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Astronomy: The Mathematician's Perspective

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Introduction

The field of astronomy offers an interesting alternative to the usual study of mathematics. Mathematical concepts used to describe the events of the universe can be taught to students using astronomy. The wonderful aspect of astronomy is that it can be used to teach to all levels of mathematics from basic algebra to geometry to calculus. An adjustment to lesson plans will allow for flexibility in proposed mathematical concepts whether it be developing an equation or constructing geometric shapes.

Student Body

Wilbur Cross High School is about 2,000 students strong. It is located in New Haven, CT. The mathematics courses, offered to these inner city school students, begin with Algebra I and three years of mathematics is needed in order to graduate. The student body, in general, is preoccupied with electronics such cell phones, i-pods, and various game units. School is boring because there are no games in mathematics classes. But, when the teacher becomes an entertainer and enlivens the classroom with variety as overhead, white board, film, slides, trips to the computer room, quiz games, and mixes the day to make time pass, the student is more interested. Hence, astronomy.

Why teach mathematics through astronomy?

Mathematical principles can be taught using astronomical data since our knowledge about the cosmos is based on mathematics and physics. Students can relate directly to the solar system because inhabiting the Earth makes us a part of our solar system. Students gaze at the stars and have questions about what they see. Some of their information is learned in elementary school when the solar system is discussed. The rest of it is obtained from higher education, if science is chosen as part of the course study, or if students become aware of news releases about discoveries or corrections made to previous astronomical data learned. In high school, students do not realize that mathematics plays a significant role in explaining nature.

Students will realize that studying mathematics is essential because it plays an important role in explaining the universe, about where we live, and how our position in the universe affects us. Therefore, the subject of astronomy will stimulate critical thinking, imagination, and motivate students to solve astronomical problems by applying mathematical principals. As a byproduct, curiosity about other aspects of astronomy and how mathematics applies to it should ensue.

Objectives

1 Students will construct graphs and compare data using graphs.
2 Students will be able to demonstrate the difference between positive and negative slope and will be able to determine the slope of a line.
3 Students will use the Pythagorean Theorem to find distances.
4 Students will determine angle measurements and distances using trigonometric ratios
5 Students will find measures of arc lengths and areas of sectors.

Overview

The astronomical data used to instruct mathematical concepts will be varied according to mathematical concept. For instance, elements such as hydrogen, helium, and carbon occur naturally in the universe. But abundance differs on each planet, Moon, and the Sun. Comparing the abundance of elements in the solar system will allow students to analyze astronomical data by using graphs.

It will be interesting for the student to note how the atmospheric gases differ from the four terrestrial planets of Mercury, Venus, Earth, and Mars to the gaseous planets of Jupiter, Saturn, Neptune, and Uranus. Of course, interest in Earth compared to the other planets always stirs conversation about habitat for life and the elements needed for it.

Other data as using diameters of planets and moons can offer interesting comparisons. Students will discover how the diameters of all the planets compare with each other and likewise the moons. They can sort the planets according to size or position in the solar system using graphs.

The Earth and Moon relationship offers a good way to teach slope. Positioning the Moon in its orbit around the Earth can demonstrate positive and negative slope. It is intriguing to note that the Moon wobbles in its orbit and is not consistently the same distance away from the Earth. This discrepancy will allow for changing data as the Moon revolves around the Earth.

A by-product of this tactic is to remove the visual and reveal the coordinate plane. [1] This will allow a smooth transition from reality to the structured and sometimes weary coordinate plane that now takes on added interest. The point can be made that the removal of the picture supports a less obstructed view of the data and the calculation of it.

On the geometry side the Earth-Moon parallax and Sun parallax can be used to teach the Pythagorean Theorem and trigonometric ratios. The Earth-Moon parallax and the Sun parallax do not offer many options to use right triangles but offer two that can be considered to be positioned in all four quadrants of a coordinate plane. Two right triangles in each quadrant is enough information to demonstrate the Pythagorean Theorem and trigonometric ratios using the Earth-Moon parallax and then emphasize the mathematical concepts by having the students use the Sun parallax. This will allow for reinforcement and questions. (Other parallax options exist by using any planetary parallax or even a stellar parallax.)

Using the orbits of planets around the Sun can teach about arc lengths. It can also be utilized to find the area of sectors. Since orbital speed varies among planets, the sweep a planet makes in a season will vary between planets and comparisons on which planets' sweep covers a larger area will be explored and may produce surprising results. It can be hypothesized to a student that an outer planet traveling at a slower speed may carve out a smaller area than expected than an inner planet traveling at a faster speed. For example, the average distance of the Earth from the Sun is 1 AU (astronomical unit) [2] and its orbital speed is 29.79 km/s whereas Saturn's average distance from the Sun is 9.572 AU and its orbital speed is 9.64 km/s. (See Appendix D Table 4 for a list of orbital speeds of the planets.) The student will find that Saturn carves out the larger area.

The circumference of the Earth can be used to reinforce using the formulas of finding the measure of an arc length and the area of a sector. Students can determine the distance they want to travel, for instance, London to Paris, determine the angle formed using the two locations and the center of the Earth and apply both formulas since the radius of the Earth will be known. This information can be compared to the distance indicated on a map.

Background Information

History

It must have been fascinating in ancient times to gaze at the stars and speculate about what they are. Ancient astronomers called some objects in the sky "planets" because of the way they moved around the Earth. Planet means "wanderer" and they named the planets after Roman gods such as Mercury which means the god of trade and profit and Jupiter which means king of the gods.

There were many mathematicians and physicists who monitored the stars and made observations. Many of these astronomers collected their data at the same time other astronomers were collecting the same or similar data and this sometimes confirmed or refuted their data or previous collected data. Names like Copernicus, Kepler, and Newton come to mind. Copernicus was the first to realize that the Earth and planets orbited the Sun. Until then and even during his time it was thought that the planets orbited Earth. Kepler's contribution is the laws of how planets orbit the Sun and Newton's contributions are laws of motion that allowed for eventual space travel. Kepler's and Newton's laws are still prominent today. Mathematicians held high status in political circles and were sought to make predictions about celestial events such as eclipses. As the need for more exact measurements and describing more celestial events became essential so did the need for more sophisticated mathematics. And, derivatives soon followed to explain the various changes in the universe. It is unbelievable but true that new mathematics is sometimes invented in order to explain the events of nature. [3]

Today, there still exist many misconceptions about the solar system. There is recorded data [4] that confirms that there are individuals who believe that Mercury is the hottest planet when in fact it is Venus because of its greenhouse effect. Their temperatures compare 425 $^{\circ}$C. to 470 $^{\circ}$C. Mercury has virtually no atmosphere and therefore the heat is not trapped about the surface as is the case with Venus which has a thick atmosphere.

Movies also reinforce misconceptions. A chase through the asteroid belt is more trilling if the asteroids are in proximity to cause collision than if they are at great distances as is the case. This wide distance between asteroids is how our space probes are able to travel out to distant planets and not be annihilated.

The Solar System

Looking at the solar system, there are eight planets. Pluto is no longer a ninth planet, it has been re-designated as a "dwarf" planet along with the asteroid Ceres and the outer solar-system object called Eris. [5] Pluto's composition, first of all, never fit the characteristics of the other eight planets. It is mostly composed of rock and ice. Its orbit is not in the same plane as the other planets. Pluto's orbital inclination of 17.15 $^{\circ}$ is higher than the almost collective pancake orbits of the other planets. The inclination of the orbits of the other planets range from 0.00 $^{\circ}$ for Earth to 7.0 $^{\circ}$ for Mercury using Earth as the base planet to measure all other planets against. (see Figure 1)

(image available in print form)

Figure 1 The figure shows the orbits of Pluto and Neptune. The orbits of the other seven planets are in almost the same plane as Neptune. The orbital inclination to the ecliptic (Sun's path among the stars) ranges from 0.00 $^{\circ}$ for Earth to 7.00 $^{\circ}$ for Mercury. Pluto at 17.15 $^{\circ}$ is obviously out of place. Figure may not be to scale. [6] Source of figure is www.Wikipedia.org, and the figure is in the public domain.

All the planets orbit the Sun in the same direction and in more or less in the same plane. Their orbits are nearly circular about the Sun but have enough eccentricity to be elliptical as was shown by Kepler. (Kepler's Laws are reviewed below.) It is common knowledge that the Earth spins on a tilted axis and so do the other planets. One planet, Uranus, is tilted on its side and spins horizontally.

The Sun contains more than 99% of the total mass in the solar system. It is composed of 99.8% hydrogen and helium and .2% other elements. The Sun is hottest at its core where nuclear fusion occurs. The Sun is gaseous throughout with no solid surface or solid core.

The inner planets, Mercury, Venus, Earth, and Mars, are called terrestrial planets because they have rocky surfaces. Terrestrial means Earth-like and the four planets are all small, dense, have abundant metals, a few moons, and no rings. It is also interesting to note that the small cores of these planets are metallic.

The outer planets, Jupiter, Saturn, Uranus, and Neptune, are called Jovian planets because they are Jupiter-like. These planets are larger in size, have smaller densities, have many moons, and have rings. Some have prominent rings, others very fine rings. They are also called gas giants. As with the inner planets it is interesting to note that it is not exactly known what the core of these planets are composed of. It is suspected that the core could be composed of compressed carbon and be diamond-like [7]. However, the gas giants do have a surface, though not a solid one. The planets' gravity holds together the gases about the core.

The formation of the solar system began 4.6 billion years ago and during that time the elements and debris that were strewn into the solar system formed into planets by crashing into each other. Eventually all the debris was either formed into planets or knocked out of the solar system by these crashes (see Figure 2).

(image available in print form)

Figure 2 Shown at left and then right is how the solar system must have looked at its beginning as debris and elements joined together by crashing or by gravity and created the celestial bodies. Some debris was knocked out of the solar system. Eventually, the planets cleaned out their areas. The ability to clean out respective planetary areas is a characteristic of planets. Source of figure: www.Wikipedia.org.

The resulting solar system was devoid of the numerous particles and the planets successfully cleared their areas. The only significant band of rocks in the solar system is the asteroid belt located between Mars and Jupiter (see Figure 3). What was left was an almost flat solar system (refer to Figure 1). The Sun is the main gravitational pull on the planets and holds them in place.

(image available in print form)

Figure 3 The asteroid belt between Mars and Jupiter was not cleaned out by the planets. It was sufficiently far enough away from Mars and Jupiter not to be absorbed by either of them. Also, the Sun's gravitational pull holds the planets in their orbits with varying escape velocities (the speed required to break the gravitational pull of a celestial body and leave orbit). Source of figure: www.Wikipedia.org.

Strategies

Graphs

Students will learn how to graph using astronomical data.

Elements (Gases) and Diameters of Planets

The elements, or gases, present in the atmosphere of a planet or Sun are dependent upon its mass, gravity, and temperature. If the temperature is high enough, as with Earth, gases such as hydrogen, which is a light element, can easily escape the atmosphere. That is why Earth has no hydrogen in its atmosphere. But, on

Jupiter, where it is cold and the mass and the gravity of it are smaller than Earth's, Jupiter then, will keep an abundant amount of hydrogen in its atmosphere.

Description of elements in the atmospheres of the planets:

Hydrogen is the lightest of all elements and the most abundant in the universe.
Nitrogen is present in all living things.
Oxygen is critical for all life on Earth.
Argon is a highly stable chemical element.
Carbon dioxide is an important greenhouse gas.
Helium is the second lightest and most abundant element in the universe.
Methane is a good fuel because of its clean burning process. [8]

The diameters of the planets vary according to size with Mercury's equatorial diameter being the smallest at 4,880 km and Jupiter's equatorial diameter being the largest at 142,984 km (See Table 2). Moon data, in terms of radii, can be found in Appendix A.

Gases will be used to instruct about graphs because it presents numerous data to work with, there are intriguing combinations of gases in the atmosphere on each planet, and there are fascinating comparisons of gases between planets, including the Sun. Students will be motivated to learn how to graph and to make conjectures about planets using the data from the graphs.

Diameters of planets and moons will be utilized because it offers different measurements, interesting comparison of measurements of planets in relation to the order of planets in the solar system and of moons in relation to their planets, and after graphing the atmospheres there are notable conjectures to be made about planet size and gases found.

Students will read tables, construct a graph using the horizontal and vertical axis of the first quadrant, label and increment the horizontal and vertical axis, determine the height of the graph, construct bar and line graphs, read and record data, and compare two graphs for similarities and differences.

Mercury: essentially none

Venus: 96.5% carbon dioxide, 3.5% nitrogen, 0.003% water vapor

Earth: 78.08% nitrogen, 20.95% oxygen, 0.035% carbon dioxide, 1% water vapor

Mars: 95.3% carbon dioxide, 2.7% nitrogen, 0.03% water vapor, 2% other gases

Jupiter: 86.2% hydrogen, 13.6% helium, 0.2% methane, ammonia, water vapor, and other gases

Saturn: 96.3% hydrogen, 3.3% helium, 0.4% methane, ammonia, water vapor, and other gases

Uranus: 82.5% hydrogen, 15.2% helium, 2.3% methane

Neptune: 79% hydrogen, 18% helium, 3% methane

Sun: 92.1% hydrogen, 7.8% helium, 0.1% other elements

Table 1 Atmospheric composition by number of molecules. Source: Universe, Freedman and Kaufmann, seventh edition, 2005.

Mercury - 4,880 km

Venus - 12,104 km

Earth - 12,756 km

Mars - 6,794 km

Jupiter - 142,984 km

Saturn - 120,536 km

Uranus 51,118 km

Neptune - 49,528 km

Table 2 Listed are the equatorial diameters of the planets. Source: Universe, Freedman and Kaufmann, seventh edition, 2005

Slope

Students will learn about positive and negative slope and how to find slope using the Earth-Moon system.

The Earth-Moon System

Both the Earth and Moon orbit around their center of mass [8] . The Earth revolves around the Sun and the Moon revolves around Earth. Because the Moon orbits the Earth in the exact amount of time that it rotates on its axis, the Moon keeps the same side or face to us at all times. The Earth and the Moon both exert the same amount of gravitational pull on each other. The Moon's orbit is slightly elliptical and therefore wobbles as it spins in orbit. Because of the wobble, more than half the Moon, almost 60%, can be seen with careful monitoring.

In theory, it is surmised that the Moon was formed when an object impacted the Earth and the debris from the collision coalesced into the Moon and the Earth reshaped itself after impact.

The Earth-Moon system will be utilized because it is a fascinating idea that the Moon revolves around the Earth, in the first place, and that that revolution does not play a role in the formation of the solar system. Also, the Moon wobbles as it travels around the Earth allowing for varied distances from the Earth, positive and negative slope can be positioned by its revolution around the Earth, and it can be related to a coordinate plane.

Students will determine the imaginary lines from the Earth to the Moon as it orbits. The average distance of the Moon from the Earth is 384,400 km. [10] The range of wobble is 357,000 km to 407,000 km. Students will transfer their drawn lines to a coordinate plane and find the rise and run which determines slope.

Pythagorean Theorem and Trigonometric Ratios

Students will find distances between two points and measures of angles using the Pythagorean Theorem and trigonometric ratios. This will be demonstrated by the Earth-Moon parallax and the Sun parallax.

Parallax

Parallax is how the position of an object appears to change depending on the vantage point taken. For example, if one is standing before a television set below two paintings, standing to the right of the set will yield the left painting as being the background to the set. However, when standing to the left of the television set the right painting will become the background. Now, if the television set is positioned away from the wall of paintings to the center of the room each respective painting will still be the background but appear smaller. Therefore, the closer the object is to the background, the larger the parallax. Parallax is used to find measurements of angles.

The Earth-Moon parallax is based on the position of the observer on the Earth with relation to the center of the Earth and the path of the Moon. The angle of the parallax is created by the distance to the Moon from the observer and back to the center of the Earth. The Moon just above the horizon will create a larger angle parallax than the Moon at a 45° angle from the horizon. A zero parallax will be created when the Moon is directly overhead. The Sun parallax is similar but the distances from the Earth to the Sun will be greater and the parallax angles will be smaller. (See Figure 4)

It is imperative to discern the distances of celestial bodies such as the Sun, the planets, and the stars. For example, the distance from the Earth to the Moon allows for calculation of space travel to the Moon, and, information about light and heat is provided by understanding our distance to the Sun. Also, stars, which include our Sun, emit energy to varying degrees. By knowing the distance of stars to Earth astronomers can calculate the luminosity of a star and compare it to the Sun for intensity. Knowledge obtained by this comparison reveal how hot the star is. The best way to measure distances to celestial objects is the parallax.

(image available in print form)

Figure 4 The diagram shows the Moon traveling around the Earth and the various angles the Moon creates with the Earth as a result of the shifting positions of the Moon. Source: author.

Parallax will be used to instruct the Pythagorean Theorem and trigonometric ratios because it has an engaging diagram, and, it also has the appealing notion that the changing of position will change the perspective of an object. A minimum and maximum of angle measurement can be determined, it can be related to a coordinate plane, it allows for simple calculation, and grasping of concept.

Students will determine the right angles that can be formed using the Earth-Moon parallax. Students will apply the Pythagorean Theorem; the square of the hypotenuse is equal to the sum of the squares of the legs, and the trigonometric ratios, sine, cosine, and tangent, to find distances and angle measurements. (The same can be applied to the Sun parallax.)

Arc Lengths and Area of Sectors

Students will determine measures of arc lengths and areas of sectors using Kepler's Laws and the equatorial circumference of the Earth.

Kepler's Laws

Johannes Kepler could not ignore small discrepancies in the orbits of the planets when predicting planetary positions. He tried unsuccessfully to prove that the orbits of the planets were circular and almost succeeded with Mars, however, the small discrepancies led him to conclude that the orbits must be elongated. He abandoned the circular theory and based his work on ellipses. He was now able to predict planetary positions with more accuracy.

Kepler's first law concerns the orbits of planets around the Sun. Until this time it was thought that the planets' orbits were circular. But, Kepler's first law states that the planets orbit in an ellipse (see Figure 5a and Figure 5b). The eccentricity of the ellipse is quite small but not small enough to be circular. In fact, the use of their near circular path will allow the assumption of circular orbits to determine measurements of arc lengths and areas of sectors that will be used in the student lessons. The eccentricity (the deviation of an ellipse from a circle) for Venus is .007, Earth .017, Neptune .010 and so on in ascending order. The smaller the eccentricity the nearer to a circle is the orbit.

(image available in print form)

Figure 5a An ellipse is an elongation that has two foci (plural for focus) that creates its shape. An eccentricity of zero, e=0, will yield a circle. The solar system has only one focus, the Sun, as illustrated above. Source: author.

(image available in print form)

Figure 5b Take another more familiar example of an ellipse: the circular cone. A plane intersects the top cone at an angle creating a conic section called an ellipse as opposed the plane intersecting the bottom cone creating a conic section that is a circle. Source of figure: www.Wikipedia.org.

Kepler's second law deals with how the planet moves around its orbit. Each planet cuts out the same area in the same amount of time whether the planet is further away from the Sun or on the opposite side of the Sun and nearer to it. (See Figure 6 below)

(image available in print form)

Figure 6 In the same time interval, when a planet is furthest away from the Sun, called the aphelion, it carves out an area that is equal to the area carved out when the planet is nearest the Sun, the perihelion. Source of shape and shaded areas: www.Wikipedia.org. Descriptive enhancements and removal of one focus were done by the author.

Kepler's third law states that the more distant the planet is from the Sun the slower its orbital speed. Refer to Figure 6. (Note that speed differs from velocity in that speed is the rate at which an object moves, speed = distance/time, and velocity is both speed and direction, for example, the velocity is 50 mph due east.)

Planet: Distance from the Sun (106km)

Mercury: 57.1

Venus: 108.2

Earth: 149.6

Mars: 227.9

Jupiter: 778.3

Saturn: 1,427.0

Uranus: 2,870.0

Neptune: 4,497.0

Table 3 Scale for lesson plan 3: 1 inch=57.1, 1.89 inches=108.2, 2.62 inches=149.6, 3.99 inches=227.9, 13.63 inches=778.3, 24.99 inches=1,427, 50.26 inches=2,870, 78.76 inches=4,497 Source: The Cosmic Perspective, Bennett, Donahue, Schneider, Voit, fourth edition, 2007.

Kepler's Laws are utilized because the planet's sweep carves out an arc and sector that is measurable. There is a notable relationship between what happens on both sides of the Sun as the planet orbits the Sun, there are curious comparisons between arcs and sectors of planets regarding the measurement of the arc and the measurement of the area of the sector based on orbital speed of the planet and when a planet speeds up or slows down, and there is intriguing seasonal data regarding measurements of arcs and areas of sectors. Students will be motivated to apply formulas to get results.

Students will create a model of one planet's orbit using astronomical data scaled to workable numbers, map out a seasonal sweep, calculate the arc length, calculate the area of the sector, and compare it to the other planets. Conjectures will be made regarding measurements of arc lengths and areas of sectors between planets.

Circumference of the Earth, Great Circle

The circumference of the Earth was first estimated by Eratosthenes by the formula 7/360*circumference of Earth=5000 stadia in 240 B.C. Applying modern day measurements it is amazing how accurate he was. He estimated the circumference of the Earth to be 250,000 stadia based on the Greek stadium. Today, we figure a stadium to be 1/6 of a kilometer and Eratosthenes' estimate to be 42,000 km. Current measurements estimate the circumference to be 40,075 km at the equator and 40,008 km from North to South Pole. [11] (The Earth is not a completely circular.)

The equatorial circumference of the Earth is used to reinforce using the formulas to find measures of arc lengths and areas of sectors.

Students will determine the angles formed between two locations and the center of the Earth and apply the formulas. The equatorial circumference of the Earth will be used as the Great Circle in all examples in this exercise.

Classroom Activities

Sample lesson plans.

Lesson 1

This lesson is the introduction to the astronomical unit for the instruction of right triangles in geometry. It will lead to lesson 2 - the Pythagorean Theorem.

Objective: Students will become motivated to learn about the next concept, the Pythagorean Theorem, have a greater understanding that mathematics is important in comprehending our daily lives, and have a deeper appreciation of what mathematics can accomplish to that comprehension.

Procedure:

1. Students will place astronomical discoveries in the order of occurrence.
2. Students will determine what is common to all discoveries.
3. Students will discuss how mathematics plays a part in each discovery.
4. Discussion will include limited history on astronomy and how mathematics played a part in it, and a few examples of misconceptions about the solar system.
5. Students will evaluate a problem and note that it is unsolvable unless they learn the next concept-the Pythagorean Theorem. This problem will be revisited after the Pythagorean Theorem is discussed.

Materials: Two student worksheets and one teacher discussion notes sheet (see Appendix B).

Assessment: Student questions and student participation in the discussion.

Lesson 2

Students will use the Pythagorean Theorem to find distances. (Note: Lesson 2 can also be used to teach trigonometric ratios by focusing on angle measurements and by substituting solving right triangles by trigonometric ratios wherever Pythagorean Theorem appears.)

Objective: Students will use the Pythagorean Theorem, the Earth-Moon parallax, and the Sun parallax to find distances between two points on a right triangle.

Procedure:

1. State and explain the Pythagorean Theorem. (Power point or overhead may be used.)

2. Demonstrate the Pythagorean Theorem by using the Earth-Moon parallax.

(See Appendix C)

3. Reinforce the concept by having the students apply the Pythagorean Theorem using the Sun parallax. (Use the figure in Appendix C and substitute the word Sun for the word Earth and the word Earth for the word Moon and elongate the lines. The distance from the Earth to the Sun is $149.6*10^{6}$ km and the Sun's radius is

$6.95*10^{5}$ km.

4. Students will also solve for distances using the following astronomical situations.

This can be presented to the students in worksheet form.

> a. Two people live in two different cities. They are both looking through a telescope at the Hubble space station 579 km above Earth. How far are they looking if they are equal distances apart from each other and how far are they looking if the distances from each other vary? (Draw a triangle with the two people as two vertices on the ground and the space station as the third angle. Drop a perpendicular from the space station to the ground to form two right triangles. Solve for distances. Other measurements for distances may be selected from a textbook or another resource. Offer different scenarios to solve for the legs of the right triangle.)
> b. Two people in two different cities are talking to each other using a cell phone. The cell phone satellite [12] is 643 km above Earth. What is the distance from the satellite to the ground where the people are using the cell phones? What is the distance when the people's locations change as they drive their cars to other destinations? (Use the same type of triangle as in a. above and create scenarios to solve for the legs of the right triangle.)
> c. Revisit the problem presented in lesson 1 and solve.

Materials: Projector for power point or overhead projector for concept presentation, worksheets for the Earth-Moon parallax, the Sun parallax, and the above problems, and rulers.

Assessment: Class participation and submitted worksheets.

Lesson 3

Objective: Students will determine the measures of arc lengths and the areas of sectors.

Procedure:

1. Kepler's Laws will be explained (power point or overhead projector may be used).
2. Students will works in groups of two or three.
3. Students will create a model of their planet's orbit around the Sun.
4. Students will map out their planet's seasonal orbit around the Sun.
5. Students will calculate the arc length and area of the sector of their season using formulas.
6. Students will compare their calculations to the other planets.
7. Students will make conjectures about orbital paths and orbital speed of planets.
8. Additional exercises: Students will find the measure of an arc length and the area of a sector between two locations on a globe and compare the measure of the arc length to the same distance on the equatorial circumference of the Earth and to the distance listed on a map. Since the Earth is not completely round and all circumferences taken around the Earth may not all be equal, there will be some discrepancies in answers when only the equatorial circumference of the Earth is utilized as the Great Circle and comparisons made. Examples of locations and angles determined: the distance between the North and South Pole will yield a 180° angle, the distance between the North Pole and the equator will yield a 90° angle, the distance between the South Pole and a location in Patagonia in South America will yield a 45° angle, etc. (See Table 2 to determine the radius of the Earth.)

Materials: Data sheet listing orbital speeds and planet distance from the sun (see Table 3 and Appendix D), comparison of measure of arc lengths and area of sectors of planets according to position from the Sun worksheet, heavy poster board, small knife to carve out the arc of the season, balls for planets and Sun, protractors, rulers, and globes.

Assessment: Construction of model and completion of worksheet.

Appendix A

(table available in print form)

Table 5 Mercury and Venus do not have Moons. Mars has asteroid type satellites whose radii cannot be determined and are listed in the source textbook below by dimensions. For example: Phobos is listed by dimensions 13*10*9 km. There are many other satellites not listed above either by radius or by dimensions. Source: The Cosmic Perspective, Bennett, Donahue, Schneider, Voit, fourth edition, 2007.

Appendix B

Geometry Warm-up Worksheet

Put the following events in order:

___ 1. Thales - Proposed the first known model of the universe that did not rely on

supernatural forces.

___ 2. Plato - Asserted that planet motion must be in perfect circles.

___ 3. Eratosthenes - Accurately established the circumference of the Earth.

7/360 * circumference of the Earth = 5,000 stadia (pl. Greek unit of distance)

___ 4. Ptolemy - His Earth-centered model of the universe remained in use for some

1,500 years.

___ 5. Copernicus - His model of the solar system provided a geometric layout for the

Sun-centered solar system.

___ 6. Tycho - His naked-eye observatory worked like a giant protractor.

___ 7. Kepler - His three laws of planetary motion describe the orbits of planets as

being elliptical and not circular.

Perihelion distance = a(1 - e)

Aphelion distance = a(1 + e)

___ 8. Newton - Force = mass * acceleration

F = ma

___ 9. Einstein - $E = mc^2$

Energy = mass of the object * the speed of light squared

The Sun converts mass to energy through a similar process of nuclear

fusion. The reverse of Einstein's equation is also true. Energy can be

transformed into mass.

PQ _|_ QR

Your pen pal calls you on the telephone and tells you to go outside and look at the full moon. It is especially bright tonight. The moon is above your pen pal but it is at an angle for you. How much further are you looking in the distance to see the moon?

(image available in print form)

Figure 7 The figure above is not drawn to scale. The diagram was drawn as such for clarification purposes only. Consider the Earth a globe and that you and your pen pal are not necessarily on the edges of the planet. Source of numerical data: The Cosmic Perspective, Bennett, Donahue, Schneider, Voit, fourth edition, 2007 for noted distances and the figure was drawn by the author.

Teacher discussion notes:

1. What is common for each entry in the list of the geometry warm-up?

2. Tie in the following during class discussion:

a. Ancient mathematicians were politically sought and valued.
b. Mathematicians were needed to accurately explain the events of the universe. For example: the eclipse.
c. Mathematicians worked out the mathematics to predict celestial happenings.]
d. Mathematicians needed to invent new mathematics to explain astronomical events.
e. Planet means "wanderer" and the planets were named after Roman gods.
For example: Jupiter means the king of the gods.

3. Misconceptions:

Did you know . . .

there are eight planets?

the Moon wobbles?
the asteroids in the asteroid belt are great distances apart from each other?
the hottest planet is not Mercury?

Appendix C

(image available in print form)

Figure 8 The Figure simulates the Earth-Moon parallax using only the right triangles. Right triangles can be drawn in the lower half of the Earth to receive four more. The distances of the legs are the same as in the figure of lesson 1 for all right triangles. In order to save the figure in lesson 1 as the challenge problem to be solved at the end of this section scale down the numbers so that, for example, on paper, 1 inch will equal 1,000 km. (Also, the school textbook can supply additional data and consider that the Moon wobbles and is not always the same distance from the Earth.) Instruction can be presented by power point or using the overhead projector.

Appendix D

Average Sidereal Period

Planet Orbital: Speed (km/s) (years) (days)

Mercury: 47.9 0.241 87.969

Venus: 35.0 0.615 224.70

Earth: 29.79 1.000 365.256

Mars: 24.1 1.88 686.98

Jupiter: 13.1 11.86

Saturn: 9.64 29.46

Uranus: 6.83 84.10

Neptune: 5.5 164.86

Table 4 Students will scale down the orbital speed to 1 inch equals 20,000,000 km/s ratio in order to measure out the distance of the arc on their model. Students will also need Table 3. Source: Universe, Freedman, Kaufmann, seventh edition, 2005.

Appendix E: Implementing District Standards

New Haven, Connecticut District Standards

The mathematics vision statement for the New Haven Public Schools District envisions students as becoming mathematical problem solvers. Students will be instructed so that they will learn to apply reason, make connections of mathematical ideas to real-life situations and to other subjects, have an active participation in subject material, and solve problems as a team.

Students lessons are designed to provoke thought using astronomical concepts. Students will have hands on participation, and will, in some lessons, work in teams.

Standards for first level algebra include number concepts. They specifically require students to be able to make comparisons and determine patterns. They also set standards for students to work in the real number system. Setting up graphs accomplishes the first goal. Finding slope will accomplish the second goal by having students work with lines in a coordinate plane and evaluate for positive and negative slope.

Standards for geometry include the applications of formulas, trigonometry, and the Pythagorean Theorem. Students will work with parallax to apply the Pythagorean Theorem and trigonometric ratios. Students will work with planets and their orbits in order to apply measurement of arc length and area of sector formulas.

Resources and Bibliography

Teachers

Peterson, Ivars. Newton's Clock. W. H. Freeman & Co., 1993.

Newton's Clock is a good book to read about the history of astronomy. It is not a textbook, and, therefore, is easy reading. It provides the useful background information needed to understand how astronomy evolved which students may question about. Ivars is a mathematics and physics writer for the Science News.

Comins, Neil F.. Heavenly Errors. Columbia University Press, 2001.

This is another good book that provides insight to what students and people think about the universe and some of the misconceptions they may have. The book shows, in light of corrected information, that some people and students never correct their erroneous beliefs. The book's strength is that it relates the incorrect beliefs and then follows it with correct information. Comins is a professor of physics and astronomy at the University of Maine.

Bennett, Jeffrey, Megan Donahue, Nicholas Schneider, and Mark Voit. The Cosmic Perspective. 4th ed. Addison Wesley, 2007.

A resourceful college textbook that is a good complement to Universe by Freedman and Kaufmann. The reading is easier in this textbook than the latter.

Freedman, Roger A., and William J. Kaufmann III. Universe. 7th ed. New York: W. H. Freeman & Co., 2005.

A resourceful college textbook that is a good complement to The Cosmic Perspective by Bennett, Jeffrey, Donahue, and Schneider. The reading is a bit more difficult than the latter textbook.

Sagan, Carl. Contact. New York: Pocket Books, 1985.

This is an enjoyable book with lots of speculation to stimulate the imagination and to consider the possibilities.

Students

Note

Check your school library for suitable books if students are interested in obtaining more information about astronomy. All school library books may not have the same titles as listed in the student resources below. Student books should offer varied information that includes facts, colorful pictures, and projects to do.

Bramwell, Martyn. Mapping the Planets and Space. Minneapolis: Lerner Publishing Co., 1998.

This book shows how to make a telescope, how to listen to space, and has facts about the solar system.

Garlick, Mark A.. Astronomy A Visual Guide. Firefly Books, 2004.

This book has good colorful photographs, facts, and some statistics.

Moore, Sir Patrick. Guide to Stars and Planets. Firefly Books, 2005.

This book focuses on constellations.

Camp, Carole Ann. American Astronomers Searchers and Wonderers. Enslow Publishers Inc., 1996.

This is an inspiring book about people and their work in astronomy. Some biographies include Edwin Hubble, Carl Sagan, and Annie Jump Carmon.

Sagan, Carl. Contact. New York: pocket books, 1985.

A good book for students as well as adults to stimulate the imagination and the possibilities.

Websites for teachers

http://solarsystem.nasa.gov/profile.cfm?Object=Dwarf

The focus of this NASA website is on the solar system. It features a kids and education webpage.

http://www.perthobservatory.wa.gov.au/information/planet_defn.html

This web address is to the Australian Observatory. It releases news and its heavenly sittings.

http://pds.jpl.nasa.gov/planets/welcome.htm

The focus of this NASA website is on the planets, and, the explorers with audio.

http://www.nineplanets.org/

This website was designed by a software engineer. Its focus is on the solar system and it offers lesson plans and films.

http://www.nasm.si.edu/etp/

This is the Smithsonian National Air and Space Museum website that focuses on the solar system.

http://school.eb.com/eb/print?articleId=58395&fullArticle=true&tocId=9058395

This is the Encyclopedia Britannica online school edition website. It features a search engine.

http://en.wikipedia.org/wiki

This is a free encyclopedia website.

Websites for students

www.eb.com

This is an Encyclopedia Britannica Online School and Library Site.

www.iconn.org

This is a Statewide Library Catalogue.

Notes

1. Removing a visual to reveal the coordinate plane was shown in a class I attended and is not an original idea by the author.

2. Universe, Freedman, Kaufmann III, seventh edition, 2005.

3. Newton's Clock, Peterson, W.H. Freeman & Co., 1993.

4. I would like to call to attention the book Heavenly Errors, authored by Neil F. Comins. This book is also listed in the Teacher's Resources section and annotated. The book is excellent reading for misconceptions about the universe. It mainly dwells on incorrect information that people have about the universe and their reluctance to change their opinion or their unwillingness to relearn correct facts. The same is true for students in universities as the book reveals. The author corrects all misinformation so the book is great learning and fun.

5. Dr. Sarbani Basu, Yale University Professor and Director of Graduate Studies Astronomy Department.

6. All figures are illustrations duplicated from a website or were drawn or enhanced by the author for clarification purposes only. They are not to scale. Figures from the website were cropped. Other representations of the solar system can be found in any astronomy textbook or on any astronomical website.

7. I would like to thank Dr. Sarbani Basu, Yale University Professor and Director of Graduate Studies Astronomy Department and professor of the seminar, for her lectures and interesting ways of looking at the universe.

8. Website: www.Wikipedia.org

9. Center of mass is the point around which two or more objects orbit. The Moon follows the center of mass of the Earth as it orbits the Earth and the Earth follows the center of mass of the Sun as it orbits the Sun. The center of mass is not always at the center of the object but where the object is the heaviest.

10. The Cosmic Perspective, Bennett, Donahue, Schneider, Voit, fourth edition, 2007.

11. www.NASA.org

12. Jennifer Esty, Science Teacher at McCabe Center, coordinator and participant of the astronomy seminar, gave me the idea of using a cell phone satellite to teach the Pythagorean Theorem.