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Modeling Natural Disasters with Mathematical Functions

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Introduction

Mathematics is an extraordinary exercise of the human mind in abstracting the results of observation to find similarities and differences between phenomena. These relations between phenomena make it possible to organize the natural world into discrete sets of objects that can be studied using similar mathematical objects and methods.

Nature, as an object of mathematical study, bridges the gap between the concreteness of the everyday environment and the abstraction of mathematics. Mathematics, in turn, allows us to summarize, formalize, interpolate, and extrapolate from observations that have been recorded.

Purpose

The mathematical concept of a function is critical to defining relations between variable phenomena, particularly phenomena that can be described using two variables. Mathematical functions can be used for both summarization and prediction. Through this unit, students will explore several types of natural phenomena, and use several types of functions and representations of functions in order to describe the phenomena. Students will use these models to make predictions and solve problems related to natural disasters.

Goals and Objectives

By completing this unit, students will be able to perform the flowing tasks:

Mathematical Topics

1) Discuss and provide examples of how mathematics is used to describe nature, and natural disasters in particular.
2) Define the following: variable, algebraic expression, equation, function, and mathematical model.
3) Use mathematical models to describe observed relationships between characteristics of natural disasters when values of some of the variables are known.
4) Derive mathematical models from existing equations and relations (solve multivariate equations for one of the variables).
5) Compare different classes of mathematical models and functions.
6) Provide examples of applications of each type of function to the study of natural disasters
7) Provide examples of dimensions and the units used to measure them
8) Represent functions as equation, tables, graphs, and verbal descriptions
9) Describe the domain and range of a function used to model a natural disaster
10) Draw a picture of a wave, and label the associated quantities or characteristics

Natural Science Topics

1) Describe the relationship between earthquake magnitude and energy in at least two different ways (multiple representations of functions)
2) Define seismic moment, local magnitude, moment magnitude
3) Describe the relationship between seismic moment and moment magnitude in at least two different ways
4) Describe the relationship between earthquake magnitude and frequency of occurrence in at least two different ways
5) Calculate wavelengths, periods, and velocities of near-shore ocean waves
6) Describe the relationship between water depth and velocity of near-shore (shallow) ocean waves in at least two different ways
7) Make calculations using mathematical models of natural disasters phenomena
8) Classify the relationship between pairs of variables associated with various natural phenomena as one of the following: linear, direct variation, inverse variation, quadratic, or exponential.
9) Pose questions about natural disasters that could be answered by using a mathematical model

10) Describe the difference between a natural hazard and a natural disaster
11) Define tsunami, volcano, earthquake, and tornado

Key Concepts

This section describes the main mathematical and scientific concepts that will be discussed in this unit. Terms that may be included in a vocabulary list are italicized. These concepts will be referred to in the individual lesson plans. The scientific concepts are related to the mathematical concepts that will be used to study the behavior of each natural phenomenon.

Mathematical Concepts

Measurement: Magnitude, Dimensions and Units

Since natural disasters include large *magnitudes* of energy and mass, *scientific notation* will be used to represent vary large and very small numbers. Discussion and use of *measurement units* will be included in comparisons of magnitude. The following provides some examples of different types of quantities or dimensions, and the units used to label them. Most physical phenomena can be described by using the following dimensions: mass M, force F, length L, time t, and temperature T (Banks, 1998). These dimensions could be made concrete for students by having them measure quantities using a scale, ruler or tape measure, clock, and thermometer. Students should be prompted to list the units they are familiar with for each dimension.

Other phenomena can be described by using derivations of these dimensions, most commonly a ratio of one of two of these dimensions. Some derived quantities can be calculated after students simultaneously make measurements with two different instruments. Calculating derived dimensions would be a good exercise for student group measurement activities. Some examples of derived quantities include the following: velocity equals length per unit time, pressure equals force per area, density equals mass per unit volume, energy equals Newton meters or joules, and power equals joules per second or watts (Banks, 1998). Some examples of units include the following: kg for mass, Newtons for force, meters for length, seconds for time, and degrees Celsius or degrees Kelvin for temperature. Some examples of derived units include the following: meters per second for velocity, Newtons per square meter, kilograms per cubic meter, Newton meters, Joules per second (Banks, 1998). All of the derived dimensions are simply *rates* .

Mathematics is used to describe nature in several ways. Numbers are used to describe the relative magnitudes of measured phenomena, and units are used to specify the dimension and perhaps the measurement instrument that was used. Measurements recorded with the same units are used to compare magnitudes of similar events. Mathematical functions are used to describe relationships between different variables, in this case, the characteristics of certain natural disasters. Mathematical models are used to summarize relationships between the characteristics of natural disasters. They are ultimately used to answer questions that humans have about natural disasters, and predict the results of events that have not occurred, but may be possible or even likely. Specific examples of mathematical models for natural disasters are discussed in the Natural Science Topics section.

As prerequisite knowledge, students should be familiar with the use of the following mathematical concepts. A *variable* is a symbol, commonly a letter, used to represent a quantity in an algebraic expression or equation. A variable may be used to denote many values, quantities, magnitudes or numbers. An *algebraic expression* consists of arithmetic operation(s) on number(s) and variable(s). An expression also can represent many values. An *algebraic equation* is two expressions linked by an equals sign. An equation may represent any number of values for the variable(s). A *function* is an equation with two or more variables, where one of the variables (the dependent variable) appears alone on one side of the equation. Also, for each value of the input variable, there is a maximum of one value of the output variable. A function represents a relationship between two sets of numbers, each of which measures the magnitude of a type of quantity or dimension.

Multiple Representations of Functions

A *mathematical function* is a relationship of two or more variables. One of the variables is the output, or *dependent variable* . The other variable(s) are known as input(s) or *independent variable(s)* . A *mathematical model* is a function that is used to describe a real situation (Connally et. al., 2001). In using a mathematical model, it is important that any variables have been defined as a quantity that can be measured, the dimension of each variable is known, and the *measurement units* are known. Mathematical functions, and thus mathematical models, can be represented in several different ways: *equations, tables, graphs* , and *verbal descriptions* . Examples of equations include $y = kx$, $y = a(x - h)^2 + k$, $y = Ae^{bx}$. Students should be familiar with vertically and horizontally oriented tables, as well as hand-drawn and calculator graphs. Many students have difficulty with verbally describing equations and functions. Students will be encouraged to verbalize functions using phrases such as "y varies directly with x", "y varies as the square of x", "y varies as the square root of x", or "y is a power function of x". When creating or using a mathematical model, students will be required to use at least two different representations of the model.

There are several types of functions that will be studied and applied in this unit, including the following: linear functions (including direct variation), *quadratic functions* (square root functions), and *exponential functions* (logarithmic functions). If the students are in a Precalculus or Algebra 2 course, it may be appropriate to discuss the definition of an *inverse function* . In this case, the derivation of the mathematical models used in this unit can be included as explanation or perhaps even as exercises. These derivations would necessitate *solving multivariate equations for one of the variables* , i.e. "isolating a variable", which some Algebra 1 students have proven capable of executing, while others have problems solving one variable equations. Regardless, Algebra 1 students should at least be introduced to the processes of solving a bivariate equation for either variable, e.g. taking the square root, log, or square of both sides of an equation. Perhaps these processes are best introduced by providing univariate examples with numbers on one side of the equation.

As a prerequisite, students should be able to describe similarities and differences between types of functions, including comparison of inputs and outputs and the shape of the graphs. Direct variation and linear functions are the same in that they both have constant differences of the dependent variable. They are different because direct variation can be modeled by a proportion. Another difference is that a direct variation graph crosses the origin of the coordinate plane. Linear and quadratic functions differ in that there is no constant rate of change in a quadratic, best seen with a table. A linear function has no maximum or minimum, has no axis of symmetry. The range of an exponential function is the same as the general quadratic. Many students have difficulty describing the domain and range of continuous function. By using models containing continuous function to study natural disasters, these sets may be thought of as discrete, measured quantities. This may be aided by using data from tables in (Abbott, 2004), and discussion of magnitude scales for all types of natural disasters.

Mathematics can be used to solve problems involving natural disasters through the use of mathematical models. Questions humans have about natural disasters can be answered using *interpolation* or *extrapolation* from a table or graph, or through the use of a mathematical model per problem situation, and solving the resulting equation(s).

Natural Science Concepts

A *natural hazard* is a situation in the natural environment that exhibits "clear signs of danger" (Abbott, 2004). A *natural disaster* is a naturally occurring event that exceeds the ability of a region to "rescue and care for it's people, to clean up the destruction, and to begin reconstruction" (Abbott, 2004). Clearly, a natural hazard exists before it becomes a natural disaster. Also, not every natural hazard becomes a natural disaster; for instance, an earthquake that occurs in a sparsely populated area would not constitute a natural disaster, since it would not destroy much property or hurt many people. The following describes several types of natural disasters whose characteristics, causes, and results will be discussed and explored in this unit.

Waves

Since both tsunami and earthquakes involve different types of waves, a discussion about waves is a necessary prerequisite to the discussions of tsunami and earthquakes. Since Algebra 1 students have probably not studied periodic functions, it may be useful to graph a sine wave on an overhead graphing calculator, and label the wavelength and wave height, and briefly discuss period and amplitude. Discussion may be prompted by having students list the types of waves with which they are familiar, e.g. ocean waves, sound waves.

Tsunami

A *tsunami* is a high-speed sea wave of seismic origin created by "an underwater earthquake, landslide, or volcanic eruption"(Johnston, 2001). A *shallow water wave* is a water wave in which the wavelength is larger than the water height, or ocean depth (Banks, 1998). Since humans are concerned with a tsunami primarily at the shore, where the water is not deep, tsunami are explored as a shallow water wave. That is, people are most concerned with the height of a wave as it hits shore. Tsunami will be discussed in the context of water depth, wave velocity, period, wavelength and energy.

The wavelength, velocity and period of a shallow ocean wave are related by the direct variation equation $L = CT$, where L = wavelength, C = velocity, and T = period (Bryant, 2005). This is an example of a linear relation between velocity and period. Period or velocity may also be expressed as a ratio of the other two quantities. Students may be asked to find any one of the three variable quantities, provided the other two.

A square root function that models tsunami velocity as a function of water depth is

$v = (gD)^{1/2}$, where $g = 9.8 m/s^2$ and D = water depth. Alternatively, this can be a quadratic model to find the depth of water if the wave velocity is known: $D = v^2 / g$ (Abbott, 2004, Banks, 1998). The energy of a water wave, particularly a tsunami, can be modeled as a function of wave height and wavelength, using the following quadratic equation: $E_W = 0.125 pgH^2 L$, where E_W = wave energy in joules, p = density of water, g = $9.8 m/s^2$, H = wave height, L = wavelength (Abbott, 2004). Alternatively, wave height can be modeled as a square root function of wave energy and wavelength: $H = 2(2E_W / (pgL)^{1/2})$. Students will use this relation to find both water depth and velocity. They should also be asked to compare the energy of shallow waves of different heights and wavelengths, in order to differentiate between linear and quadratic relationships. This is probably most easily found through using a tabular representation of the relation, then comparing the respective energies.

Volcanoes

A *volcano* is "a cone shaped mountain or hill, formed by the accumulation of hardened magma" with a "hole . . . from which lava and/or hot ash and gases erupt from deep underground" (Johnston, 2001). Volcanoes are commonly located near *subduction zones* between *tectonic plates* (Abbott, 2004). During an eruption, *pyroclasitic* bombs may be ejected from the volcano (Scheidegger, 1975, Abbott, 2004). A relation between the maximum distance a bomb may shoot and the initial velocity as it leaves the chute is given by the model $D = v_0^2 / g$, where D = maximum distance, $g = 9.8 m/s^2$ and v_0 = initial velocity (Scheidegger, 1975). This model is simplified in that it assumes the bomb is ejected from the volcano at a 45^o This function is quadratic, and may be used to find the distance is the velocity is known. More commonly, this relation has been used in the square root form $v_0 = (gD)^{1/2}$ to estimate initial velocities of the bombs. Students will be asked to use this relation to find both the maximum distance of the projectile and the initial velocity, in different situations.

Earthquakes

An *earthquake* is shaking of the earth caused by *seismic waves* (Johnston, 2001), of which there are several types. *Seismic waves* are waves that travel from the *focus* of an earthquake (Johnston, 2001). The *epicenter* of an earthquake is the point on the earth's surface directly above the earthquakes *focus* , or where the actual sliding of a *fault* occurred (Abbott, 2004). Earthquakes will be discussed in the context of different magnitude scales, amount of energy released, and frequency of occurrence.

Most people are familiar with earthquakes as being classified according to the *Richter scale* , which was developed in the 1930s (Abbott, 2004). However, in the 1970s Hiroo Kanamori developed a more precise measurement of earthquakes, based on the physical properties of an earthquake, as opposed to seismic measurements (Abbott). The *seismic moment* is the product of rock rigidity, length of fault, relative movement of fault, and the magnitude (Scheidegger, 1975). The *moment magnitude scale* describes the moment magnitude as a function of seismic moment using the logarithmic model $M_W = 2/3 \log_{10} (M_0) - 6$, where M_W = moment magnitude and M_0 = seismic moment. Alternatively, the moment can be made a function of moment magnitude using the exponential function $M_0 = 10^6 10^{3/2 Mw}$. Students will be asked to find both the seismic moment and the moment magnitude by using this relation.

The total energy released in an earthquake can be related to the Richter magnitude using another exponential function: $E = 10^{11.8+1.5M}$, where E = energy in ergs and M = Richter magnitude (Bercovici & Brandon). Alternatively, Richter magnitude can be a function of the amount of energy released: $M = (2/3)\log_{10}(E) - 7.86$. Students will convert between Richter magnitude and energy.

Like most natural disasters, earthquake magnitude and frequency are inversely related, i.e. larger magnitude earthquakes occur much less frequently than smaller earthquakes (Scheidegger, 1975). The relationship between earthquake magnitude and the frequency of occurrence (recurrence interval) is modeled by a logarithmic equation: $\log(N) = 5.3-0.93M$, where N = the cumulative number of earthquakes and M = minimum Richter magnitude (Bolt, 2004). This can easily be written so that N is an exponential function of M: $N = 10^{5.3} - 10^{0.93M}$. Similarly, magnitude can be a logarithmic function of the cumulative number of earthquakes: $M = 5.7 - (\log_{10}(N) / 0.93)$. This relation will enable students to calculate recurrence intervals of earthquakes of given Richter magnitudes.

Tornadoes

A *tornado* is a very fast wind vortex, usually about 20 feet wide, extending downward from a cloud. The wind speeds in a tornado are among the fastest wind speeds measured (Johnston, 2001, Scheidegger, 1975, Abbott, 2004). The common scale used to measure tornadoes is the Fujita scale, which is listed in Abbott, p295. The wind velocity and Fujita scale number are related by the function $v = 6.3(F + 2)^{3/2}$, where v = wind velocity, F = Fujita scale number. This is the velocity as a square root function of the Fujita scale number. This relation can also express the Fujita scale number as a quadratic function of the wind velocity, although it involves a cube root: $F = (v^{2 / 36.7})^{3/2} - 2$. Students will use this relation to find both velocity and Fujita scale numbers.

Population growth

Students should be presented with a table of global population values for different years, and then graph a discrete data set to appreciate the rapid growth rate of the global human population. An exponential function that describes population as a function of time is $p = p_0(1 + r)^t$, where p = population, p_0 = initial population, r = percent growth rate, and t = time, usually in years (Connally et. al., 2001). In this case, r would be negative if there existed a decline in population. Otherwise, r can be found as a ratio of the birth rate to the death rate. If we let 1 + r = b, the base of the exponential expression, the function becomes $p = p_0 b^t$. An alternative version of this relation describes the time it would take for the population to reach a given amount p: $t = \log_b(p / p_0)$. Students will use both functions to predict the future global population, provided the current growth rate or 1.3% (Abbott, 2004). This relation will also be used to find out how long it will take the global population to reach a certain level.

Students

This unit is designed for a class of 10 to 20 students in grade 9 in an Honors Algebra 1 class. Lessons may be applicable to 10th, 11th, or 12th grade students in an Algebra 2 or Precalculus class as well. This unit should take approximately 3-5 weeks, if taught continuously. However, the lessons have been designed so that they are independent of each other and may be used in the context of another unit covering similar objectives and having covered the same prerequisite skills and concepts.

The following describes some statistics of state, district and school students. Included are standardized test performance, ethnicity, pupil to teacher ratio, and annual expenditure per pupil. The percent of Connecticut students performing at goal on the CAPT is 46.3. For the district this percent is 11.6. At James Hillhouse High School, the percent proficient on the CAPT is 4.8. This shows that local students need much help with preparation for the type of content that the CAPT assesses, as well as the actual format of the test. 13.8% of Connecticut students are black, while 53.7% of New Haven Public Schools students are black. At James Hillhouse High School, 89% of students are black. Thus this unit is designed with black students as an overwhelming majority of the student audience. The number of students per teacher in the state of Connecticut is 13.6, while this ratio for the New Haven Public Schools district is 14, slightly higher than the state average. The amount of money allocated for each student per year in the state is $11579, while the district annual expenditure per pupil is $10382 (Nations Report Card, Connecticut No Child Left Behind (NCLB) State Report: 2005-06 School Year, Mayo, Garris). This is evident by a lack of resources with which to instruct students. One of the goals of this unit is to be able to describe the mathematics of natural disasters through hands on learning with limited resources.

Rationale

The topic of nature is used as a means to show the capacity of functions to model actual data. The most dramatic events in our environment are natural disasters. Students relate to examples of extreme weather and natural disasters. Some may have experienced a disaster, most will have heard of these types of disasters, and all should appreciate the power of the forces of nature and their manifestations.

Each lesson assumes the students have been exposed to skills and concepts involved in the type of function that is discussed, i.e. inverse functions, solving multivariable equations for one variable. Prerequisite skills are listed, and it is assumed that students have had practice with these skills, some discussion of the concepts, and are ready for applications problems. The classroom environment will use several different structures in an effort to stimulate student discussion. Vocabulary is listed and separated into the mathematical terminology and symbols, and the scientific terminology.

An introduction to the scientific concepts will be incorporated into each mathematical lesson. The unit is structured this way to allow the lessons to be taught independently of each other. Each lesson is to take up about a week's worth of instructional time (at 45 min per session, 5 days). This allows time for the inevitable adjusted instruction that may take place regarding the prerequisite skills and the scientific content. It also allows time for the demonstrations and activities (2 days each) and the assessment (1 day).

Teaching Strategies

Teaching mathematics requires both the science of mathematics and the art of relating the content to the students. In executing this unit, the classroom environment will be regularly modified to enhance student learning and socialization. Topics will be presented, practiced, and assessed in multiple modalities, but most commonly via the most salient modality. Presentations of fundamental science concepts will be used to interest students with each lesson's question(s). Where possible, demonstrations and activities using manipulatives and student-centered activities will be used. Involving all students in demonstration or activity incorporates the equity principle of instruction. The use technology is incorporated by the use of the graphing calculator, both by students in investigation, and by the teacher in presentation.

The main focus of the content is on two of the four content standards: number and operations, algebra, and some measurement as an introduction to units. The NCTM process standards emphasized include the following: problem solving through the use of models to answer problems to real or possible scenarios, communication through small group and whole-class discourse and verbalization of mathematics, connection through using natural disasters as a context in which to show the application of mathematics to nature, and representation through the requirement that all mathematical models and their use must be shown in at least two different ways (Executive Summary: Principles and Standards for School Mathematics).

For each type of natural disaster, a case study will be presented to the class either as an introduction, or in the context of solving problems. Hopefully, each case study will be an event that they have heard of. Data known about the case study could be used to theorize about different possible scenarios. The unit may be motivated by an open ended writing assignment about each student's most significant experience with a natural disaster or weather.

The following details which case study of each type of natural disaster will be used in either introducing the disaster, presenting the characteristics of the disaster, or student research of the type of disaster. Where possible, the most deadly disasters have been chosen. The tsunami case study will be of the Moro Gulf in the Phillipines in 1976, which killed 8000 people. The volcano case study with be of the Mt. Pelee in Martinique in 1902, which killed 30000 people. Mt. St. Helens in 1980 is an alternative, since it was much more recent. The earthquake case study will be of Tanshan, Beijing and Tianjin in China in 1976, which killed 242000 people. The tornado case study will be of Manikganj, Dhaka and Tangail districts of Bangladesh in 1989, which killed 800 people (Kovach & McGuire, 2003). Population growth could be exemplified by a case study of Africa, whose growth rate is 2.4% (Abbott, 2004). Another interesting case study for a discussion on population is Easter Island, although it was population decrease that was the situation.

Classroom Activities - Lesson Plans

Lesson Number 1 - Functions as Mathematical Models of Natural Phenomena

Objective(s) - students will be able

1) To discuss and provide examples of how mathematics is used to describe nature, and natural disasters in particular
2) To define and provide an example of a function and a mathematical model
3) To provide examples of dimensions and the units used to measure them
4) To apply the concepts of function, independent and dependent variable to describe relationships between natural phenomena using multiple representations of functions.
5) To classify the relationship between pairs of variables associated with natural disasters as linear, quadratic, or exponential
6) To pose questions about natural disasters that may be answered by using a mathematical model
7) Describe the difference between a natural hazard and a natural disaster
8) Define tsunami, volcano, earthquake, and tornado

Prerequisite skills

graphing two-variable data, evaluating expressions, translating verbal phrases and sentences to algebraic expressions and equations, solving equations, creating and interpreting tables and graphs on coordinate plane

Materials

Ruler or tape measure, thermometer, clock, scale

Worksheet or text with tables of different types of function, without the equations

Vocabulary - Terms & Symbols

function, domain, range, independent variable, dependent variable, f(x) = y, vertical line test, mapping diagram, earthquake, volcano, water wave, tornado, tsunami, dimension, unit

Strategy

Students should be prompted to discuss different types of phenomena that are measured. They should also discuss what types of instruments are used for the measurements. This discussion should result in defining *dimension* as in (Banks, 1998). As a class, we list different units that are used to measure the dimensions of mass, force, length, time and temperature. Review the definition of function. Use the student as function input metaphor - can't go to two places at once. Encourage students to come up with measureable quantities and measurement units. Prepare a measurement activity that involves a simple linear function. Prepare tables of different types of functions related to natural disaster for review of function types. Prompt students to classify each table as a type of function. Describe natural hazards and disasters, the different types of natural disasters, and their characteristics. Use the case studies to provide an example of each type of disaster to be studied. An overhead slide show would nicely show the results of the disasters. Ask questions about the characteristics of natural disasters, i.e. "are these two related? How so?" Provide encouragement and feedback.

Student tasks

Form small groups

List examples of quantities that we measure in the environment

Classify each type of quantity by its dimension, and give an example of units

Discuss how some pairs of quantities may be related

Find two quantities that have a functional relationship

Describe the functional relationship in words using "is a function of" and in symbols using a = f(b)

Present one of your functions to the class, with both verbal and symbolic representations

Engage in measurement activity involving length, time, and force, using ruler, clock, and scale A distance, rate, and time problem would suffice.

Classify a function represented with a table as linear, quadratic or exponential.

Answer elementary questions about the natural disasters discussed, define terms.

Questions

For each of the following pairs of natural phenomena, determine the dimension and possible units of each variable, and determine if there may be a functional relationship between the two. Then identify the independent and dependent variables, and write a sentence that describes the relationship.

time of day & temperature

month & temperature

location & weather

seismic wave (moment) amplitude & Richter scale magnitude

number of deaths & earthquake magnitude

number of deaths & building materials

Give examples of dimensions and units

What is the difference between a natural hazard and a natural disaster?

What questions can we ask about these natural disasters?

What type of mathematics would help us answer these questions?

Lesson Number 2 - Modeling Tsunami Height, Tornado Wind Speed, and Safe Distance from Volcanic Bombs with Quadratic and Root Functions

Objective(s) - students will be able

1) To use mathematical models to describe observed relationships between characteristics of natural disasters when values of some variables are known
2) To derive mathematical models from existing equations and relations (solve multivariate equations for one of the variables)
3) To provide examples of the application of a quadratic or square root function to natural disasters
4) To represent functions as equations, tables, graphs, and verbal descriptions
5) To describe the domain and range of functions used to model natural disasters
6) To draw and label a simple pictoral representation of a wave
7) To calculate wavelengths, periods, depths, and velocities of near-shore ocean waves
8) To describe the relationship between water depth and wave velocity of shallow water waves in at least two different ways

Prerequisite skills

Vertex form of quadratic function, translations of parabolas, solving equations using square roots

Materials

graphing calculator, graph paper, worksheets with student tasks, questions, and a place to write answers

Vocabulary - Terms & Symbols

Parabola, vertex, axis of symmetry, x-intercepts, maximum, minimum, Wave, water depth, wave velocity, run up, velocity

Strategy

A wave can be modeled by having two students hold a piece of rope in front of the class. One student propagates a wave, and the other students can be prompted to estimate the height of the wave. Another student propagates a larger wave, and again the students can estimate the height of the second wave. A discussion explaining the definition of amplitude follows. The amplitude here can be related to water depth instead of wave height. Provide one form each of the equations for tsunami and tornado. Prompt students to derive the other form of the relation. Provide students with specific calculation questions that can be solved using the presented equations. Provide encouragement and feedback

Student tasks

Describe a wave, and its characteristics.

Give examples of waves

Give examples of waves involved with natural disasters

Take notes on tsunami and tornado equations

Attempt to solve equations for the independent variable

Answer questions using either form of the given relations

Questions

Define a wave, tsunami, volcano and tornado

Find the wavelength of a tsunami with given velocity and period

Find the period of a tsunami with given velocity and wavelength

Find the velocity of a tsunami with given wavelength and period

Find the height of a tsunami with given velocity

Find the velocity of a tsunami with given water depth

Find the energy of a tsunami with given water depth and wavelength

Find the wavelength of a tsunami with given water depth and energy

Find the height of a tsunami with given wavelength and energy

What is a shallow water wave?

Find the safe distance from a volcano that shot a pyroclasic bomb with a given initial velocity

Find the initial velocity of a bomb that lands a given distance from the chute of a volcano

What is the difference between a tornado and a hurricane?

What scale is used to measure tornadoes?

Find the magnitude of a tornado with given wind speed

Find the wind speed of a tornado with a given magnitude

Compare the energies in two shallow water waves with two given sets wavelength and height

What is the relationship between these energies compared to their heights?

Which natural phenomena have quadratic relationships?

Write a sentence describing the relationship between tsunami height, wavelength and velocity

Write a sentence describing the relationship between initial velocity of volcanic bomb and the safe distance from the volcano

Write a sentence describing the relationship between tornado wind speed and magnitude

Lesson Number 3 - Modeling Earthquake Magnitude and Population Growth using Exponential and Logarithmic Functions

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Objective(s) - students will be able

1) To physically model an earthquake and describe how friction and surface area relate to earthquake magnitude
2) To describe the relationship between earthquake magnitude and energy in at least two different ways
3) To define seismic moment, Richter magnitude, and moment magnitude
4) To describe the relationship between seismic moment and moment magnitude in at least two different ways
5) To describe the relationship between earthquake magnitude and frequency of occurrence in at least two different ways
6) To apply exponential and logarithmic functions to the relationship between earthquake magnitude and energy released
7) To apply exponential and logarithmic functions to the relationship between time and population size

Prerequisite skills

order of operations, rules of exponents, graphing functions manually and with graphing calculator, solving equations by taking logarithms and exponentiating

Materials

Sponges of different sizes, all with a flat side, string, weights

graphing calculator, graph paper, worksheets including discussion prompts and calculation exercises

Vocabulary - Terms & Symbols

friction, acceleration due to gravity, surface area, energy (concept & different units), logarithm, exponent

Strategy

The cause of an earthquake can be simply modeled using a sponge, a piece of string, and some weights. If only one sponge is available, this exercise can be done as a classroom demonstration of a force overcoming friction. Otherwise, students may be divided into groups, each of which has one type of sponge, a piece of string, and some weights. Each group places a saturated sponge on a table a given distance from the edge. Students then incrementally place weights on the end of the string, which is hanging off the edge of the table. As a class, we estimate the surface areas of the sponges that are in contact with each table. As a class we discuss and record how much weight or force it takes for each sponge of different area to be moved initially. The weight resembles the amount of force required to build up along a fault before it gives way and yields an earthquake. The area of the sponge represents the area of rock that moves during an earthquake. Students should conclude that it takes more force to slide the larger sponge. Students should be prompted to generalize that the larger the area of moving rock, the larger the earthquake. This discussion segues into the discussion of earthquake magnitude. The effects of an earthquake on a building may be shown by shaking a tall rectangular gelatin on a table. The table is bumped from the side, and the "building" should shake considerably. Begin discussion of population by asking students what the local, state, national, and global populations are. These would be good questions for students to research independently. Abbott includes many nice graphical representations of population fluctuations and growth. Provide notes on earthquakes and population growth. Provide one version of each equation that will be used in the exercises. Encourage and provide feedback.

Student tasks

Participate in sponge and weight exercise

Record data

Participate in concluding discussion

Take notes on earthquakes and population growth, particularly equations

Attempt to solve each equation for the independent variable

Practice calculations on worksheet

Respond to and discuss open-ended questions

Questions

Define an earthquake, seismic moment

Name two scales used to measure earthquake magnitude

Find the seismic moment of an earthquake with a given moment magnitude

Find the moment magnitude of an earthquake with a given seismic moment

Find the Richter scale magnitude of an earthquake that released a given amount of energy

Find the amount of energy released in an earthquake of a given Richter magnitude

Find the recurrence interval of an earthquake with a given minimum Richter magnitude

What is the current population of the United States?

What is the current population of the earth?

How can the growth rate of a population be found using the birth rate and death rate?

At the current growth rate, in what year will the world population be 10 billion?

At the current growth rate, what will the world population be in 2050?

What effects does the human population have on other species of animals and plant?

on natural resources?

Which natural phenomena have exponential relationships?

Resources

Teacher Bibliography - used in writing this unit, and suggested for use in further research by educators.

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Banks, Robert B. *Towing Icebergs, Falling Dominoes, and Other Adventures in Applied Mathematics.* Princeton University Press. Princeton, NJ., 1998.

Used to define shallow water waves and discussion of dimensions and units.

Bolt, Bruce. *Earthquakes 5th ed.* W.H. Freeman & Co. New York, 2004.

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Bercovici, David and Brandon, Mark. *G&G100 Natural Hazards, Homework 2* . Question two is used for the formula that relates earthquake magnitude to energy in ergs.

Bryant, Edward. *Natural Hazards 2nd ed.* Cambridge University Press. Cambridge, 2005.

Connally, Hughes-Hallet, Gleason, et al. *Functions Modeling Change, 2nd ed* . John Wiley & Sons. Hoboken, NJ, 2004.

Used for definition of exponential function.

Johnston, Moira, ed. director. *The Facts on File Earth Science Handbook.* Diagram Visual Information Ltd. New York, 2001.

Used to define earth science terms.

Kovach, Robert, Bill McGuire. *Guide to Global Hazards.* Firefly books LTD. Buffalo, NY, 2003.

Used to select case studies of the different types of natural disasters.

Murdock, Kamischke, Kamischke. *Discovering Algebra* . Key Curriculum Press. Emeryville, CA. 2002.

Rubenstein, Craine, Butts, et al. *Integrated Mathematics 2.* McDougal Littell/Houghton-Mifflin. Evanston, Illinois, 1995.

Scheidegger, Adrian E. *Physical Aspects of Natural Catastrophes.* Elsevier. New York, 1975.

Used for frequency and magnitude of earthquakes, velocity and water depth of tsunami, volcanic bombs and maximum reach, for the relation of Fujita scale to wind speed.

Tallack, Peter, ed. T *he Science Book.* Weidenfeld & Nicolson. London, 2001.

Websites

Connecticut No Child Left Behind (NCLB) State Report: 2005-06 School Year,

www.csde.state.ct.us/public/cedar/nclb/dist_school_nclb_results/ayp_sip_list/ReportCard_state_2005-06.pdf, 2006.

Used for CT AYP.

Connecticut State Department of Education, Connecticut Academic Performance Test (CAPT) Generation 3 Handbook for Mathematics, www.yale.edu/tprep/about/currentstudents/capt/capt_g3_math_handbook2006.doc, 2006. Contains CAPT generation 3 handbook.

Executive Summary: Principles and Standards for School Mathematics, www.nctm.org/uploadedFiles/Math_Standards/Principles_and_Standards_for_School_Mathematics/12752_exec_pssm-1.pdf

Used to establish relation to national goals established by NCTM.

Garris, Lonnie, Jr., Strategic School Profile 2005-06 High School Edition: James Hillhouse High School, www.csde.state.ct.us/public/der/ssp/SCH0506/sr075.pdf

Used for school profile.

Mathematics Curriculum Framework Prek-12 Matrix, http://www.sde.ct.gov/sde/cwp/view.asp?a=2618&q=320872, 2005.

State curriculum framework

Mathematics Subject Classification System, www.ams.org/mathweb/msc2000/, 2000.

Mathews, Kenneth R., NHPS Curriculum, www.nhps.net/hillhouse/Curriculum/Math/NHPS_Curriculum.htm, 2007.

Contains New Haven High School Math curriculum, see algebra 1.

Mayo, Reginald, Strategic School Profile 2005-06: New Haven School District www.csde.state.ct.us/public/der/ssp/dist0506/dist060.pdf, 2006.

Used for district profile.

National Center for Education Statistics: Search for Public School Districts, nces.ed.gov/ccd/districtsearch/, 2007.

Used for student teacher ratios

click on search for "search for public school districts"

type "new haven" in "district name"

click new haven school district in new haven ct

Nations Report Card, nces.ed.gov/nationsreportcard/states/

Used for CT data.

www.ecs.org/clearinghouse/67/07/6707.pdf

Describes high school grad requirements by state.

www.sde.ct.gov/sde/lib/sde/pdf/curriculum/math/cmtgrade8.pdf

Contains grade 8 CMT information..

Student Reading List - material appropriate for high school students researching topics covered in this unit

Science for Kids, mynasadata.larc.nasa.gov/las4/servlets/dataset

Has nice applet for weather inquiry (would be nice to have one that queried by zip code).

www.uwgb.edu/dutchs/EarthSC202Notes/quakes.htm

Has nice cartoon earthquake demo, lots of visuals.

Appendix: Implementing National, State and District Standards

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- see (Executive Summary: Principles and Standards for School Mathematics)

NCTM principles - focus on equity, teaching, learning, technology, assessment

NCTM content standards - focus on number & operations, algebra, measurement

NCTM process standards - focus on Problem Solving, Communication, Connection, and Representation.

CT State Department of Education Curriculum Framework (Mathematics Curriculum Framework Prek-12 Matrix). This unit focuses on the three content areas of Algebraic Reasoning - Patterns and Functions, Numerical and Proportional Reasoning, and Working with Data - Probability and Statistics. Both core and extended items are included.

The following lists the power standards, concepts and objectives from the New Haven Algebra 1 curriculum, which are developed in or related to this unit (NHPS Curriculum).

Unit 1 Functions - power standards 1,2,3, concepts A,B,C, objectives 1,3,7
Unit 2 Graphing - power standards 2,3,4, concepts A,B, objectives 1-6,8
Unit 3 Linear Functions and Inequalities - power standards 1,2,3, concepts B,C, objectives 2,3,4
Unit 4 Slope and Linear Functions - objective 10
Unit 5 Applications of Linear Functions - power standards 2,3,4, concept A, objectives 1,2,3
Unit 6 Quadratic Equations and Functions - power standards 2,3,4, concept C, objectives 3,4,5
Unit 7 Exponential Equations and Functions - power standards 2,3, concept B, objectives 2,3,4,5

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