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Wave Math

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INTRODUCTION

Ocean surface waves are surface waves that occur in the upper layer of the ocean, and vary in size from ripples to tsunamis. While ocean waves typically result from wind or geologic effects, they may travel thousands of miles before encountering land. As these waves travel there is little actual motion of individual water particles in the direction of propagation, despite the large amount of energy and momentum waves may ultimately deliver at their destination.

BACKGROUND

Wind wave size formation

The majority of large breakers on an ocean beach result from distant winds far from that shoreline. Three factors influence the *size formation* of *wind waves* :

1. The speed at which the wind travels.
2. The *fetch* or distance of open water over which the wind has blown.
3. The time during which the wind has blown over a given area.

These three factors work together to determine the size and shape of ocean waves: the greater each of the variables, the larger the waves. Waves are characterized by:

1. Wave Height (from trough to crest).

2. Wavelength (from crest to crest).
3. Wave Period (arrival time intervals of consecutive crests at a stationary point).
4. Wave propagation direction (with respect to north).

Waves present multiple heights in the same oceanographic area however, for weather reporting and for scientific analysis of wind wave statistics, their characteristic height over a period of time is usually expressed as *significant wave height* . This figure represents the average height of the highest one-third of the waves in a given time period (usually chosen somewhere in the range from 20 minutes to twelve hours), or within a specific wave or storm system. Consequently, the largest individual waves are likely to be about twice the reported significant wave height for a particular day or storm, due to the variability of wave height.

Wind wave type formation

Three unique *type formations* of *wind waves* develop over time:

1. Ripples, or capillary waves.
2. Seas.
3. Swells.

Ripples appear on smooth water when the wind blows, but will subside as the wind ceases. The restoring force that allows them to propagate is surface tension. Seas are the larger-scale, often irregular motions that form under sustained winds. They tend to last much longer, even after the wind has died, and the restoring force that allows them to persist is gravity. As seas propagate away from their area of origin, they naturally separate according to their direction and wavelength. The regular wave motions formed in this way are known as swells.

Some waves undergo a phenomenon called “breaking.” A breaking wave is one whose base can no longer support its top, causing it to collapse. A wave will break when it surges into shallow water, or when two wave systems oppose and combine forces. When the slope, or steepness ratio, of a wave is too great, breaking is inevitable. A 1:24 slope may be a long, shallow swell found in deep waters. A 1:14 and higher slope is a wave that is too steep to remain coherent and maintain its integrity. Waves will also break if the wind forces escalate sufficiently to blow the wave crest off the base of the wave.

Spilling type of surf

Three main types of breaking waves are identified by surfers or surf lifesavers. Their varying characteristics make them more or less suitable for surfing, and each breaking wave type presents different dangers.

1. *Spilling* , or *rolling* : these are the safest on which to surf; they can be found in relatively sheltered areas.
2. *Plunging* , or *dumping* : these break suddenly and can “dump” swimmerspushing them to the bottom with great force. Strong winds can cause dumpers; they can also be found where there is a sudden rise in the sea floor.
3. *Surging* : these may never actually break as they approach the water’s edge, as the water below them is very deep. These waves can knock swimmers over and drag them back into deeper water.

Since ocean surface waves are mechanical waves that propagate along the interface between water and air and the restoring force is provided by gravity, they are often referred to as surface gravity waves. As the wind blows, pressure and friction forces perturb the equilibrium of the ocean surface. These forces transfer energy from the air to the water, forming waves. In the case of monochromatic linear plane waves in deep water, particles near the surface move in circular paths, making ocean surface waves a combination of longitudinal (back and forth) and transverse (up and down) wave motions. As the wave amplitude or height increases, these particle paths no longer form closed orbits: after the passage of each crest particles are displaced a little forward or backward from their previous positions, a phenomenon known as Stokes drift.

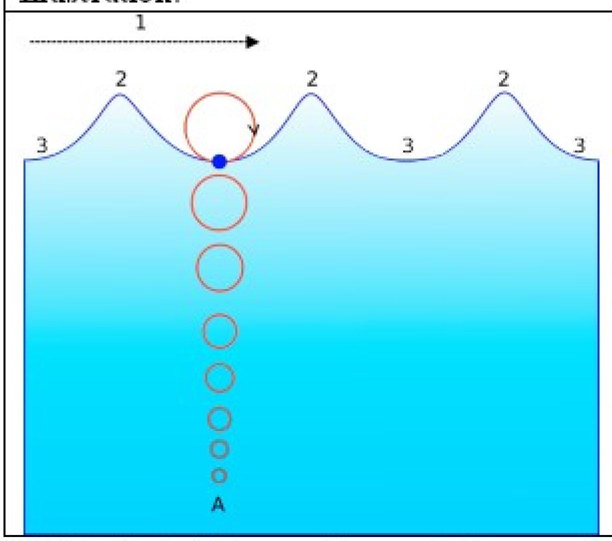
Illustration:	**Description:**
	**CIRCULAR PARTICLE MOTIONS:** the motion of a particle in an ocean wave. **FIGURE A** = In deep water. The orbital movement of fluid particles remains circular as the water depth increases below the surface. **1** = Propagation direction. **2** = Wave crest. **3** = Wave trough.

FIGURE A: Science of waves in deep water

While the ocean depth increases and in deep water as illustrated above [FIGURE A], the radius of the circular motion decreases in length, reducing the orbit. Whereas when a depth equal to half the wavelength Λ is reached, the orbital movement decays to nearly zero. The celerity or phase speed of the surface wave is well approximated by

$$c = \sqrt{\frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi d}{\lambda}\right)}$$

where:

c = phase speed;
Λ = wavelength;
d = water depth;
g = acceleration due to gravity at the Earth’s surface.

In deep water, where

$$d \geq \frac{1}{2}\lambda, \text{ so } \frac{2\pi d}{\lambda} \geq \pi$$

and the hyperbolic tangent approaches 1; ***c*** is computed in m/s and approximates 1.25 $\sqrt{\lambda}$, when Λ is measured in meters. This expression demonstrates that waves of different wavelengths travel at different speeds.

Wave velocity also increases with wave depth and the relative difference in density. For an ocean depth of 4000m, a wave’s celerity or speed would be about SQR (10 x 4000) = 200 m/s = 720 km/hr, and similar to the cruising speeds of jet aircraft.

The general formula required for these calculations is

c = $\sqrt{gd}$

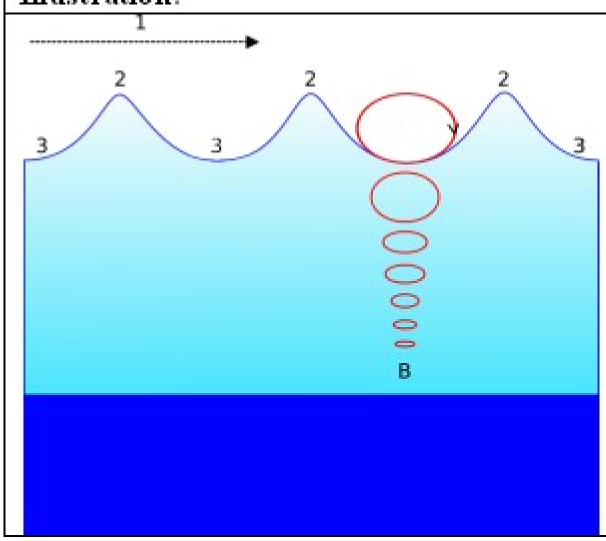
Illustration:	**Description:**
	**ELLIPTICAL PARTICLE MOTIONS:** the motion of a particle in an ocean wave. **FIGURE B** = In shallow water. The orbital movement of fluid particles remains elliptical as the water depth decreases below the surface. **1** = Propagation direction. **2** = Wave crest. **3** = Wave trough.

FIGURE B: Science of waves in shallow water

When waves propagate in shallow water as illustrated above [FIGURE B], where the depth is less than half the wavelength, the particle trajectories are compressed into ellipses and elliptical orbits, subject to Boussinesq equations or shallow water equations. As the water depth *d* decreases towards the coast and in shallow water, the velocity of the crest and the trough of the wave will also be transformed: the crest moves faster than the trough, causing surf and a breaking of the waves.

Shallow water equation models have only one vertical level, so they cannot directly encompass any factor that varies with height. Vertical pressure gradients are nearly hydrostatic, and horizontal pressure gradients are due to the displacement of the pressure surface, implying that the velocity field is nearly constant throughout the depth of ocean water.

FIGURE C: Wave Groups

Wave velocity also has implications during a storm, where the fastest waves are the ones with the longest wavelength. Consequently, the first storm waves to arrive on the coast are the long wavelength swells. When several wave trains are present, as is typical across the surface of the ocean, the waves form groups. In deep water the groups travel at a group velocity, which is half of the phase speed. Each wave within a group appears to begin at the back of group, increase in size and eventually disappear at the front of the group as illustrated above [FIGURE C].

The movement of ocean waves can be captured by wave energy devices that track wave groups. The energy density per unit area of regular sinusoidal waves depends on the water density P, gravity acceleration *g* and the wave height *H* , which is equal to twice the amplitude, *a* :

$$E = \frac{1}{8}\rho\ gH^2 = \frac{1}{2}\rho g a^2$$

The velocity of propagation of this energy is the group velocity. A wave group of larger amplitude moves at half the average speed of its component waves; and the wave's energy spectrum does not move at the speed of the waves but at the group velocity.

When distant storms send long waves out over great distances, they arrive at a time that corresponds with the group velocity, not the wave velocity. Therefore, a group of waves with a period of 14 s would travel at a group velocity of 11 m/s, not 22 m/s, and take approximately 24 hours, not 12 hours, to travel to a shore at 1,000 km distance. The important practical application of this wave science is solving the 3D wave equations back through time. These methods allow tracking the signal source from the measurements, performed in several places that are frequently quite far from the point where the wave has originated. Most ocean-based wave systems are comprised of many more than two components, each having generally different wave amplitudes and wave periods. This does not alter the group-based behaviors and velocities however; the interference between wave trains of different periods, moving in the same direction forms an irregularity of waves.

Disproportionately large coastal waves are referred to as *Sneaker Waves* , a term used to describe what can often appear in a wave train without warning. Sneaker waves are formed when the energy of a number of smaller waves becomes *focused* , and that several smaller waves have run into each other and created constructive interference. Constructive interference means that the individual wave peaks coincided and created a new wave that is the sum of those waves that were superimposed. Because of their larger size than preceding waves, sneaker waves can catch unsuspecting swimmers, washing them out to sea. While sneaker waves are a universal coastal phenomenon, a higher incidence of sneaker waves has been traced to several states in the United States: Northern California, Oregon and Washington. While these ideas have no scientific basis, one common belief that has wide circulation and has entered popular culture through music and literature predicts that out of a certain number of waves, one will be much larger than the others: every seventh wave is the popular predictable pattern.

While tides that are caused by the moon and sun’s gravitational forces, tsunamis that are caused by underwater earthquakes or landslides, and waves generated by underwater explosions or the fall of meteorites have a quantitative origin, other individual waves that travel outside of groups do not. These waves are distinctly different as they traverse across the ocean independently as “freak waves,” “killer waves,” “king waves,” “monster waves,” or “rogue waves” and can attain the extreme height of approximately 30 meters. In oceanography, they are more precisely defined as waves whose height is more than twice the significant wave height (SWH), which is itself defined as the mean of the largest third of waves in a wave record. These huge deep ocean waves are believed to form in a similar process to the way Sneaker Waves are thought to form, only on a much more massive scale.

Ocean wave measurement

Seafaring observations of waves have been recorded for over 130 years. This long record of the wave climate is complemented by indirect measurements of wave activity found in the Earth recorded by seismometers. More accurate quantitative measurements can be made using a wave pole on a fixed structure. An observer stands on the shore in a designated spot and sights the wave alongside a pole positioned between them and the waves. Such poles are often part of weather monitoring stations located along coastlines, particularly those associated with lighthouses. Electronic poles known as wave staffs are often used for precise engineering applications, and are operated on some research platforms such as the Aqua Alta tower in the Adriatic Sea, offshore of Venice. Wave staffs are usually replaced by radar (widely used in the Netherlands) or laser altimeters (such as found on some U.S. NDBC stations) for routine measurements.

A more common and robust way of measuring waves uses a buoy that records the motion of the water surface, which does not require a fixed platform. The buoy-recorded motions provide a time history of the water elevation for that location and statistics can be calculated including the significant and maximum wave heights and periods. Modern wave rider buoys usually measure their movement along the three dimensions and so also give information about wave direction. For the south east Queensland coastline there are wave rider buoys about every 100 km along the coast. The wave rider buoys are typically positioned off the entrances of major ports or major recreational surfing or swimming beaches. A network of wave rider buoys properly positioned can allow the interpolation of the wave climate for that region. Wave rider buoy data is a typical input for coastal modeling, the wave rider wave train is typically the deep water wave climate that is then refracted across the seabed contours into the wave breaking zone. In coastal areas, the wave-induced velocities and pressure fluctuations can also be recorded using pressure gauges and current meters, similar to measuring tides.

Wave heights can also be measured from space, at least in a statistical sense, using the change in the form of radar pulses reflected off the sea surface by altimeter radars as found on the French/U.S. Topex/Poseidon and Jason satellites. Other radar techniques, either ground-based wave radar or airborne systems such as real or synthetic aperture radars, can also provide measurements of wave statistics. These radar systems are best suited for long period waves or swells, allowing the tracking of swells over very long distances.

Ocean wave models

Surfers are very interested in a predictable wave climate. Predictions of the surf quality for upcoming days and weeks are available from many websites that also provide weather forecasting. The Ocean Wave models are driven by more general climate models that predict the winds and pressures over the oceans.

Ocean wave models are also an important part of examining the impact of shore protection and beach management. For many coastal areas more information about the wave climate would improve contemporary environmental understandings, therefore estimating the effect of ocean waves is important for managing littoral environments.

LESSON PLANS

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[1] Plan one: 45 minutes

Topic: Ocean Surface Waves In Deep Water

Objective: SWBAT define and analyze Forming Deep Water Surface Waves

Lesson Starter: Surface Waves are generated by wind. Offshore storms generate winds which blow on the surface of the sea and create ripples, similar to the ripples in a cup of coffee or tea form when you blow on it to cool it down. The wind can be seen on weather maps as low pressure areas. The more tightly positioned the isobars, or meteorological contour lines of equal or constant pressure on the weather map, the stronger the winds blow. Consequently, small Surface Waves or capillary Surface Waves are initially generated in the same direction that the wind is blowing.

As the wind continues to blow and the Surface Waves generated remain under the influence of a constant wind the smaller Surface Waves will increase in size. The stronger and longer the wind blows the greater the effect on these ripples and the larger they will become, eventually creating a Larger Swell. Initially the Surface Waves will just be small but these will soon increase in size because the wind more effectively impacts the small Surface Waves than the calm flat sea surface.

While the size of Surface Waves is dependent on the generating wind speed, the seas are only considered *fully formed* once the largest Surface Waves that can be generated for a specific wind speed have been attained.

All generated Surface Waves have a signature, presenting differing speeds and wave periods. The longer period Surface Waves move faster and overtake the shorter period, slower Surface Waves. As these Surface Waves propagate, traveling farther away from the original wind source, they start to organize themselves into Swell lines: *Wave Trains* form. These sets of Surface Waves inevitably reach the beach as *Ground Swell* , since the original wind source that generated them has ceased to influence them.

Lesson: For deep water, the relationship between speed and wavelength is given by the formula:

l = g x t x t / (2 x pi) l = t x c for all kinds of waves, substitute in above equation: t x c = g x t x t / (2 x pi) c = g x t / (2 x pi) or t = c x 2 x pi / g or t = c x 0.641 (s)

where t= wave period (sec), f= wave frequency, l= wave length (m) and pi=3.1415... to calculate c and l from wave period t (in sec): c = t x 1.56 m/s= t x 5.62 km/hr = t x 3.0 knot

l = 1.56 x t x t (meters)

Thus waves with a period of 10 seconds travel at 56 km/hr with a wave length of about 156m. A 60 knot (110 km/hr) gale can produce in 24 hours waves with periods of 17 seconds and wave lengths of 450m. Such waves travel close to the wind's speed (97 km/hr). A tsunami travelling at 200 m/s has a wave period of 128 s, and a wave length of 25,600 m.

The general formula required for these calculations is

c = $\sqrt{gd}$

Lesson Review: FIGURE A. ORBITAL MOTIONS: Wave Particle Displacement.

Lesson: Apply the formula for a circular orbit and calculate:

$(x - h)^2 + (y - k)^2 = r^2$ or $\mathbf{x^2 + y^2 = r^2}$.

Lesson Closer: Summarize for students that surface waves could theoretically travel much faster on larger planets, in media denser than water.

[2] Plan two: 45 minutes

Topic: Ocean Surface Waves In Shallow Water

Objective: SWBAT define and analyze Forming Shallow Water Surface Waves

Lesson Starter: The relationship between wave speed or phase velocity and depth of long surface waves in shallow water is computed by the formula c x c = g x d x (p2 - p1) / p2 or c x c= g x d for water/air

where

c= wave speed, g= acceleration of gravity (9.8066 m/s/s), d= wave depth or upper layer depth, m), p2= density of water (=1) and p1= density of air (= 0.00125).

Lesson: The formula states that wave velocity increases with wave depth and the relative difference in density. For an ocean depth of 4000m, a wave's celerity or speed would be about SQR (10 x 4000) = 200 m/s = 720 km/hr. Students should determine that these wave velocities are comparable to the velocities of commercial jet aircraft.

The general formula required for these calculations is

$$c = \sqrt{gd}$$

Lesson Review: FIGURE B. ORBITAL MOTIONS: Wave Particle Displacement.

Lesson: Apply the formula for an elliptical orbit and calculate:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Lesson Closer: Summarize for students that surface waves could theoretically travel much faster on larger planets, in media denser than water.

[3] Plan three: 90 minutes

Topic: Ocean Surface Waves

Objective: SWBAT define and analyze what determines the size of Swell

Lesson Starter: Three main factors determine the size and quality of a wave in open sea:

1. *Wind Speed:* The faster the wind speed the larger the wave.
2. *Wind Duration* **:** The longer the wind blows the larger the wave.
3. *Fetch* **:** The greater the wind-affected area the larger the wave.

Once Surface Waves are no longer influenced by wind they begin to forfeit their energy. Surface Waves will travel as far as they can while being decreased by friction from the sea bed and obstacles in their path, such as a big island.

There are different factors affecting the wave size at a certain surf break:

· Swell Direction **:** Will the cresting Surface Waves break open in the same direction as the current swell direction.
· Ocean Floor: A swell coming straight from deep sea up onto a reef will generate big, barreling Surface Waves. Whereas a long, shallow ledge up to the shore will slow down the Surface Waves and deplete their energy, rendering Surface Waves with less power.
· Tide **:** Some Surface Wave formations are totally tide dependant.

On The Coastline: Shoaling Alters Surface Waves

As a swell approaches the coastline and comes into contact with the sea floor the waves will start to dissipate. Some of the wave's energy is lost through contact with the sea floor. The shallower the water becomes the slower they move. As they slow down they have to squash together, shortening their wave period. This process is called *shoaling* and an increase in wave height results. The steepness of the sea floor gradient is directly proportionate to the resulting increase of Surface Wave height. This pronounced increase in wave height starts to happen at depths of approximately one half of the wavelength.

Breaking Waves

As the wave moves into increasingly shallow water the bottom of the wave decreases speed. There comes a point where the top of the wave overtakes it and starts to spill forward and the wave starts to break. A wave will start to break when it approximately reaches a water depth of 1.3 times the wave height.

The type of wave that is produced is dependent on different factors:

· Type Of Swell
· Wind Direction
· Slope of Sea Bed
· Sea Floor Features

Type Of Swell

Ground Swell creates a more dramatic wave. The longer wavelength waves will move quickly and get into shallow water before starting to break. The breaking waves will be steeper and faster. Whereas Wind Swell will tend to break in deeper water and with less force and configuration.

Wind Direction

An *Offshore* wind is capable of generating more prominent waves. The wind blows against the top part of the wave and delays the top from overtaking the bottom part. These Surface Waves break later than they normally would in calm conditions, exhibiting huge plumes of spray blowing back over the top of the wave.

Whereas an *Onshore* wind will have the opposite effect: the onshore wind pushes the top of the wave forward causing the wave to break before the normal breaking depth is attained. Surface Waves tend to be less uniform and with less amplitude.

Slope Of The Sea Floor

The contrast to the gently sloping sea floor is a steep slope or a reef. The swell approaches the beach / reef at a greater speed. From the general formula required for these calculations

c = $\sqrt{gd}$

it can be seen that the wave “jacks up” due to the rapid change in depth creating a higher wave. The breaking depth is reached much later that on the gently sloped bottom. The top of the wave quickly overtakes the bottom and pitches forward (often taking the inexperienced surfer with it). The waves created by the rapid change in depth are much steeper and hollower, forming the well-known tube.

Reef breaks such as Pipeline in Hawaii are examples of this type of break.

WEBSITE

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http://www.kwsi.com/ynhti2008/

GLOSSARY OF TERMS FOR WAVES AND SURFING

AMPLITUDE The vertical distance from still water level to wave peak (half the height of the wave).

BARREL The same as Tube. The hollow part of a breaking wave where there is a gap between the face of the wave and the lip of the wave as it curls over. One of the highlights for any surfer is catching a tube ride.

BATHYMETRY The features of the sea floor. The measurement of water depth at various places in a body of water.

BOUSSINESQ In fluid dynamics, the Boussinesq equations for ocean waves is an approximation valid for weakly non-linear and fairly long waves. The approximation is named after Joseph Boussinesq, who first derived them in response to the observation by John Scott Russell of the wave of translation (also known as solitary wave or soliton). The Boussinesq equations and Boussinesq-type equations were first introduced and published in 1872.

CAPILLARY WAVES The first small waves created when the wind blows on the sea.

CELERITY The rapidity or speed of motion or action or propagation of an ocean surface wave, or other wave on the surface of a liquid.

CHOP Moderate local winds form little waves known as chop, or Wind Chop.

CLOSEOUT A wave that breaks along its entire length at the same time making it unstable for surfing. Closeouts can either be caused by a strong offshore wind or sea floor topography and are also called shutting down.

CORDUROY Swell lines that look like corduroy.

CREST The highest part of the wave (above still water level). Same as the Wave Peak.

CRUMBLE / CRUMBLY WAVES The lip of waves weakened by onshore wind are said to crumble, unstable for surfing.

DECAY OF WAVES / WAVE DECAY The decrease in wave height and increase in wavelength of a wave once it is outside the fetch.

DIFFRACTION When the wave comes into contact with an obstacle or barrier such as a breakwater, the energy of the wave is transmitted along a wave crest. Diffraction is the 'spreading' of waves into the sheltered region within the barrier's geometric shadow.

DOPPLER SHIFT The Doppler shift or effect, named after Christian Doppler, is the change in frequency and wavelength of a wave for an observer moving relative to the source of the waves.

FETCH The area of sea surface where the wind generates the waves/swell. Fetch is one of the key areas in the quality of a swell and the size of the waves.

FULLY DEVELOPED SEA Waves that have reached the maximum size possible for a fetch, wind speed and wind duration.

GLASSY Waves that have incredibly smooth faces due to the lack of local wind or a slight offshore wind. Have a look at this picture of a glassy Huntingdon Beach wave.

GROUND SWELL Waves no longer being affected by the winds that generated them. Waves outside the Fetch.

LEFT A Left is a wave that breaks from left to right as you are looking from the beach.

LIP The upper most part of the breaking wave where a surfer will do maneuvers such as a floater.

NEAP TIDE Smaller than normal tides occurring then the gravitational pull of the Sun and Moon are at right angles to the Earth.

OFFSHORE WAVE This in surfing terms relates to the wind blowing from the shore. A ground swell mixed with offshore winds makes for cracking surf.

ONSHORE WAVES The opposite to offshore. The wind blows toward the beach and as a result the waves lose their shape and crumble.

PEAK The highest part of the wave (above still water level) and synonymous with the Crest.

PEAK DIRECTION The wave direction at the frequency at which a wave spectrum reaches its maximum.

PEAK PERIOD / WAVE PERIOD The time taken for consecutive wave crests or wave troughs to pass a given point and the greater the wave period the better the swell.

PROPAGATION Wave propagation is any of the ways in which waves travel through space and time, usually with transference of energy.

REFRACTION The tendency of wave crests to become parallel to underwater contours as waves move into shallower waters. Waves moving in shallow waters move more slowly than waves moving in deeper water. Refraction can be seen where waves 'wrap' round a point and their direction seems to change.

RIGHT A Right is a wave that breaks from right to left when viewed from the beach.

SIGNIFICANT WAVE HEIGHT How significant are your wave heights? You are likely to have seen significant wave height on surf reports. The Significant Wave Height is the average height of the one-third highest waves of a given wave group.

SHOALING Waves being forced to bunch together as they enter shallower water and slow down.

SPRING TIDE Larger than normal tides occurring when the gravitational pull of the Sun and Moon are combined in line. The opposite effect to Neap Tides.

STOKES DRIFT The Stokes drift is the difference in end positions, after a predefined amount of time (usually one wave period), as derived from a description in the Lagrangian and Eulerian coordinates. This nonlinear phenomenon is named after George Gabriel Stokes, who derived expressions for this drift in his 1847 study of water waves.

SURF Waves breaking near the shore.

SURF ZONE The area from the shore out to where the waves start breaking, and surfers lineup.

TIDE The increase and decrease in sea level resulting from the Moon's and to a lesser extent Sun's gravitational pull.

TROUGH The lowest part between two successive waves (or the part between two successive waves below still water level).

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