Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
2007 Volume III: The Physics, Astronomy and Mathematics of the Solar System

The Mathematical Dynamics of Celestial Navigation and Astronavigation

Curriculum Unit 07.03.09
by Kenneth William Spinka

INTRODUCTION

Astronomical reference points have prevailed as universal reference points for position-fixing derived with a variety of mathematical methods to determine the position of a ship, aircraft or person on the surface of the Earth until quite recently, with the advent of inexpensive and highly accurate satellite navigation receivers or GPS. The Algebra, Calculus, Geometry, and Trigonometry processes of Celestial Navigation or Astronavigation are the subject for this presentation of the Math Curriculum.

This curriculum unit will assist in teaching about these subjects in the high school classroom. Each mathematical lesson plan will address one of three mutually exclusive methods for calculating a navigator's position on earth using the astronomical references of celestial navigation: the **Intercept Method** , or **Marc St Hilaire Method** ; the **Longitude by Chronometer Method** ; and the **Ex-Meridian Method** . Each mathematical lesson plan will address one of the four New Haven Math Curricula: Algebra, Calculus, Geometry, or Trigonometry. These twelve (12) lessons will reference common celestial objects: the Sun; Moon; other planets; and fifty-seven (57) "navigational stars" described in nautical almanacs.

The **Intercept Method** , or **Marc St Hilaire Method** , delineates a position line on which the observer is situated by taking two sets of sights at a time interval of approximately three (3) hours and *run-on* the earlier position line to the time of the second observation to give a "fix." This method compares the true zenith distance and calculated zenith distance of an astronomical object to locate the intercept or exact position on a position line.

The **Longitude by Chronometer Method** , known by mariners as "Long by Chron", delineates a position line on which the observer is situated, by also taking two sets of sights at a time interval of approximately three (3) hours and *run-on* the earlier position line to the time of the second observation to give a "fix." This method uses an assumed latitude and calculates the longitude crossing that position line.

The **Ex-Meridian Method** delineates a position line on which the observer is situated and is usually used when the sun is obscured at noon or as a result a meridian altitude is not possible. The navigator measures the altitude of the sun as close to noon as possible and then calculates where the position line lies. This method uses an assumed longitude and calculates the latitude crossing that position line.

To explore these historical and mathematical accomplishments in navigation, students will first construct a sextant. Then they will participate in twelve (12) lesson plans that assist teaching the aforementioned navigational methods in the classroom with the astronomical content and dynamics from the Frontiers of Astronomy seminar, 2007, at the Yale New Haven Teachers Institute, Yale University, New Haven, CT.

BACKGROUND

Celestial Navigation , also known as A **stronavigation** , is a position-fixing process that has enabled sailors to cross featureless oceans with certainty and target unsighted land with precision. Astronavigation uses angular measurements or sightings between the horizon and a common celestial object to perform navigational problem-solving. Although the Sun is the most often measured celestial object, more sophisticated navigators are prepared to use our Moon, other planets, or one of 57 "navigational stars" described in nautical almanacs to observe the positions of these celestial objects relative to the observer and a known location. In ancient times, the vessel's home port or home capital was used as the known location. With the rise of the British Navy and merchant marine, the Greenwich Meridian or Prime Meridian at Greenwich, England eventually became the starting location for most celestial almanacs. Astronavigation calculates angles between objects in the sky (celestial objects) and the horizon to locate one's position on the globe. At any given instant of time, any celestial object such as the Earth's Moon, the planet Jupiter, or the navigational star Spica, one of the brightest stars in the nighttime sky, will be located directly over a particular geographic position on the Earth. This geographic position is known as the celestial object's sub-point and its location, defined by latitude and longitude, can be determined from the tables of nautical or air almanacs.

These almanacs describe the positions and movements of celestial bodies, including the sun, moon, planets, and 57 stars chosen for their ease of identification and wide spacing. The Almanac specifies for each whole hour of the year the position on the Earth's surface at which each body is directly overhead. The Sun, Moon and Planets are perceived independently and are therefore specified separately however, only the star Aries is specified, while the other stars are assigned a set angular distance. The navigator can extrapolate by means of navigational tables to acquire the position of each object for each minute of time.

In Great Britain a nautical almanac has been published annually by the HM Nautical Almanac Office, ever since the first edition was published in 1767. Also commercial almanacs were produced that combined other information. A good example would be Brown's - which commenced in 1877 - and is still produced annually, its early twentieth century subtitle being "Harbour and Dock Guide and Advertiser and Daily Tide Tables". This combination of trade advertising, and information "by permission... of the Hydrographic Department of the Admiralty" provided a useful compendium of information. More recent editions have kept up with the changes in technology - the 1924 edition for example had extensive advertisements for coaling stations.

The "Air Almanac" of the United States and Great Britain tabulates celestial coordinates at 10 minute intervals. The Sokkia Corporation's annual "Celestial Observation Handbook and Ephemeris" tabulates daily celestial coordinates (to a tenth of an arc-second) for the Sun and nine stars, Polaris and eight others.

To determine the position of a ship or aircraft by celestial navigation or astronavigation, the navigator uses a sextant to take a "sight" to measure the apparent height of the celestial object above the horizon, and notes the time from a marine chronometer. The object's position is then looked up in the Nautical Almanac for that particular time and after allowance for refraction, instrument error and other errors, a position circle on the Earth's surface is calculated.

Previously, **latitude** was measured either at noon (the "noon sight") or from the angle between Polaris, the North Star or the Pole Star, and the horizon. Polaris always stays within 1 degree of celestial North Pole. At the North Pole, which is 90 degrees latitude, Polaris is directly overhead at an altitude of 90 degrees. At the equator, which is zero degrees latitude, Polaris is on the horizon with zero degrees altitude. Between the equator and the North Pole, the angle of Polaris above the horizon is a direct measure of geographic or terrestrial latitude. If a navigator measures the angle to Polaris and finds it to be 10 degrees from the horizon, then he is on a circle at about North 10 degrees of geographic or terrestrial latitude. The latitudinal angles are reversed in the Southern hemisphere, where the equator is still zero degrees however; the South Pole is at an altitude of 90 degrees. In the Southern Hemisphere, a constellation called the Southern Cross, or Crux, points to the place the southern stars circle around. The Southern Cross is a key constellation for navigating south of the equator. Navigational angles are measured from the horizon because locating the point directly overhead, the zenith, is difficult. When haze obscures the horizon, navigators use artificial horizons, which are bubble levels reflected into a sextant.

Azimuth Latitude can also be determined by the direction in which the stars travel over time. If the stars rise out of the east and travel straight up you are at the equator, but if they drift south you are to the north of the equator. The same is true of the day-to-day drift of the stars due to the movement of the Earth in orbit around the Sun; each day a star will drift approximately one degree. In either case if the drift can be measured accurately, simple trigonometry will reveal the latitude.

Comparatively, **longitude** can be measured in the same way. If one can accurately measure the angle to Polaris, a similar measurement to a star near the eastern or western horizons will provide the longitude. The problem is that the Earth turns about 15 degrees per hour, making such measurements dependent on time. A measure only a few minutes before or after the same measure the day before creates serious navigation errors. Before precision chronometers were available, longitude measurements were based on the transit of the moon, or the positions of the moons of Jupiter. For the most part, these were too difficult to be used by anyone except professional astronomers.

The longitude problem took centuries to solve, as presented in the book **Longitude: The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time by Dava Sobel** . As opposed to a degree of latitude, which is always sixty nautical miles or about 111 km (69 statute miles, each of 5280 feet), a degree of longitude varies from 0 to 111 km: it is 111 km times the ***cosine*** of the latitude, when the distance is laid out on a circle of constant latitude. More precisely, one degree of longitude = $(111.320 + 0.373\sin^2\Phi)\cos\Phi$ km, where Φ is latitude).

Two useful methods evolved during the 1700s, and both are still practiced today: lunar distance without a chronometer; and lunar distance with a chronometer, or an accurate timepiece.

The first method, called "lunar distances," was refined in the 18th century. Although it is only used today by sextant hobbyists and historians, this method is dependable, and can be used when a timepiece is not available or the accuracy of a timepiece is suspect during a long sea voyage. The navigator precisely measures the angle between the moon and a body like the sun or a selected group of stars lying along the ecliptic. That angle, after it is corrected for various errors, is the same at any place on the surface of the earth facing the moon at a unique instant of time. Old almanacs listed angles in tables, from which the navigator could look up the measured angle, and then the time at Greenwich. Modern handheld and laptop calculators can perform the calculation in minutes, allowing the navigator to use other acceptable celestial bodies than the original nine. After identifying Greenwich Time (GMT), the navigator can calculate longitude. While GMT could refer to either an astronomical day starting at noon or a calendar day starting at midnight, Universal Time (UT) was established in 1928 as more precise. The term Greenwich Mean Time persists however, and is in popular usage to this day in reference to civil timekeeping, whereas there are several versions of Universal Time [see glossary].

The considerably more popular method was, and still is, to use an accurate timepiece to directly measure the time of a sextant sighting. The need for accurate navigation led to the development of progressively more accurate chronometers in the 18th century. Today, time is measured with a chronometer, a quartz watch, a shortwave radio broadcast from an atomic clock, or the time displayed on a GPS display. A quartz wristwatch normally maintains time within a half-second per day. If it is worn constantly, keeping it near body heat, its rate of drift can be measured with the radio, and by compensating for this drift, a navigator can keep time to better than a second per month. Navigators check chronometers from his sextants, at geographic markers surveyed by professional astronomers. This is now a rare skill, and most harbor masters cannot locate harbor markers.

Historically, three chronometers were kept in gimbals, so that they remained level, in a dry room near the center of the ship. They were used to set a watch for the actual sight, so that no chronometers were ever risked to the wind and salt water on deck. Winding the chronometers was a crucial duty of the navigator, logged as "chron. wound." for checking by line officers. Navigators also set the ship's clocks and calendar.

Early navigators on the northern hemisphere could determine their latitude by measuring the angular altitude of the North Star. The earliest sailors simply used measurements of hand or finger widths to determine latitude; later, cross-staffs and astrolabes were developed to increase the precision of the sighting. Eventually quadrants, octants, and sextants were invented, along with the introduction of printed tables of the positions of the sun, moon, and stars for various times and days of the year. Determining latitude by the sun is more complicated, since one has to measure the sun's altitude at noon (or: the sun's highest point in the sky for a given day) which changes during the year for a given location.

The celestial line of position concept was discovered in 1837 by Thomas Hubbard Sumner when, after one observation he computed and plotted his longitude at more than one trial latitude in his vicinity--and noticed that the positions lay along a line. Using this method with two bodies, navigators were finally able cross two position lines and obtain their position, determining both latitude and longitude. Later in the 19th century came the development of the modern *Marc St. Hilaire* intercept method; with this method the body height and azimuth are calculated for a convenient trial position, and compared with the observed height. The difference in arc-minutes is the nautical mile "intercept" distance that the position line needs to be shifted toward or away from the direction of the body's sub-point. Two other methods of reducing sights are the longitude by chronometer and the ex-meridian method.

(image available in print form)

Figure 1 , *Source:* [8]

Celestial navigation usually requires a *marine chronometer* to measure time, a *sextant* to measure the angles, an *almanac* giving schedules of the coordinates of celestial objects, a set of sight reduction tables to help perform the height and azimuth computations, and a chart of the region. Small handheld computers, laptops and even scientific calculators enable modern navigators to "reduce" sextant sights in minutes, by automating all the calculation and/or data lookup steps.

A number of increasingly accurate instruments were developed over many years to measure navigational angles, including the kamal (a rectangle of wood cut to fit the distance from the horizon to the star with a piece of knotted string attached, which could be held in the teeth, guaranteeing that an "arm's length" distance would remain constant), astrolabe (a compact instrument used to observe and calculate the position of celestial bodies before the invention of the sextant), octant (an instrument for observing altitudes of a celestial body from a moving ship or aircraft) and sextant (an instrument for measuring angular distances used especially in navigation to observe altitudes of celestial bodies, as in ascertaining latitude and longitude). The sextant and octant are most accurate, and an improvement over the astrolabe, because they measure angles from the horizon, eliminating errors caused by the placement of an instrument's pointers, and because their dual mirror system neutralizes relative motions of the instrument, showing a steady view of the object and horizon.

(image available in print form)

Figure 2 , *Source: Spinka, 2007. The marine **SEXTANT*** measures the altitude of a celestial object above the horizon.

Navigators measure distance on the globe in degrees, arc-minutes and arc-seconds. A nautical mile is defined as 1852 meters or 1.150779 miles, but is also (not accidentally) one minute of angle along a meridian on the Earth. Sextants can be read accurately to within 0.2 arc-minutes. So the observer's position can be determined within (theoretically) 0.2 miles, about 400 yards (370 m). Most ocean navigators, shooting from a moving platform, can achieve a practical accuracy of 1.5 miles (2.8 km), more than close enough to navigate safely when out of sight of land.

The U.S. Air Force and U.S. Navy continued instructing military aviators on the use of astronavigation until 1997 as a redundant method to GPS, because:

1 it can be used independently of ground aids
2 has global coverage
3 cannot be jammed (except by clouds)
4 does not give off any signals that could be detected by an enemy[4]

Astronavigation was used in commercial aviation up until the early part of the jet age; it was only phased out in the 1960s with the advent of inertial navigation systems.

A variation on terrestrial celestial navigation was used to help orient the Apollo spacecraft traveling to and from the Moon. To this day, space missions, such as the Mars Exploration Rover use celestial navigation to guide spacecraft throughout the solar system.

As early as the mid-1960s, advanced electronic and computer systems had evolved enabling navigators to obtain automated celestial sight fixes. These systems were used aboard both ships as well as US Air Force aircraft, and were highly accurate, able to lock onto up to 11 stars (even in daytime) and resolve the craft's position to less than 300 feet. The SR-71 high-speed reconnaissance aircraft was one example of an aircraft that used automated celestial navigation. These rare systems were expensive, however, and the few that remain in use today are regarded as backups to more reliable satellite positioning systems. Similar systems have been used on spacecraft such as Deep Space 1. As part of NASA's New Millennium program, the primary goal of Deep Space 1 was the testing of technologies to lower the cost and risk of future missions.

Modern practical navigators nearly always use celestial navigation in combination with *satellite navigation* to correct a *dead reckoning track* , or a course estimated from a vessel's position, angle and speed. Using multiple methods helps the navigator detect errors, and simplifies procedures. When used this way, a navigator will from time to time measure the sun's altitude with a sextant, then compare that with a pre-calculated altitude based on the exact time and estimated position of the observation. On the chart, one will use the straight edge of a plotter to mark each position line. If the position line shows one to be more than a few miles from the estimated position, one may take more observations to restart the dead-reckoning track.

Dead reckoning is the process of estimating a present position by projecting course and speed from a known past position.

(image available in print form)

Figure 3 , ***Source:*** *Spinka, 2007. Dead reckoning estimates positions by projecting the course and speed from past positions.*

Dead reckoning navigation plots the 9am position as illustrated by the triangle; then by extrapolating the course from that position with a known average speed the position at 9:30am and 10am can be estimated respectively, as illustrated by the corresponding semi-circles. While this method of navigation is no longer considered primary, dead reckoning is frequently used as a backup navigation method should the primary navigation system fail. Clearly, the precision of dead reckoning can be compromised by both *set* and *drift* , which are characteristics of the current or the velocity of water over the ground in which a ship is sailing. While drift is the magnitude of the current, typically measured in knots, set is the bearing in the direction the current is flowing, typically measured in degrees clockwise from either magnetic or true (geographical) North.

THEORY OF NAVIGATION

Position determination in astronavigation is primarily a matter of converting one set of coordinates to the other. This is done by the solution of a spherical triangle called the navigational triangle.

The concept of the spherical navigational triangle is graphically shown in the illustration, a diagram on the plane of the celestial meridian. The celestial meridian passes through the *zenith* of the observer, and is therefore a vertical circle of the horizon system. Elements of both systems are shown in the illustration below, indicating that an approximate solution can be generated graphically.

(image available in print form)

Figure 4, Source: [8]. *The navigational triangle across the spherical surface of Earth.*

The vertices of the navigational triangle are the elevated pole (P_n), the zenith (*Z*), and the celestial body (*M*). The angles at the vertices are, respectively, the meridian angle (*t*), the azimuth angle (*Z*), and the parallactic angle (*X*). The sides of the triangle are the codeclination of the zenith or the colatitude (colat) of the observer, the coaltitude or zenith distance (*z*) of the body, and the codeclination or polar distance (*p*) of the body.

A navigational triangle is solved, usually by computation, and compared with an observed attitude to obtain a line of position by a procedure known as sight reduction. With the emergence of electronic computers and hand-held calculators, sight reduction has been performed increasingly with limited use or elimination of tables.

To establish a celestial line of position, the navigator observes the altitude of a celestial body, noting the time of observation. An observation is made with a sextant, the name of which derives from early instruments that had an arc of one-sixth of a circle. By means of the double reflecting principle, the altitude of the body is double the amount of arc used. The marine sextant uses the visible horizon as the horizontal reference. An air sextant has an artificial, built-in horizontal reference based upon a bubble or occasionally a pendulum or gyroscope. The sextant altitude, however measured, is subject to certain errors, for which corrections are applied. Time is repeatedly mentioned as an important element of a celestial observation because the Earth rotates at the approximate rate of 1 minute of arc each 4 seconds of time. An error of 1second in the timing of an observation might introduce an error in the line of position of as much as one-quarter of a mile. Time directly affects longitude determination, but not latitude. The long search for a method of ascertaining longitude at sea was finally solved two centuries ago by the invention of the marine chronometer, a timepiece with a nearly steady rate.

Coastal Navigation is similar to Celestial Navigation. Instead of celestial reference points, Coastal Navigation determines the location of a moving vessel with reference to a different set of fixed geographical objects such as a lighthouse as illustrated in Figure 5.

(image available in print form)

Figure 5, Source: *Spinka, 2007. Comparative navigational references: a lighthouse and a celestial object.*

1. When navigation references of a lighthouse and a celestial object are compared, the lighthouse always remains at the same geographical location, independent of time.
2. The position of the celestial object **CN** however, is dependent on time, so that the exact time at the moment of the observation and measurement is required for calculations with this variable.
3. The distance from the celestial object at **CN** to the surface of the Earth, at point **CN'** is of such a large magnitude that the results are far more difficult to map when compared to those of the lighthouse reference.

1. **Dec.** - declination (from the 'Nautical Almanac').

(image available in print form)

Figure 6, Source: *Spinka, 2007. Ecliptic declination angles.*

2. **GHA** - Greenwich Hour Angle - (from the Nautical Almanac).

3. **LHA** - Local Hour Angle.

(image available in print form)

Figure 7, Source: *Spinka, 2007. Greenwich Hour Angle and Local Hour Angle comparisons.*

DETAILS

1. **Chronometer**

Correct the deciphered value on the chronometer according to these principles:

- **Ch** - reading of the chronometer
- **+St** - state of the chronometer / $\mathbf{U_o}$ - time GMT
(+ / - as required)
- If the center half was used, add that time starting from the center half.

2. **Sextant**

- Measure the height **CN** = $\mathbf{h_o}$
- Add the constant misconception of the sextant from the certificate = $\mathbf{c_c}$
- Add the mistake of the index measured before or after the measurement = $\mathbf{c_i}$
- Resulting in $\mathbf{h_o + c_c + c_i = h_s}$

LESSON PLANS

Navigational angles between the horizon and selected celestial objects are used to locate a position on the globe; and those angles translate directly to the mathematical models of Algebra, Calculus, Geometry, and Trigonometry of my assignments and within the latest New Haven Math Curriculum. Mathematical models of Algebra, Calculus, Geometry, and Trigonometry that explore astronavigation.

Spherical geometry, the geometry of the two-dimensional surface of a sphere, is an example of a non-Euclidean geometry and has important practical uses in navigation and astronomy. In plane geometry the basic concepts are points and line, whereas on the sphere points are similarly defined and the equivalents of lines are differently defined as "the shortest paths between points," which is called a geodesic. On the sphere the geodesics are the great circles, so the other geometric concepts are defined like in plane geometry but with lines replaced by great circles. In spherical geometry angles are defined between great circles, resulting in a spherical trigonometry that differs from ordinary trigonometry in many respects, including that the sum of the interior angles of a triangle exceeds 180 degrees.

Spherical geometry is the simplest model of elliptic geometry, in which a line has no parallels through a given point. Contrast this with hyperbolic geometry, in which a line has two parallels, and an infinite number of ultra-parallels, through a given point.

An important geometry related to that modeled by the sphere is called the real projective plane; it is obtained by identifying antipodes (pairs of opposite points) on the sphere. Locally, the projective plane has all the properties of spherical geometry, but it has different global properties. If the coordinates (longitude and latitude) of a point on the Earth's surface are (Θ, Φ), then the coordinates of the antipodal point can be written as $(\Theta \pm 180°, -\Phi)$. This relation holds true whether the Earth is approximated as a perfect sphere or as a reference ellipsoid. Specifically, this is non-orientable. In mathematics, an orientation on a real vector space is a choice of which ordered bases are "positively" oriented (or right-handed) and which are "negatively" oriented (or left-handed).

Angles are the most common type of numbers that astronavigators and celestial navigators calculate. The position of celestial bodies and other points on the surface of the earth can be defined and located by a description of angles. The sextant is the preferred instrument for measuring those angles, which are measured in the units: degrees, and minutes. While the sextant is calibrated to a complete circumference that encircles 360 degrees (360°), and displays one degree as the equivalent to 60 minutes; seconds of an arc are neither measurable with the precision of a sextant nor used in the process of astronavigation and celestial navigation. Since the angle measurement instrument - the sextant - is not precise enough to measure them. The smallest unit of angle used by navigators is the tenth of minute. Recently, the popularization of GPS devices added the 1/100 of minute.

The nautical mile (=1852 m) is a unit deliberately selected to simplify the conversions between spatial angles and linear distances. One nautical mile corresponds to an arc of one minute on the surface of earth. Angles and distances on the surface of earth are, therefore, equivalent. One exception is the minute of longitude, equivalent to one mile only near the Earth Equator. Another important equivalence is between time and degrees of longitude. Since the earth goes one complete turn (360°) in 24 hours, each hour corresponds to 15° of longitude, or 900 Nautical miles (NM).

LESSON PLAN OVERVIEW

The Introductory Lesson: Construct a Sextant

1. The **Algebra** Lessons.

a. The **Intercept Method** , or **Marc St Hilaire Method**
Title: Calculating Locations by The Intercept Method, or Marc St Hilaire Method.
Time: The time allocated is approximately 90 minutes.
Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.
Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the algebra with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Intercept Method, or Marc St Hilaire Method.
b. The **Longitude by Chronometer Method**
Title: Calculating Locations by The Longitude by Chronometer Method.
Time: The time allocated is approximately 90 minutes.
Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.
Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the algebra with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Longitude by Chronometer Method.
c. The **Ex-Meridian Method**
Title: Calculating Locations by The Ex-Meridian Method.
Time: The time allocated is approximately 90 minutes.
Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.
Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the algebra with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Ex-Meridian Method.

2. The **Calculus** Lessons.

a. The **Intercept Method** , or **Marc St Hilaire Method**
Title: Calculating Locations by The Intercept Method, or Marc St Hilaire Method.
Time: The time allocated is approximately 90 minutes.
Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.
Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the calculus with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Intercept Method, or Marc St Hilaire Method.
b. The **Longitude by Chronometer Method**
Title: Calculating Locations by The Longitude by Chronometer Method.
Time: The time allocated is approximately 90 minutes.
Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.
Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the calculus with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Longitude by Chronometer Method.
c. The **Ex-Meridian Method**
Title: Calculating Locations by The Ex-Meridian Method.
Time: The time allocated is approximately 90 minutes.
Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.
Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the calculus with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Ex-Meridian Method.

3. The **Geometry** Lessons.

a. The **Intercept Method** , or **Marc St Hilaire Method**

Title: Calculating Locations by The Intercept Method, or Marc St Hilaire Method.

Time: The time allocated is approximately 90 minutes.

Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.

Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the geometry with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Intercept Method, or Marc St Hilaire Method.

b. The **Longitude by Chronometer Method**

Title: Calculating Locations by The Longitude by Chronometer Method.

Time: The time allocated is approximately 90 minutes.

Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.

Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the geometry with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Longitude by Chronometer Method.

c. The **Ex-Meridian Method**

Title: Calculating Locations by The Ex-Meridian Method.

Time: The time allocated is approximately 90 minutes.

Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.

Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the geometry with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Ex-Meridian Method.

4. The **Trigonometry** Lessons.

a. The **Intercept Method** , or **Marc St Hilaire Method**

Title: Calculating Locations by The Intercept Method, or Marc St Hilaire Method.

Time: The time allocated is approximately 90 minutes.
Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.
Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the trigonometry with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Intercept Method, or Marc St Hilaire Method.

b. The **Longitude by Chronometer Method**

Title: Calculating Locations by The Longitude by Chronometer Method.
Time: The time allocated is approximately 90 minutes.
Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.
Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the trigonometry with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Longitude by Chronometer Method.

c. The **Ex-Meridian Method**

Title: Calculating Locations by The Ex-Meridian Method.
Time: The time allocated is approximately 90 minutes.
Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.
Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the trigonometry with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Ex-Meridian Method.

LESSON PLAN DETAILS

The Introductory Lesson

TOPIC: Construct a Sextant.

OBJECTIVE: SWBAT (Students will be able to) construct a sextant, understand how it works, and apply this instrument to a variety of navigational applications.

MATERIALS:

1. 1 - CD with Jewel case.

2. 5 - Lego blocks.

    a. 1 - 2 x 4 brick
    b. 2 - 2 x 1 plates
    c. 1 - 2 x 2 brick
    d. 1 - 2 x 2 plate

3. 1 - Tube of clear adhesive (Cyanoacrylate or equal).

4. 1 - Letter-size paper with back-adhesive for printing the Vernier scale on inkjet or laser printers.

5. 2 - Small mirrors (40 mm x 22 mm x 3 mm thick).

TOOLS:

1. 1 - Ruler.
2. 1 - Scissors.
3. 1 - Paper cutter.

ASSEMBLING INSTRUCTIONS:

1. Align and adhere the circle-shaped Vernier scale to the CD.

2. Align and adhere the smaller triangle-shaped Vernier scale to the jewel case corner, adjacent to the CD.
3. Position and adhere the mirrors.
4. Position and adhere the Lego blocks.

OPERATING INSTRUCTIONS

1. Safety Warning.

2. Precision.

Mounting mirror 1 at the center of the CD:

1. While working on a flat surface, adhere the center mirror back to the large side of a 2x4 Lego brick. The mirror should also be adhered perpendicular to the flat surface for optimum results.

2. Assemble the 2x4 Lego brick over two 2x1 Lego plates. Each 2x1 Lego plate should be positioned on opposite sides of the center hole of the CD. The 2x4 Lego brick spans the space between these Lego plates as well as the center hole of the CD.

3. Adhere the mirror assembly to the CD and adjust the following:

a. Align the large side of a 2x4 Lego brick side to the 180° scale line. The silvered surface of the mirror or the back surface of the glass mirror should be over the center hole of the CD.
b. Center the Lego plates without interfering with the center hole of the CD.

Mounting mirror 2 (half-silvered) to the CD jewel case:

I used a 2x2 Lego brick mounted on a 2x2 Lego plate, to hold the CD jewel case mirror. Cut the 4 brick bumps out, because they will be visible thru the transparent part of the half silvered mirror.

1. While working on a flat surface, adhere the half silvered mirror to the 2x2 Lego brick. Make sure the mirror is perpendicular to the flat surface.

2. Assemble the jewel case mirror brick to the plate.

3. Position and adhere the jewel case mirror assembly in the CD jewel case corner. Make sure that:

a. The CD is positioned pointing more or less as shown in the layout to the left, so you will have space to place the Vernier in the other jewel case corner afterwards.

b. Place the half silvered mirror assembly parallel to the center mirror. Position it visually.
c. The assembly should be adhered to the CD jewel case.

Adhering the Vernier scale:

At this point, your CD-Sextant is almost complete. The Vernier scale should be adhered in the 0° position:

1. Cut the Vernier in a triangular form, to fit the CD jewel case corner. I did stick the Vernier on a blank sticker paper piece, in order to make the Vernier paper ticker. This is important because the Vernier edge will be unsupported.

2. Adjust the mirrors (see fine-tuning the mirrors)

3. Rotate the CD until the mirrors are parallel.

4. While holding the instrument in position for observations look through the half silvered mirror and focus on a far away object. Turn the CD slowly until the reflected image and the direct image coincide. This must be the instrument zero, so....

5. Carefully place and stick the Vernier in the CD jewel case, reading 0°00'. This means that the Vernier tick A must coincide with the 0° scale line. On the other side of the Vernier, the 60' tick must coincide with the 59° tick in the scale.

Make sure the Vernier and CD scale are very close together. The Vernier probably will be a little higher than the CD, and you might want to bend it down a bit.

SAFETY FROM THE SUN

Observing the Sun by looking directly into it can be extremely dangerous. Since excessive exposures can cause cataracts from the UV radiations and burnt retinas from the visible light, caution must be taken while observing the Sun, to protect the viewer's eyes.

A dependable Sun filter should be added to the Sextant to accommodate this preventative health measure, otherwise alternative reference objects should be viewed.

1. Observe the Sun for only a few seconds and only after the shade is in place.
2. Never stare directly at the Sun.
3. If any discomfort is sensed, discontinue observing the Sun until a stronger filter.

Materials that can be used to make the filter:

- Aluminized Mylar film - this is a material specifically developed for solar observation. Can be found in science supply stores. Probably the best material available.
- Welder's glass - strong filter, used to protect the welder eye. Difficult to cut, can be found in construction stores.
- Photography film. Use a dark negative with silver coating. This means black-and-white film. Color film does not contain silver and will not filter the UV rays (they are dark for visible light, but not for UV light).
- Dark floppy disk media.

Additional eye safety information is available at:

http://sunearth.gsfc.nasa.gov/eclipse/SEhelp/safety2.html

Photographic 35 mm dark negative film (there is one in the end of every film roll) is one possibility as shades for Sun and Moon sights. The negatives should be mounted in slide frames: two layers of photographic 35 mm dark negative film for the Sun frame and one layer of photographic 35 mm dark negative film for the Moon. Both slide frames are removable and are attached to the instrument frame using Lego pieces. Trimming the lower edge of the slide to make it thinner will ensure that the slide window matches the imaginary "tube" formed by the mirror edges.

The shade must be positioned between the two mirrors and the filter surface must be orthogonal to the line connecting both mirror centers. This is to prevent introducing a refraction error. Try to position the slide center in the line connecting the two mirror centers. The Sun observation is made by looking thru the half silvered mirror, below the shade.

(image available in print form)

Figure 8 , Source: [8] *Sun filters.*

Fine-tuning the mirror angles:

This Sextant is not equipped with screws to adjust the tilt of the mirrors. Instead, each mirror can be fine-tuned by inserting paper shims between the Lego brick and plates (or by reducing the Lego brick and plates by sanding) at meaningful locations to achieve the desired results.

First check the angle of the CD mirror (center mirror). As you look to this mirror, the reflected CD edge must be perfectly aligned with the edge you see outside the mirror. This alignment should be performed for all directions.

The half-silvered mirror should be trimmed by setting the instrument to 0°00', focusing on a distant object and adjusting the mirrors so that the direct and reflected images align. Adhere a round 1x1 Lego piece to the CD surface, to use as a turning knob.

(image available in print form)

Figure 9 , Source: [8] *Using the CD Sextant by viewing the Sun and the horizon.*

1. The **Algebra** Lessons.

a. The **Intercept Method** , or **Marc St Hilaire Method**
Title: Calculating Locations by The Intercept Method, or Marc St Hilaire Method.
Time: The time allocated is approximately 90 minutes.
Materials: Graphic Calculator; CD Sextant; Commercial Plotting Sheets.
Objectives: Students will be able to locate a position by sighting with the CD Sextant, calculating the algebra with the Graphic Calculator, and plot that position on the Commercial Plotting Sheets by The Intercept Method, or Marc St Hilaire Method.

The sextant is an instrument that is applied to measure these angles. The eyepiece is aligned to the small mirror, which is fixed in the frame of the instrument. This mirror is half transparent. Through the transparent half, the navigator can see the horizon directly. The small mirror also partially reflects the image from the large mirror, where the star is visible. The large mirror is mobile and turns with the arm of the sextant. Altering the angle between the two mirrors aligns the navigational sighting. The altitude of the star is measured in the Vernier scale. There is a drum to make the fine adjustments. Whole degrees are read in the scale and the minutes in the drum.

(image available in print form)

Figure 10 , Source: [8] *The Earth and the Celestial Sphere.*

If the Earth were the center of the universe, then around the Earth there could be a larger sphere centered in the same point, upon which the stars are fixed as if they were painted across its internal surface. The **Celestial Sphere** (*figure 10*) is this other sphere **.**

(image available in print form)

Figure 11 , Source: [8] *Earth coordinate system.*

To specify a position on the surface of the Earth, a system of coordinates has been developed that consists of two angles: latitude and longitude. **Latitude** is the angle measured from the Equator in direction North-South. **Longitude** is the angle in the Pole between the Meridian of Greenwich and that of the considered position (*figure 11*).

(image available in print form)

Figure 12 , Source: [8] *Celestial Coordinate System*

A similar system is used for the Celestial Sphere (*figure 12*). The angle analogous to the latitude in the celestial sphere we call **declination** . The declination is measured in the plane North-South, from the Celestial Equator. The analog to the longitude is named **Right Ascension** or **RA** . Like the longitude, the Right Ascension is measured from an arbitrary Meridian: the Vernal Equinox Point (a.k.a. first point of Aries).

To determine an astronomical position, draw the lines of position:

1. Plot your assumed position.
2. Using a parallel ruler, draw a line passing on the assumed position, in the direction of the Azimuth of the star.
3. Over this line, measure the error Delta of the estimate - in the direction of the star or contrary to it - according to the sign of the Delta.
4. Draw the line of position, orthogonal to the Azimuth, at this point.

For more information:

http://www.kwsi.com/ynhti/SextantWebsite/

VOCABULARY

Source [7]:

almanac
declination
orthogonal
altitude
facula or faculae
octant
astrolabe
horizon
photosphere
astronomical transit
hypotenuse

Pythagoras
azimuth
LOP
sextant
Besselian year
nadir
transit
chronometer
navigational transit
zenith

GLOSSARY

UT0 is Universal Time determined at an observatory by observing the diurnal motion of stars or extragalactic radio sources, and also from ranging observations of the Moon and artificial Earth satellites. It is uncorrected for the displacement of Earth's geographic pole from its rotational pole. This displacement, called polar motion, causes the geographic position of any place on Earth to vary by several meters, and different observatories will find a different value for UT0 at the same moment.[7]

UT1 is the principal form of Universal Time. It is computed from the raw observed UT0 by correcting UT0 for the effect of polar motion on the longitude of the observing site. UT1 is the same everywhere on Earth, and is proportional to the true rotation angle of the Earth with respect to a fixed frame of reference. Since the rotational speed of the earth is not uniform, UT1 has an uncertainty of plus or minus 3 milliseconds per day. The ratio of UT1 to mean sidereal time is defined to be $0.997269566329084 - 5.8684 \times 10^{-11}T + 5.9 \times 10^{-15}T^2$, where T is the number of Julian centuries of 36525 days each that have elapsed since JD 2451545.0 (J2000).[7]

UT1R is a smoothed version of UT1, filtering out periodic variations due to tides. It includes 62 smoothing terms, with periods ranging from 5.6 days to 18.6 years.[7]

UT2 is a smoothed version of UT1, filtering out periodic seasonal variations. It is mostly of historic interest and rarely used anymore. It is defined by the equation:

$$\mathbf{UT2 = UT1+0.0220 \bullet \sin(2\Pi t)-0.0120 \bullet \cos(2\Pi t)-0.0060 \bullet \sin(4\Pi t)+0.0070 \bullet \cos(4\Pi t)}$$

seconds, where t is the time as fraction of the Besselian year.[7]

UT2R is a smoothed version of UT1, incorporating both the seasonal corrections of UT2 and the tidal corrections of UT1R. It is the most smoothed form of Universal Time. Its non-uniformities reveal the unpredictable components of Earth rotation, due to atmospheric weather, plate tectonics, and currents in the interior of the Earth.[7]

UTC (Coordinated Universal Time) is an atomic timescale that approximates UT1. It is the international standard on which civil time is based. It ticks SI seconds, in step with TAI. It usually has 86400 SI seconds per day, but is kept within 0.9 seconds of UT1 by the introduction of occasional intercalary leap seconds. As of 2007 these leaps have always been positive, with a day of 86401 seconds. When an accuracy better than one second is not required, UTC can be used as an approximation of UT1. The difference between UT1 and UTC is known as DUT1.[7]

UTC-SLS (UTC with Smoothed Leap Seconds) is a modified form of UTC that avoids unequal day lengths. It usually ticks the same as UTC, but modifies the length of the second for the last 1000 UTC seconds of a day containing a leap second so that there are always 86400 seconds in the UTC-SLS day.[7]

UTS (Smoothed Universal Time) is an obscure form of UT used internally at IERS. The same abbreviation was for a time used to refer to UTC-SLS.[7]

STUDENT'S BIBLIOGRAPHY

[1] Latitude Hooks and Azimuth Rings, by Dennis Fisher, An International Marine/McGraw-Hill Companies Book, 1995, Camden, Maine.

BIBLIOGRAPHY

[1] Universe, by R.A. Freedman and W. J. Kaufmann III, seventh edition published by W.H. Freeman & Co.

[2] The Cosmic Perspective, by Jeffrey O. Bennett, Megan Donahue, Nicholas Schneider, Mark Voit, fourth edition published by Addison Wesley.

[3] http://www.tecepe.com.br/nav/

[4] Astronomical Almanac for the Year 2007/2008 and Its Companion, The Astronomical Almanac Online: Data for Astronomy, Space Sciences, Geodesy, Surveying, Navigation, and other applications by Nautical Almanac Office (U.S.) and Naval Observatory, Nautical Almanac Office U.S. Defense Dept. (Hardcover - Jan 17, 2007).

[5] Tools of Navigation, http://www.mindwareonline.com

[6] Latitude Hooks and Azimuth Rings, by Dennis Fisher, An International Marine/McGraw-Hill Companies Book, 1995, Camden, Maine.

[7] http://www.wikipedia.org

[8] http://www.nasa.gov

[9] http://www.kwsi.com/ynhti/SextantWebsite