the cable circumference on the Brooklyn Bridge.

The Pearl Bridge has a total length of 12,828 ft. and a span of 6,527 ft. (1991 m). This is almost a quarter mile (366 m.) longer than the second longest suspension bridge, the Storebaelt in Denmark built in 1998, 580 meters longer than the Humber Bridge in England, 692m longer than the longest US suspension bridge, the Verrazano Bridge in New York, and 710 m longer than the Golden Gate Bridge in San Francisco, CA.

Math

How many meters long is the Golden Gate Bridge? $1991 - 710 = 1281\text{m}$

This is an amazing site showing a very clear satellite map of the world and points you can click on to see a bridge photo taken there.

<http://geomimages.berkeley.edu/wwwp904/index.html>

How to Choose the Right Bridge for the Job

Bridges must be built strong enough to safely support their own weight as well as the weight of the people and vehicles that pass over it. The bridge must withstand earthquakes, weathering, strong winds, and freezing and thawing. When determining what type of bridge to build, engineers evaluate terrain, length of the needed span, cost, aesthetics, available materials and technical expertise. Bridges change as our needs and resources change. Engineers are constantly looking for ways to improve bridge designs and materials. During the recent California earthquakes, engineers analyzed overpasses to find ways to make them stronger. A new material that is very strong has glass fibers imbedded in concrete.

Class activities include first completing worksheets checking for understanding and identification of the bridge types. Then hands on activities using guiding handouts and virtual labs on the internet will enable the students to understand and discuss the forces, loads materials and shapes in building bridges.

Pictures of each type of bridge will be cut, pasted and labeled to pages in the students' bridge portfolio. Websites contain a multitude of design examples of each category.

Students will research to compare and contrast various bridges. Dimensions will be recorded on a worksheet table. Lengths of several objects and bridges will be graphed to visually show their differences. Students will write ratios of length to height of a bridge and the ratio that represents one length compared to another.

A NOVA program follows the building of the Clark Bridge in Illinois over the Mississippi River. The newest and best technology is used. There is also an interactive activity where students can test their engineering skills by matching the right bridge to the right location.

<http://www.pbs.org/wgbh/nova/bridge/>

The students will investigate the strengths and weaknesses of each bridge type by experiencing the forces and strengths of designs and materials. Use the PBS Web Sites to develop basic bridge science and math knowledge and then show what is learned. PBS has a great introductory interactive lab for simplifying the real-life conditions that affect bridges. The forces lab shows the forces of squeezing (compression), stretching (tension), bending, sliding (shear), and twisting (torsion) and vector analysis.

<http://www.pbs.org/wgbh/buildingbig/lab/forces.html>

The materials lab demonstrates and tests the strengths and weaknesses of several structural materials from wood to steel.

Its loads (forces that act on structures) lab illustrates dead, live, dynamic, wind, thermal, earthquake and settlement loads.

The shapes lab shows how different forces affect various geometric shapes. The same amount of material changes its strength and stiffness according to the shape it is.

Activity

To experiment, students will place a sheet of paper over two books standing up 8 inches from each other. You

can also use file cards and then place the books closer accordingly.

1. Place the paper on top and see how many pennies it can hold. Record your results.
2. Arch the paper so it rests on the surface the two books are on.
3. Arch the paper on top of the books and test it with pennies.
4. Crease the paper in the middle and repeat steps 2 and 3. Test again with a load of pennies.
5. Fold the sheet back and forth to make accordion pleats. Count the amount of pennies after spreading them evenly over the area and again with the pennies loaded in one area.
6. Glue another sheet on the top of the pleats and one on the bottom. Test again. Remember to record and analyze the results.
7. Combine steps 3 and 6, the principles of strength through curvature and also creasing.
8. Finally, an even stronger solution is the creased barrel. A sheet is folded up along diagonal lines and the accordion pleat lines are folded down to form an arched accordion-like form.

Activity

Another hands-on activity, "How much can I take?" will teach about forces. Gather at least 6 different materials to test for tension (pull on both ends), compression (push together on both ends) and torsion (twist the two ends in different directions). The students will make up a table to record results, making up a rating scale from 1 (very weak/crumbles easily) to 5 (super strong/does not break).

Activity

How does shape affect how well a material performs? To test the rigidity of shapes, a quick activity compares the stability of a square to a triangle. Using seven equal size straws or straw pieces and 14 small paper clips, construct the shapes, inserting a paper clip into each end and hooking them together.

Stand each shape up and push down on top. Journal how each bends and twists. Lastly add straws to the unstable shape to make it stable. The solution is to attach diagonals in the square to make rigid triangles. This is the principle behind trusses and also a great geometric concept. To expand on this see the activity included in this unit that stabilizes polygons up to hexagons. Also students can make the most rigid structure possible with 20 straws and 49 paper clips. Another team building activity is to have teams construct these in complete silence and trust their signal to each other.

<http://pbs.org/wgbh/buildingbig/educator.html>

The West Point Bridge Designer 2006 software is an excellent way to introduce the students to engineering through an authentic, hands-on design experience of modeling, testing and optimizing a steel highway bridge.

<http://www.matsuo-bridge.co.jp/english/> is a site profiling an innovative Japanese bridge company, photos of its bridges and descriptions of bridge construction methods.

<http://www.havenet.combridgink/> has a Bridge of the Month Quiz and many photos.

Geometric Shapes in Bridge Trusses

Draw 6 attached squares in a row with diagonals from opposite corners.

Color one example of each: Then find how many of each shape you can find.

Square - yellow _____ (more than 10)

Rectangle that is not square - red _____ (>20)

Parallelogram - blue _____ (> 30)

Rhombus - green _____ (> 10)

Triangle within one square - orange _____ (> 40)

Triangle inside two squares - purple _____ (> 8)

There are five main designs of trusses. Below is the Multiple Kingpost truss.

Extra credit - sketch all five types and label each with its name.

Color one example of each angle or line type:

Parallel lines - red right angle - blue

Perpendicular lines - orange acute angle - green

Intersecting lines - yellow obtuse angle - purple

Line of symmetry - black

Icebreaker activity

At the beginning of this unit, an activity or two to build trust, cooperation and team spirit, will create motivation and effort. These are easy, low anxiety activities.

Getting to Know You is an icebreaker that consists of 16 yes or no questions in equal squares on a handout given to each student.

Ask the class first who would like to win a million dollars! This question creates enthusiasm. Tell the students to mingle and ask each other the questions, making sure to write their names on the paper. When a student answers yes to a question, he or she writes their name in the proper square. The one at the end of five timed minutes who has the most signatures wins a large check for a million dollars, printed to look like the real

thing.

The questions are:

1. Can you name a bridge?
 2. Have you been on a suspension bridge?
 3. Have you seen a picture of the George Washington Bridge in NYC?
 4. Have you seen a bridge from a train or airplane?
 5. Have you been delayed by a bridge?
 6. Did the ancient Romans have bridges?
 7. Have you seen a bridge being built?
 8. Have you walk over a bridge?
 9. Do you have a relative that lives across a bridge?
 10. Have you been on a bridge with over four lanes?
 11. Have you seen a pedestrian bridge?
 12. Have you seen a bridge in another state?
 13. Have you seen a moveable bridge?
 14. Have you seen an arch bridge?
 15. Do you cross a bridge to go home?
1. Have you been over the Q Bridge in New Haven?

Human Arch - Icebreaker Activity

This is a 20-30 minute lesson that will introduce the three terms: force, compression and load. There will be 5 students in each group. Two students will form an arch by standing facing each other and pressing their palm together. They will then each back up as far as possible. A third student will very gently pull down on the arch until it collapses. The students who formed the arch describe their experience.

The three concepts of force (a pushing or pulling on an object), compression (a pushing force that squeezes, and load (dead load is the weight of the structure alone and live load is the weight of anything it carries).

The fourth and fifth students must find a way of strengthening the arch. After the arch is strengthened and tested again, students discuss their solutions. The answer is to add a buttress at each set of feet. Have students relay their experiences. Have students switch roles to feel the forces in each member of the arch.

Bridge Bingo

Students make up their own bingo cards the first time and reuse them for subsequent games. A sheet of paper is divided into 25 equal squares. A short form would have 9 squares. 30 pictures of famous bridges and bridge parts are printed by the teacher on a handout.

The students cut these out and glue them on their sectioned paper. The order has to be different from the others. The free square is in the center. The teacher calls out from the scrambled master set of the names or terms shown by the pictures. Everyone plays until one has Bridge Bingo!

Jeopardy

A game board will have to be made up. Teams of students can make up \$100 to \$400 questions (and answers) for their given category for another class, since they would have the answers. Also if a classroom is set up with centers, one center would be to make up jeopardy questions (with answers and dollar amount).

Suggested categories would be:

1. Famous Disasters 5. Connections
2. What Kind Am I? 6. The Long and Short of It
3. What's my name? 7. That Part of Me
4. Past, Present, and Future 8. The Long and Short of It

Bridge of the Day or Week and Daily Bonus Question

Display a large picture of a famous bridge with the 5W's (who, where, why, what, when) given each day or week. Students are to study the facts on their own for trivia questions each day at the end of class for points or rewards. A Bonus Question of the Day is on the board and the first student to answer it wins. This encourages students to arrive early.

Centers in the Classroom

Eight centers are spread around the classroom perimeter. The students would spend 20 minutes at each and rotate. This amount of time can be adjusted depending on the attention span of the students. They would record their experience and any experiential work in their Bridge Journal. Acceptable behavior and effort will also be explained and points will be given at the end of class. The centers would have large posters explaining the procedures and expectations of each. Handouts and worksheets will be hung in folders or pockets attached to the poster. The teacher continually moves around to coach and guide.

The eight centers are:

1. Computer research center for students to design their Bridge Notebook cover, complete

questions, scavenger hunts and find facts (5W's - who, where, why, what, when) and pictures to be displayed.

2. Interactive computer program center to show how forces act and react in bridge structures, how a bridge will hold together during an earthquake, how tensile strength is measured and other amazing programs.

3. Audio-visual center contains books, poems and videos about bridges. Opening Day of the Brooklyn Bridge is interesting and there are many others. Low level readers can listen to tapes of the reading as they follow along. Students can make up their own bridge stories and poems and they or others can illustrate them.

4. Game center includes unscramble bridge vocabulary words, crossword puzzles, hangman and word search games to learn bridge language.

5. Puzzle center contains bridge pictures cut up in squares to be put together. Also students will play Match That Piece! The teacher makes two prints of a detailed picture. One copy is cut up into some very small pieces that would be challenging to identify by themselves. A small square piece of a bridge picture has to be placed correctly on another complete picture.

6. Calculating center has mathematical bridge problems to solve, categorized in easy, medium and advanced skill levels.

7. Build a bridge center displays instructions and materials to build a small model bridge. There are many choices of materials, such as, toothpicks, spaghetti, straws, coffee stirrers, cardboard, popsicle sticks, balsa, and found objects.

8. Drawing center has materials for designing a scale model of a bridge on graph paper with side and top view, drawing a bridge from a photo and or original ideas, designing a tower for a large suspension bridge and creating a graph in color of bridge information. There is an activity included in this unit about coloring various geometric elements and shapes in a truss.

Speakers and Field Trips

A civil engineer can be invited to talk about his job. Students should write 5 questions for the engineer before the speaker arrives. Questions can be brainstormed to spark interest, such as what types of projects has the engineer undertaken, what courses in school are recommended, what advice can be given to present students, what is the best and worst part of being a civil engineer, what are other job opportunities in bridge building, what are their requirements to obtain and hold a job and what do they pay.

A field trip could be to walk the Brooklyn Bridge or to any bridge in the local city to identify the functional parts and aesthetic appeal.

The Bridge Notebook/Portfolio

Each student will keep an organized notebook of the activities and information in this unit. The cover will be designed by the student on a computer.

Page 2 will be a daily effort point sheet. A point is given to each student for being on time, coming prepared, completing the five minute focusing DO NOW, being cooperative, and cleaning up, for a total of 5 points. Another 5 points is given for working constructively all period, for a daily total of 10 points.

Page 3 is the check off sheet for completion of the activities. When a lesson is complete, the teacher signs it off. This is a clear and organized way for the student and teacher to see what is incomplete, especially after an absence. Organizational and study skills are learned and practiced. Another alternative is to have each line contain each day's work assignment and description of the lesson.

Page 4 is a rubric to assess the completed portfolio;

1. Completed activities.
2. Completed journals
3. Neat and organized
4. Cover design
5. Self-evaluation
6. Participation in discussions
7. Extra credit
8. Cooperated as part of a team

Understanding the Strength in Columns

A column is a structural member, usually upright, that supports a weight that pushes down on it. It therefore must have good compressive strength to prevent buckling and then collapsing.

Activity

Students will take a piece of paper to form different shapes of columns and test the strength of each. In general, a column becomes strong against buckling with the more material away from its central vertical axis.

Predict which shape column would buckle the least.

1. Roll a 4 x 11 inch strip of paper and paste to form a circular hollow column.
2. To form a square hollow column, cut a 5 x 11 in. strip and fold it length wise every inch, five times and glue into a square.
3. Fold another to form a triangular hollow column. Cut a strip of paper 4 x 11 in. and fold it vertically every one inch and glue.
4. Fold another to form an I-beam. Cut a two 2 x 11 inch strips. Fold them along two lines $\frac{1}{2}$ in. from the long edges each. Test each folded strip alone for strength. Then glue the middle sections back to back and notice the difference in bending under a compression load. Next glue two one-inch strips of paper, one each on the top and bottom of the "I". Test. Glue two more 1-inch strips on either side of another I-beam. Predict the result before you test again. This last model should be less strong because more material was placed closer to the central axis. The circular column will be the strongest, 20% stronger than a square one of comparable size. Circular columns are not used as much because they are difficult to fasten to beams.

The most important factor that enables a column, unless it is quite short, to resist being deformed is its load bearing capacity, or how much weight it will support. This capacity depends on four factors: (1) on the stiffness of its material, (2) the geometry of its cross-sectional area, (3) its length and (4) if the ends of the column are fixed to something.

Look at the table below of four common structural materials and their modulus of elasticity.

Compare the materials.

1. Find the factor by which a concrete column has a greater load bearing capacity than a wooden one of the same size. Round to the nearest tenth.

30 / 13 is about 2.3 times

2. Approximately what fraction of a steel column's load bearing capacity is that of a concrete column's?

30 / 200 is about 1 / 6

Material Young's Modulus (10⁹ N / m²) Newtons of force/meters

Wood (Douglas fir tree) 13

Concrete 30

Aluminum 70

Structural steel 200

Building Stability - Columns

There are many activities to show how engineers know how to choose the structural material and shape, solid or hollow of a column or beam. We will next calculate one example to spark your interest in this fascinating store of knowledge. To demonstrate how doubling the thickness of a column affects (this is amazing) its load bearing capacity, draw the cross section of two columns. One is a 2x4, and the other is a 4x4(in.) wooden column.

Using the formula above for finding I, compare the moment of inertia of each.

$$I = (4)(2)^3 / 12 \text{ which equals } 2.67 \text{ in.}^4$$

$$I = (4)(4)^3 / 12 \text{ which equals } 21.3 \text{ in.}^4$$

Therefore, we see that the 4x4 is _____ times stiffer than the 2x4. (8 times! $21.3 / 2.67 = 7.977\dots$)

You can demonstrate further how a hollow 4x4 will support three times the weight of a solid 2x2.

More activities can be found on such internet sites as <http://polymorph.net/engineer.htm>

Building Stability - Beams

How a beam acts under a load depends on how it is supported and how its ends are attached to the supports. It can be supported on one or both ends (roots). The amount of force applied to the root depends on the load size and the distance from the root. This is called the bending moment.

Make up an equation for this last statement, using variables of your choice and then compare them to the formula that is used:

$$M = P L$$

Bending moment (M) = load (P) x distance from root (L)

The bending moment is zero at the end supports and maximum in the middle.

A lower strength material can be used in a beam that is made greater in height.

The strength of a beam, simply supported and loaded in the middle, to resist sagging/bending is a function of its height squared, span, width, and E. Simply said:

1. if you double the height, the strength is quadrupled and the stresses reduced by a factor of 4.
2. if you double the length of a span, the strength is cut in half.

3. if you double the width, the strength is doubled.
4. if the modulus of elasticity is doubled, then the strength is doubled.

Finding the Forces in a Truss

In stone bridges, the arch is vital. In truss bridges, the important form is the triangle. For thousands of years, crude forms of the truss bridge were built but without knowing how to give a truss the most strength. A square shape can be bent out of shape, as we have seen. It won't hold anything up. Any weight on it could cause it to collapse. A triangle, however, cannot be distorted. You cannot change this basic shape. This is why triangles are so important in construction. Before designing a model bridge that will hold the greatest load, students will learn how to choose the sizes of individual members in the most efficient proportion to the magnitudes and characters of the forces they must carry.

The book *Shaping Structures*, by Waclaw Zalewski and Edward Allen, has a lot of great information on the effects and distribution of forces on bodies and the art of shaping structures. In the companion CD, there are eight excellent step-by-step models for finding and designing forces in a truss. Skill level for students for this CD program should be at least strong Algebra 1.

What happens when you change the depth of a truss? The Force Polygons show that if you reduce the depth of a truss by half, member forces in the top and bottom chords exactly double! To keep forces low, make the truss as deep as possible. Longer trusses mean more material, however, so a depth has to be chosen that is optimal for given conditions.

Final Project: Designing a Community Friendship/Peace Bridge

Cities like New Haven have cross-cultural issues to address. We need to build bridges of understanding and break down barriers. After completing the research and lessons on this topic, the students will design a friendship bridge for their community.

The mission of this bridge is to increase respect among people from different races, cultures and religions. This bridge will be the site of community dialogue that will enhance mutual understanding and encourage everyone to participate in community problem solving through action groups. There are abundant websites about building relationship bridges.

"Never doubt that a small group of thoughtful, committed people can change the world; indeed, it is the only thing that ever has." Margaret Mead

"Not everything we face can be changed, but nothing can be changed until it is faced."

James Baldwin.

A bridge builder is one who has made a commitment to understand people with different backgrounds. Say to the students, "Imagine receiving a phone call informing you that you have been selected to become a New Haven Ambassador. What an Honor! What is your plan?" How can our lives build bridges to the world? We need education to construct better bridges. Three of the greatest bridge builders of the 1800's grew up poor but realized reading and learning was the only way to accomplish their dreams.

Project steps:

Using all your research and a creative mind, design a friendship bridge for your community on graph paper. Consider the:

- purpose
- site choice
- size
- type
- materials
- colors
- detail
- weather
- traffic
- other constraints you have studied

Final Project Rubric

Ask the students to help create a rubric. Ask them what attributes a friendship bridge should have. List these on the board and consolidate them into a rubric.

Some suggestions are:

Sense of beauty (aesthetics), visually pleasing appearance and style

Creativity

Relationship to the environment

Graceful lines, proportion, balance, and detail

Economical, low maintenance and a reasonable budget for construction

Safety

Anchorage

Design matches the purpose (mission statement)

Shows investment of time and effort

Presentation is neat and clear

Drawing is accurate and professional in appearance

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