West Nile Virus and Lyme Disease: Making Sense of the Numbers

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Introduction

Here in New Haven, we live at the epicenter of two separate disease explosions; Lyme disease and West Nile Fever. These infections are among a class of emerging pathogens that are vector borne and linked to recent changes in our environment and society. The rapid growth of these infections leads to many questions. What is the current state of these diseases in New Haven? What does the current rate of infection predict about future spread of disease in our area? Are these diseases preventable? What are the environmental pressures that are causing these diseases to spike? What can we do as a city and state to reduce exposure of our citizens to these diseases?

My interest in the evolution of these diseases goes back to my own experience. In 2000, I was working as an organic farmer in Bethany, CT. Our fields were only 8 miles from downtown New Haven, yet without being surrounded by 9 foot tall deer fencing, we would have lost most of our crops to the tremendous deer pressure. In late May I developed a 104º fever, stiff neck, and muscle and joint aches. It became apparent that I had Lyme disease. I never had the classic bull's-eye myself; however I saw a few magnificent versions of the bull's-eye rash among fellow farm workers. I have also commiserated with many friends who have not only suffered the illness but the cure. Doxycycline makes the patient extremely sun sensitive. Three weeks of this antibiotic, while wearing hats, longs sleeved pants and shirts in mid summer is almost as memorable as the illness itself.

My farming background put me in frequent touch with The Connecticut Agricultural Experiment Station. Not only did I use their soil lab, and plant pathology services regularly, I had many wonderful interactions with the scientists who work there. In considering material for this unit, I decided to return to CAES as a teacher, to seek resources for my students. I visited and interviewed Dr. Kirby C. Stafford III, Vice Director and Chief Scientist at the station in early April. In addition to sharing information and resource material with me, Dr. Stafford also pointed me toward a curriculum development team at the Peabody Museum. I met with Curriculum Specialist Terri Stern in late April. Terri discussed with me the unit they developed around these diseases as part of a Science Education Partnership Award (SEPA). I was also fortunate to attend their special event: Backyard Bloodsuckers: Biodiversity Bites Back! There, I spoke to John Shephard of the mosquito collection team from CAES. I also met scientist Maria Diuk-Wasser, who is observing robin roosting patterns. Her research has shown major roosting areas of robins in the city and resultant concentrations of infected mosquitoes. This includes one such "hot spot" along the West River, less than a mile from the high school.
where I teach.

The Connecticut Agricultural Experiment Station (CAES) has played major roles in the identification of both pathogen and vector in both of the diseases we will study. Scientists at CAES were the first to isolate the West Nile Virus and identify its major vectors in North America. They continue to conduct mosquito surveillance and to test between 100,000 and 200,000 mosquitoes a year for the virus. CAES has also identified the American robin as an important reservoir of WNV. Dr. Louis Magnarelli and John Anderson M.D. from CAES worked on the tick vector for Lyme disease throughout the late 1970's and early 1980's. Yale School of Medicine in New Haven was first to send a team of doctors to study an unusual outbreak of arthritis in Lyme Connecticut in 1975. This investigation led to the classification of Lyme disease in 1977. These groups continued to collaborate and all participated in the First International Symposium on Lyme disease in New Haven in 1983.

Numbers begin to tell their story here. The accumulation of data from these dedicated scientists, doctors and epidemiologists have left us a trail of information to dissect, analyze and draw conclusions regarding the state of these diseases and their future in our communities. With access to the internet, we can follow developing information on each disease from the websites of CAES, the Connecticut Department of Public Health (CTDPH) and the CDC (Centers for Disease Control). As members of this community, and stakeholders in our own well being, we cannot help but be interested in these questions. As mathematicians and statisticians, we can look at the data and begin to find some answers.

The goal of this learning unit will be to look at these two emerging diseases, explore their epidemiology and their evolution into diseases of concern to New Haven. I am excited that so much material that we will be able to look at will be primary source. Much of it will be found on the internet, but I hope to include field trips to CAES, the Peabody Museum, and perhaps labs at Yale or the New Haven Health Department. We will be able to collect data as we learn about the epidemiology of these diseases. These data will help us to create a clearer picture of the state of these diseases in Connecticut. Ultimately, as mathematicians, our objective will be to use the tools at our disposal to analyze the data as best as we are able, and use the trends that we see, to predict the future in the way that only mathematicians can.

**Vector Cycles of Diseases**

Both diseases that we will study are vector borne. WNV is an arbovirus, or a virus spread through the bite of an arthropod, in this case a mosquito. Lyme disease is caused by a bacterium called Borrelia burgdorferi (Bb), which is carried to humans in the bite of a tick. Both exist in a complex matrix of vectors, hosts and reservoirs. To understand this cycle, we need to understand the specific vocabulary of this phenomenon. A vector is the organism (mosquito or tick) that carries the disease in its body and transmits it to the host. In our study, we are generally concerned with the human hosts, although both diseases affect other animals who could be considered hosts as well. A reservoir, sometimes known as an intermediary host, is an organism that carries the pathogen in its body but is usually unaffected by the disease itself. This host maintains a supply of the pathogen from year to year to be picked up by new generations of the vector's organisms.
Figure 1: Borrelia burgdorferi and flavivirus

On the east coast, the Lyme cycle follows the two-year life cycle of the black legged tick (Ixodes scapularis). This tick, commonly known as a deer tick, hatches as a larvae from an egg in midsummer of the first year and seeks a blood meal from a small animal such as a mouse, bird or chipmunk. These small rodents are reservoirs of Bb. The infected larva falls off the rodent and will wait for the following spring to molt into a nymph. In spring, the nymph seeks a host, this time perhaps a larger mammal such as a human, raccoon or deer. If the nymph was not initially infected, it may become infected at this meal, while attached to an infected host. If the nymph is infected (as many are) it may pass Bb to the host. The nymph molts into an adult, who again seeks to feed on a large mammal, yet another opportunity to transfer the bacteria. The adults will mate, feed, and fall off to over winter and lay eggs in the spring, completing the cycle. 1

Figure 2: Deer tick (Ixodes scapularis) - Adult on left, nymph on right

In the New Haven area, the mosquito most commonly associated with the transmission of WNV is the common species Culex pipiens. This mosquito has a statewide presence in Connecticut. The life cycle of all mosquitoes are tied to water, but unlike swamp dwellers, this mosquito can grow readily in artificial containers of stagnant water and is adapted to polluted water. Mosquitoes grow from egg to larva to pupa, all in the water. The process can take from a couple of weeks to a couple of months to pass through all three stages. At the end of the pupa stage, a mosquito emerges as an adult that can fly. Most female mosquitoes live for 1-3 weeks, while males have a much shorter life. It is, however, the females who are of concern because they seek blood in order to reproduce. The adults acquire the virus from reservoir species like the American robin. Culex pipiens prefers birds as hosts, but will bite humans, particularly in the evening. 2 A previously infected mosquito can
transfer the virus to a human host through a bite.

Figure 3: Culex pipiens mosquito

**Lyme and West Nile: Emerging diseases**

First discovered in Uganda in 1937, West Nile Virus made an abrupt debut in the Western Hemisphere 1999. The first cases centered in Queens, NY. In the 10 intervening years West Nile Virus has spread to 47 states and DC. To date there are 28,943 reported cases in the United States, with an estimate of upwards of 1.5 million actual cases. Within our state, the disease prefers urban settings to rural. In 2008, all cases reported to the CT state Department of Health were in Fairfield County, the majority in the cities of Stamford and Bridgeport.

Lyme Disease's history is pure Connecticut, although similar syndromes have been recorded in Europe since the late nineteenth century. The concrete definition of Lyme disease arose from a 1975-77 Yale study of the incidence of arthritis in Lyme, CT. While completing that study, Dr. Alan Steere consulted one of the country's leading experts in ticks, Dr. Willy Burgdorfer. Several years later, while studying Rocky Mountain Spotted Fever, Dr. Burgdorfer made the serendipitous discovery of spirochetes in the bodies of ticks he was dissecting. He related this to the Lyme study. Immediately, tests were done on samples from Lyme sufferers that confirmed the presence of matching antibodies to the bacteria samples Burgdorfer had discovered. The identification of the pathogen Borrelia burgdorferi that causes the syndrome has allowed effective treatments and diagnostic testing to be developed. Although Lyme disease has developed and spread in the rural parts of our state, the infections in urban New Haven County totaled 400 in 2008. According to Kirby Stafford III, Chief Scientist/State Entomologist at the Agricultural Experiment Station in New Haven, these reportable cases may only represent 10-16 percent of the total cases, due to the strict reporting criteria. Even so, Lyme disease is the most common vector borne disease in the United States.

The World Health Organization (WHO) defines an emerging disease as "one that has appeared in a population for the first time, or that may have existed previously but is rapidly increasing in incidence or geographic range." WNV and Lyme disease have short and intense histories within our state. Both have increased in
frequency and range in a short period of time and have quickly made themselves diseases of concern to our state. These diseases are "reportable", which means that physicians who diagnose the disease, as well as labs who detect presence of the disease in blood samples, are required to report these findings to the state. As the scrutiny of these new diseases continues, we will begin to see how they have evolved to their current level of threat. Chavers and Vermund in "Emerging Infectious Diseases" describe eight factors that contribute to the evolution of new diseases. We will look at four of these which have a great effect on the development of both diseases in our state. These are:

1. Demographic factors including population growth and distribution
2. Social and Behavioral changes
3. Climate and environmental changes
4. Microbial evolution

**Evolution of Emerging disease**

**Demographic factors**

Connecticut is much more populated today than it was 60 years ago. The population in Connecticut in 1990 was 3,287,116 contrasted with a population in 1950 of 2,007,280. In addition to significant population growth and increased density, the population has shifted to rural areas as a place to live, not to farm. The number of acres converted to developed use from rural use was 40,800 in the 5 year period from 1992-1997. These demographic shifts have increased the contact between humans and forested areas and therefore with the insects and mammals that thrive in these forests. Statistically, we can study historical census data to understand the change in residential patterns in Connecticut, and relate them to the growth of Lyme disease.

**Social and Behavioral changes**

Some of the social attitudes that have caused a rural migration in Connecticut, include a growing affection and protection for wildlife in the face of increased development. As farming has been replaced by other industries in Connecticut, the rural countryside has become a refuge for the urban weary. It is not a working landscape but a pure and bucolic haven for peaceful living. This has also been made possible by inexpensive cars and gas and an increase in commuting. The city is no longer a modernist's haven, but a dirty, toxic environment. The country living is healthy and robust. Outdoor recreation is also very popular, giving more contact among people who are hiking, walking, swimming and the insects that are causing disease. Despite the rise in vector borne illness, this perception of the relative healthfulness of the countryside persists. This was one of the troubling aspects of the original Lyme puzzle. Many who were affected thought that they were raising children in the safest, most natural environment possible: Why were their children being stricken with debilitating illness?

**Climate and environmental changes**

This may be perhaps the most compelling factor in the evolution of WNV and Lyme disease in CT. The flora of...
Connecticut has changed drastically in the past 100 years. Young forests have replaced the mostly deforested, agricultural landscape of 19th-century Connecticut. These young forests meet the suburbs and form a transitional environment or ecotone, in which deer, ticks and rodents thrive. According to Arno Karlen, in Biography of a Germ Connecticut is currently two-thirds secondary-growth forests and ecotone. Mark Walters' book "Six Modern Plagues" quotes Rutgers ecology professor Edmund Stiles: "Three hundred years ago there were probably about 25 deer [sic] per square mile. Now there are something like 200 per square mile." Stiles goes on to say that generalist species like deer and mice thrive in disrupted forests, while many of their original predators are more specialized and do not. According to Kirby Stafford at CAES, "rates in Connecticut generally run around 30-60 deer per square mile, which is still a lot of deer." Additionally, a rise in average temperature and precipitation, attributed to global warming has paralleled a rise in tick population.

In WNV, global warming has been implicated in the change in avian migratory patterns that have expanded the reach of this disease. With a delay in the onset of winters, the lifespan of mosquitoes can increase and the numbers of infected vectors within the season will increase. Increases in precipitation leads to exponential increases in mosquito breeding and populations. The deaths of large populations of bats due to a fungus called white nose syndrome may lead to a sharp uptick in the mosquito population. Using these factors as predictors for the growth of these diseases, we can draw on climate, rainfall and animal population records as part of our study.

**Microbial evolution**

The evolution of these pathogens is an area of current scrutiny by scientists. Although it is beyond the scope of this unit to study the microbial evolution of flavivirus and Borrelia burgdorferi, it can be said that evolutionary changes in these pathogens have allowed their adaptation to the vectors and environmental conditions in the Western Hemisphere. Lyme disease may have evolved from a less virulent version of the Borrelia bacterium, or it may be more pervasive as a result of recognition and environmental exposure. We recognize its existence in Europe, the midwest and the West coast as a syndrome caused by a borrelia spirochete. However, in each area it is carried by different vectors. Likewise, WNV is spread by a different set of vectors in Europe and Africa, though the virus is similar enough in both places as to be recognized as the same virus.

**Symptoms and Management of Lyme disease and West Nile Virus**

Medically, we know quite a bit about these two diseases. In Lyme disease, the tick transmits the bacteria Bb as it feeds on the host's blood. After the tick attaches itself to the human, it uses its mouthparts to open a tiny wound in the skin, into which blood will pool and from which the tick will drink. The tick has special secretions in its saliva to anesthetize the wound and to keep the blood from clotting. The tick drinks blood, and pumps out these secretions as it stays attached to the person. As the blood collects in the tick's midgut, the Bb inside of the tick multiplies in the presence and nutrients of the blood. This process creates a mini factory for Bb to multiply exponentially and be injected into the person's body. As a result, the usual acquisition of Bb occurs only after an attachment of 48 hours.

Once inside a human the bacteria can create three stages of Lyme disease. The acute stage occurs 7-14 days
from the tick bite and presents itself as a rash called erythema migrans. In light skinned people it resembles a "bull's-eye" pattern - in dark skinned people it resembles a bruise. Flu-like symptoms may begin to occur at this point. Stage 2 occurs several days to weeks after the initial infection and may include fever, fatigue, head and neck pain and stiffness in joints. Stage 3 is a persistent infection in untreated or inadequately treated patients. This includes arthritis, persistent pain, swelling (often in the knees) and neurological symptoms.

Treatment for Lyme disease is an antibiotic such as Doxycycline, for 14 -21 days. Most infections respond to treatment, although there remains a controversy over lingering infections. Some believe that Bb can evade antibiotics and live silently in the body for some time, causing problems after many months or years. This controversy led to major schisms about treatment and eventually to Senate hearings in 1993.

West Nile Virus can cause a range of disease in humans. It is caused by a virus called a flavivirus, and is related to several other disease agents such as Japanese encephalitis virus and St. Louis encephalitis virus. The incubation from the time of bite is 2-24 days. It has three potential clinical outcomes. One is asymptomatic. It is believed that 80% of infections result in no noticeable symptoms. The second is known as West Nile Fever. This disease is a flu-like illness that can present sore throat, fever, and even vomiting in a third of cases. The third possible outcome is West Nile Neurological Disease (WNND). This syndrome is the most severe and can include neurological impairment, encephalitis and death. Forty-one percent of reported infections are WNND; however only 4% of all WNV cases are fatal. It is believed that the number of WNND disease cases is an excellent indication of overall infection rates. The estimate is that for every case of WNND there are 28 cases of WN Fever and 140 infections with the virus, again most being asymptomatic.

Treatment is for relief of symptoms and supportive therapies such as hydration and fever reduction.

**Teaching Strategy**

The idea of this unit is to teach math around a purpose. The goal is for students to seek out math applications on an "as needed" basis to answer questions about the diseases we are focused on. The students who can utilize this unit will likely be high school students in an integrated math course that exposes students to statistics as well as functions. The goal of the unit is to illuminate the bridge from univariate to bivariate data and integrate the interpretation of statistics with algebraic representations of real life relationships.

Principles of statistics will be used to collect, vet, analyze and display information about Lyme and WNV. Single variable distributions will be evaluated using measures of center and spread. We will apply evolutionary thinking to predicting environmental pressures that affect the health and growth of particular organisms, such as the tick. This leads us to connect two factors, x and f(x), and enter the world of bivariate data. We will test our assumptions using correlation and regression. We can analyze and predict the growth of disease by studying the range of our x variable.

This is an exploration model, with the teacher serving as facilitator and guide. Lessons have concrete tasks to complete. Students will explore materials to seek out information needed to accomplish the tasks. Students will be provided with materials (films, research writings, newspaper articles and websites) and a list of the assignments (assessments) required for each section of the unit. The teacher guides the groups to fulfill the assignments. Following each completed assignment, students will share the information they have collected with the other groups.
Math cannot help but be deep and abstract. Understanding its depth requires a student’s ability to apply the tools they have learned. Like reading, it must become a meaningful symbolic language, not simply a page of ciphers and symbols to be manipulated. Great literature is full of nuance, subtexts and symbolism and so too are the nuanced uses of math, and ways to interpret numerical information that we have collected. Once applied in a specific way, students are able to approach synthesis of mathematical ideas by generalizing the application to a broader range of cases. This unit strives to connect the depth of mathematical reasoning with the students’ curiosity. The subjects of disease, death, plague and survival capture our collective imaginations and can make for some intriguing examples to explore and analyze. In addition, we focus on two diseases that are geographically and temporally relevant. Restricting our focus to two diseases is intended to make the manipulation of numerical data about the diseases clear. It is an excellent interdisciplinary exploration into the sciences of biology, public health, medicine and environmentalism.

**Unit Objectives**

*Enduring understanding*

Math can help us study the process of evolving diseases. The engine of evolution is random mutation, mimicked by probability. The pressure of environment leads to success or failure of a mutation. Statistics helps us look at variation and possibilities and find patterns. Through our study of data and statistical analysis, we may find patterns created by the data. Writing a function that emulates the data we have collected is a powerful tool, known as modeling. When we discover correlations between variables such as environmental influences and populations we can explore these relationships through linear functions. Modeling, combined with evolutionary prediction and analysis can help us to understand and reduce the risk of diseases.

*Mathematical objectives*

1.1 Use principles of Statistics to collect organize and analyze information that relates to West Nile Virus and Lyme disease.
1.2 Sort information categorically and numerically
1.3 Create a collection of values (distribution) about the diseases of interest that can be analyzed and compared to other distributions using
   a. Measures of center
   b. Measures of spread
1.4 Using the previous univariate data, suggest a relationship using an independent variable (x) and an x-dependent f(x) variable. Plot as a scatterplot.
1.5. Using the scatterplot, find the mean of x and mean of f(x). Using that point as a pivot, manually draw a line of best fit.
1.6 Use a statistics utility to create a regression model for the data. Discuss the relationship between the visual representation (graph) and algebraic representation (equation).
1.7 Use correlation to test the relationship proposed in 1.5. Discuss correlation and best fit.
1.8 Use data, regression models and evolutionary thinking to suggest public and personal actions and policies around these two diseases
Biological objectives

1.1 Analyze historical information about the diseases of West Nile Fever and Lyme disease and their presence in New Haven County and in Connecticut. Create a disease profile for each illness that describes symptoms and outcomes and identifies the pathogen that causes these illnesses.
1.2 Identify vectors, hosts and reservoirs in vector-based disease systems and create diagrams that follow the life cycle of the disease vectors.
1.3 Collect information about environmental factors that affect the health of the various organisms in the life cycle of these pathogens.
1.4 Using evolutionary thinking, predict environmental and social factors that have contributed to the growth of these diseases in our current situation.
1.5 Test your hypothesis using a scatterplot to visually check for a relationship between the variables you have chosen to investigate.
1.6 Evaluate and discuss the linearity of the relationship between the variables investigated. Evaluate any outliers or unusual observations.
1.7 Discuss exponential growth and disease, looking at historical disease models and looking at the mechanics of bacterial and viral reproduction.
1.8 Use data, regression models and evolutionary thinking to suggest public and personal actions and policies around these two diseases.

Assessments

In relation to each objective in the unit, students will submit the following work as an ongoing assessment of their understanding of and attainment of the objectives. Students will be divided into working groups to research either West Nile Fever or Lyme Disease Students will:

1.1 Interpret several types of tables and charts with data about our target diseases. Read historical information about the diseases. Create a profile of the disease including symptoms as well as outcomes of these illnesses and describes the pathogen that causes the illness. Shape a targeted question for further research.
1.2 Construct a pie chart or histogram showing regions of Connecticut and their rates of disease
1.3 Using Lyme disease incidence in two counties in Connecticut and the rates of West Nile Virus-infected mosquitoes in several Connecticut counties, compare measures of center and spread by using box and whisker plots.
1.3.2 Create a diagram showing the life cycle of the pathogen that they are studying.
1.4 Collect data from primary sources (government websites/weather almanacs) related to the life cycle drawing from 1.3. Using the data, your previous graphs, and evolutionary thinking to support your writing, suggest a hypothetical relationship between disease rates increase or decrease and environmental factors and create a scatterplot of the data collected about variable x, and population, f(x).

1.5 Manually fit a line of best fit. Discuss the slope and approximate fit of the data points and write a descriptive model that includes the hypothesis, research and the mathematical graphic.

1.6 Use the data from 1.5 and a statistics utility to create a regression model for the data. Predict data points outside of the collected data, using your function and line of best fit. Write a comparison of the line of best fit and the regression model.

1.7 Perform correlation tests to demonstrate or disprove predicted relationships.

1.8 Write policy suggestions for New Haven County to reduce future disease cases. Use all of your work from the unit to create a supporting document for your policy suggestions.

**Example Lessons**

**Lesson 1**

*Objectives*

Interpret several types of tables and charts with data about our target diseases. Read historical information about the diseases. Create a profile of the disease including symptoms as well as outcomes of these illnesses and describes the pathogen that causes the illness.

Shape a targeted question for further research. Identify variables and populations to be examined in order to answer these questions.

*Strategy*

Students will work in groups. Using the indicated websites and/or packet of information about Lyme and WNV, students will use the following questions as prompts, to discuss what areas of interest they have for further study around this disease.
Are you at risk for catching this?
How sick will you/can you become from this illness?
Are there deaths in CT attributed to this disease?
What precautions would you be able to follow to avoid catching this disease?
How does where you live affect your chances for getting this disease?
What kind of information would you need to answer these questions?
Who is tracking this disease?
Is the rate of infections increasing or decreasing?

Students should discuss the research material and prepare a group question, or topic for further exploration. A written response should be prepared that includes the following outline:

1. A list of symptoms and outcomes for humans infected with Lyme or West Nile Virus
2. A list of risk factors for becoming ill from this pathogen.
3. A question that you intend to research further related to this illness.

Groups should present their work to the class and the class should open up to discussion.

Prompts for class discussion during presentations

What will you measure to find out? The quality you choose to measure is a variable.

Who will you look at in order to measure that quality above. This is your population.

Why measure that population?

When will you measure them (in all contexts or only a specific one)?

Where is the population?

Are there agencies studying this that you could look to for your information?

Discuss the importance of context to attach meaning to numbers in order to know how to correctly interpret the numerical information on the retrieved (or teacher-provided) charts.

Numbers in context:
Value or quantity

Nominal: Numbers can be names such as a passcode/phone number/upc code

Ordinal: place on a wait list, batting order, class rank

A nominal code sometimes includes order, ex. room number in a building

Frequency: the number of occurrences. This is tricky when it is the frequency of a value.

Ex. In a deck of cards there are 4 10's. 4 is the frequency, 10 is the value.

Lesson 4

Prior knowledge

Students will look at univariate data in Lessons 1, 2 and 3. They will discuss ways to display and compare data with an emphasis on measures of center and spread. There will be discussions and comparisons of skewed and symmetric distributions. Students will discuss outliers and their effect on a distribution.

Objectives

Using the univariate data previously examined and the life cycle drawing from 1.3, suggest a hypothetical relationship between increase or decrease of disease rates and environmental factors. For example students could propose that tick and mosquito rates are responsive to changes in rainfall or deer populations.
Collect data from primary sources (government websites/weather almanacs) that pairs the variables in your hypothesis.
Create a scatterplot with x and f(x) axis, and the paired data points (x,f(x)).

Strategy

Students will use previously prepared charts depicting the life cycle of their pathogen of concern. Through online, or resource research, students will identify factors that can create pressure in favor or against populations of organisms in the vector cycle. Students will identify specific populations and variables to examine (x) (for example: rainfall) and will collect data about the variable's effect on disease rates or populations of organisms [f(x)]. Students will collect data and create a scatterplot using an x, f(x) coordinate plane. Students will present their findings and explain the relationship they have chosen to graph.

Prompts for class discussion during research and presentations

What environmental factors affect the creatures in the life cycle of Lyme?
How might we determine if these factors are related to disease rates?
Do infection rates tell us anything about the environments in which these diseases thrive?
What can we infer about the relative prevalence of these diseases based on geography? Based on year?
What does the shape of your graph suggest about the relationship between your x-variable and the f(x) variable?

Bivariate data is a pairing of quantitative data. We can use a visual strategy of plotting the intersection of these pairs and examining their spatial distribution for a pattern.

**Lesson 5**

**Objectives**

- Manually fit a line of best fit. Discuss the slope and approximate fit of the data points and write a descriptive model that includes the hypothesis, research and the mathematical graphic.

**Strategy**

Students should find the mean of the x values (mean of x) and mean of the f(x), (mean of f(x)) values and mark the point (mean of x, mean of f(x)) on their scatterplot. Using this point as a pivot point, students should work in their groups to decide on a "line of best fit". Alternative ways to model the data as a line should be discussed. Students may propose more than one such line.

**Prompts for class discussion during presentations**

- What shape is suggested by each graph?
- What do you think this means about the variables you are examining?
- What line would you draw to connect this data? Recall that slope means vertical change divided by horizontal change. What slope describes this data the best? Students should compare methodology of drawing a line of best fit.
- What is an actual dependent result f(x) from the data? Using the same x, what is f(x) predicted by the line we drew?
- What is the difference between the f(x) value predicted on your line: f(x) and the actual f(x)? This is (f(x) - f(x)) or residual. It is the measurement of the amount that our model missed.
Students should try to interpolate a data point. To interpolate means to look at an x value point that is not a member of the data set but falls within the range of the observed data. Using the line of best fit and that x-value, predict an expected f(x)-outcome. Each student should prepare a written report based on the group data. The report should be a verbal description of a model illuminated by the scatterplot and/or the line of best fit. Students should draw conclusions about this relationship based on the research, as well as the linearity of their results. They should also discuss the data sets for outliers or unusual points.

![Example of Scatterplot with Regression Line](image)

Figure 4: An example of a scatterplot from CDC Lyme data with regression line

**Lessons 6-8**

Continuing with the exploration model, students will learn how to find a regression equation on the calculator, and how to read the values for correlation. Emphasis will be on looking at slopes and comparing trends for accuracy. Outliers and/or influential points will be examined and discussed. Correlation values will be used to evaluate how well the data fits the line. Residuals will be examined in the calculator and discussed in terms of the graph. Students will spiral back to the questions raised in lesson 5 about the differences between observed and expected values using the line of best fit and relate these to residuals. Cautions about inference from the regression will be discussed.

**Summative Assessment**

A final report will be written that includes information the vector cycle and life cycle of the disease agent. Also included will be previous writings about the group’s question, hypothesis, data, mathematical model and graphical models. Conclusions about correlation should also be included. Recommendations based on the investigation should be written with an eye toward personal safety and or/public policy that could result in reduction of infections in the future.
Appendix

This unit follows standards from the 2005 CT Mathematics Curriculum Framework:

**Algebraic Reasoning: Patterns And Functions**

Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies. How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?

1.1 Understand and describe patterns and functional relationships.

9-12 core: Describe relationships and make generalizations about patterns and functions

9-12 extended: Model real-world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Represent and analyze quantitative relationships in a variety of ways

9-12 core: Represent and analyze linear and nonlinear functions and relations symbolically and with tables and graphs.

9-12 extended: Relate the behavior of functions and relations to specific parameters and determine functions to model real-world situations.

**Numerical and Proportional Reasoning**

Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. How are quantitative relationships represented by numbers?

2.1 Understand that a variety of numerical representations can be used to describe quantitative relationships.

9-12 core: Interpret and represent large sets of numbers with the aid of technologies
Working with Data: Probability and Statistics

Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies. How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?

4.1 Collect, organize and display data using appropriate statistical and graphical methods.

9-12 core: Create the appropriate visual or graphical representation of real data

9-12 extended: Model real data graphically using appropriate technologies and strategies.

4.2 Analyze data sets to form hypotheses and make predictions.

9-12 core: Analyze real-world problems using statistical techniques.

9-12 extended: Describe and analyze sets of data using statistical models.

Endnotes

1 Stafford III, Ph.D., Kirby C Tick Management Handbook New Haven: The Connecticut Agricultural Experiment Station, 2007
4 Stafford III, Ph.D., Kirby C Vice Director, Chief Scientist/State Entomologist, interview on Thursday April 2, 2009
5 WHO website http://www.who.int/topics/emerging_diseases/en/
7 www.census.gov/population/www/censusdata/files/urpop0090.txt
8 http://www.farmlandinfo.org/agricultural_statistics/index.cfm?function=statistics_view&stateID=CT
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