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Using Mathematics to Explain the Spread of Diseases

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Overview of the Unit

In today's society there is an increasing demand for understanding how mathematics is relevant to everyday activities. As this demand for the application of mathematics increases teachers and curriculum writers are faced with providing real life examples of the connection between mathematics and daily real world phenomena. As a result more and different subject areas are introduced in the mathematics curriculum to create problem situations.

This unit will provide problems that are embodied in the content matter from the topic diseases. The history and the spread of diseases will be explored and the mathematical models that can be used to explain the spread of diseases will be discussed. The unit will also incorporate the new directions stated in the professional standard for teaching mathematics. These standards state that teachers should create an environment that fosters critical thinking, create worthwhile mathematical tasks, and provide the tool for enhancing discourse. To respond to these demands, the teacher needs to provide an atmosphere in which the student can develop mathematical proficiency, and in which the student is constantly encouraged to collaborate with others and also to be an independent learner.

The rationale and objectives of the unit

The unit is written to:

1. Provide a link between mathematics and the biological sciences
2. Develop problems that are based on the content from the spread of diseases
3. Use mathematical and statistical models to solve problems based on the spread of disease.

The objectives of the unit:

1. The unit is designed to provide a series of lessons that can be used to enhance the application of mathematics and provide a connection of mathematics to other curricula areas
2. The unit will provide background information to problems in both algebra II and A.P Calculus classes during the study of topics such as exponential functions, rate of change and growth and decay functions.
3. The unit will provide the background information to statistical problems in the content area of probability, and descriptive statistics.
4. The unit will satisfy both the state and school district's curriculum goals by providing connections between mathematics and other content areas, and thus making mathematics more applicable to real world situations.
5. The unit will also satisfy the Reading Initiative, by suggesting areas of interest that students can do research and connect their findings to mathematics.

This curriculum unit is written in three sections. The first section gives an overview of bacteria and viruses their origin and spread; the role of viruses and bacteria in the spread of diseases; a discussion of emerging contagious diseases; malaria parasites and their impact on society and an explanation of pandemics. The second section will discuss mathematical and statistical models and their application to the growth of bacteria and the spread of diseases. The third section will consist of unit plans and the connection to the New Haven curriculum.

SECTION I

The Origins of Microbes

Microbes have a much longer evolutionary history than plants and animals, and therefore have had more time to evolve into diverse forms [1] . Since they were here first, they had unchallenged access to all the sites both on the surface and in the interior of the Earth. These microbes have been challenged to survive under cataclysmic conditions that are unknown to animals and plants, because plants and animals have only been around for a relatively short span of evolutionary time.

Micro organisms have proven their ability to withstand challenges that have never been experienced by man. [2] They are responsible for transforming the atmosphere through their chemical activities and thus making the Earth habitable for plants and animals. The catastrophic impacts that were experienced on the Earth during the early formation of life did not completely obliterate emerging life, microorganisms survived this period. After this traumatic period micro organisms multiplied and occupied most of Earth's environments, from the bottom of the ocean to the temperate regions and the polar ice. Different adaptations to these various places contribute to the diversity of traits developed by the micro-organisms. There are micro organisms that can withstand radiation 3000 times that which humans are able to withstand.

Bacteria

Bacteria are considered to be the smallest free living micro organisms, and are an ancient life form. Bacteria can reach and occupy all habitats that support life. They can live in fluids and some are airborne. They have a variety of shapes that enable them to occupy diverse habitats. Most bacteria have simple shapes, such as rods, spheres, or spirals. The average size of bacteria is about 1-5 m long and 1-2 m in diameter. They have rigid cell walls, this rigidity of the walls is necessary because the high concentration of salts and other molecules packed into the small cell interior exert powerful outward pressure. The strong walls prevent the bacteria from exploding. The cell walls also give bacteria their shape [3] .

The growth of bacteria can be defined as an increase in size or mass. The cell increase in size then divides. In general growth in a bacteria population is the increase in mass, the increase in number through replication. There is therefore a distinction between increasing in size, the growth, and increasing number of cells through multiplication. Bacteria have the most rapid growth rate of any free living organism; under ideal conditions they can double in number every twenty minutes.

Bacteria reproduce asexually by binary fission. The first step is the replication of the chromosomes, secondly the cell walls begin to elongate, and the two chromosomes separate. At that time a septum begins to form between the two cells. After the septum is formed, the two cells separate.

Bacteria usually have only one or two chromosomes. This lack of complexity gives them the ability to mutate their genomes rapidly and add new DNA segments acquired from other micro-organisms. They have shorter generation time and this allows them to evolve rapidly such as becoming resistant to the antibiotics humans used against them to treat disease. Not all the changes in bacteria are caused by mutations. Bacteria can acquire DNA from other micro-organisms and from other species and incorporate it into their genomes. This process is called horizontal gene transfer. This horizontal gene transfer has played an important role in their evolution.

Viruses

Viruses are smaller than bacteria, and are generally too small to be seen under an ordinary light microscope. One other feature of viruses when compared to bacteria and other organisms is that their genomes are not necessarily composed of DNA. Some viruses have genomes composed of ribonucleic acid (RNA). The genomes of viruses are usually much smaller than those of bacteria. Viruses use the compounds in the cells they infect to reproduce. Their genes enable them to take over infected cells biosynthetic machinery for this purpose [4] .

Viruses have very simple structures. The virus' genome is enclosed in a tightly packed protein coat called a capsid. The capsid can protect the genome while the virus is outside the cell it invades. A virus infects a cell by first attaching to the surface, and then releasing its genome into the cell's interior. This attachment of the virus to the target cell is very specific, because a virus can only attach to cells that have the right type of surface receptors for that virus. Viruses that attach to one type of cell may not attach to other types of cells. The second phase of the virus infection process is the take over of the biosynthetic machinery of the infected cell by the proteins produced by the virus' genes. The virus makes many copies of its genomes and its protein. These components are assembled into intact viruses, which then leave the infected cell. Viruses do not multiply by binary fission, but are assembled from component parts, the nucleic acid and protein.

The replication of a virus is done in five stages, the attachment, the internalization, the transcription of the genome, the viral assembly and the virus release. Viruses are described as, helical, icosahedral or complex in shape.

How Diseases Spread

The majority of microbes that human encounter on a daily basis do no harm. Some are considered beneficial in most circumstances. Only a minority of these organisms cause diseases when they interact with the human body. These disease- causing bacteria and viruses are called pathogens. Some of these pathogens cause mild diseases while some cause life threatening infections. Some are chronic, and develop over a period of time, while others are rapidly harmful or fatal. Virulence is the term used to describe the degree to which an organism can cause disease.

Diseases that are spread among the community passing from one person to another are called infectious diseases. Diseases can be transmitted by air borne droplets, or through contaminated water, via clothes, bedding, domestic utensils, or other items that have been in contact with an infected host. Some diseases are spread through the involvement with an intermediate agent. This agent is called a vector. These vectors are usually parasites that feed from the human body. Fleas, lice, and mosquitoes are examples of vectors.

If the infection occurs within a population it is said to be sporadic. If the disease spreads continuously in a community it is said to be endemic. An epidemic occurs when the number of cases of a particular infection rises above the endemic level. If the infection spreads throughout the world then it is called a pandemic [5] .

Infections in Humans

Infections spread from a source known as the reservoir of infection. These reservoirs may be human beings or animals, water, or soil. The source of infection is the individual or location from which the infection is acquired. There are five phases in the infection process, the entry into the host; the primary replication; the spread within the host; the exit from the host; and the host's response.

There are three general pathways:

1. Entry through mucous membrane, namely the respiratory tract, the urinary tract, the gastro-intestinal tract, and the conjunctiva.
2. Entry through the skin, this can be done through insect bites, animal bites, and by skin to skin contact.
3. Parenteral transmission, vertical transmission, direct inoculation into the blood or through blood trauma.

There is a distinction between the ability of an organism to cause disease and its ability to infect. Infection is different from having a disease. A person can be infected, but not have a disease. Infection expressed as an equation is: Infection + signs of symptoms = disease. If an organism infects a host and then become a normal part of the flora then it is called colonization. Infection is measured by the number of organisms on the host, and not the number of hosts that has died. Infectivity is therefore, the ability of an organism to establish itself on a host [6].

Mode of Transmission

Vertical transmission is a term is used to describe an infection that is transmitted from the mother to the child while in utero. Perinatal infections and post natal infections are not considered vertical infections.

Horizontal infections are infections transmitted from human to humans. There are significant differences in the number of infections that occur between horizontal infections and vertical infections. In general the rate of infection by horizontal infection tends to be greater than the rate of vertical infections. In vertical infection only one person is infected from the host, while in horizontal infection a number of persons can be infected by a single host.

Vector - borne infections: These are infections transmitted by a vehicle or a carrier. These vehicles are living animals such as lice, fleas or mosquitoes. These vectors transit organisms from the bloodstream of infected persons to other persons (susceptible).

Zoonoses: Zoonoses are infections that can be transmitted between animals and humans. The host of the disease is the animal. There are three modes of transmission from animal to human:

1. Direct transmission occurs via direct contact with the animal. The organism does not undergo any developmental change between the two hosts.
2. The micro organism has an intermediate host in its life cycle. Humans can only be infected with the infective stage of the organism.
3. The organism infects a human from a non living site.

Food - Borne Infections: Food - Borne diseases due to microbes are divided into two categories. These are intestinal infections and intoxications. Intestinal infections are caused by the multiplication of bacteria within the intestine causing diarrhea. Intoxications occur because the organism has multiplied in the food and released entertoxins that are ingested [7].

Emerging Diseases: These are diseases that have recently appeared in a population, or are diseases that have existed in the past, but are rapidly increasing. A few of these diseases that fall into that category are described in this unit.

Cowpox Virus

The cowpox virus is usually transmitted from pet rats to human. The virus belongs to the family Poxviridae and is closely related to other species, such as the variola virus. Cows are not the only animals infected by the virus. Wild rodents are considered to be the reservoir. Cats, zoo animals, and humans, are incidental hosts. This disease is usually rare. It presents as a localized skin lesion. The virus is transmitted to human by direct contact with the infected animal, usually a rat or cat [8].

Chronic Wasting Disease Prions in Elk Antler Velvet

This is a contagious fatal prion disease of deer and elk. A prion is an infectious agent that is composed entirely of proteins. Disease is caused by the protein interacting with normal proteins in the host, causing these normal proteins to change shape to match the infectious protein. In the process this can destroy vital tissues such as brain tissue. It is believed that the disease is transmitted by prions in the antler velvet of the affected elk. Humans who eat antler velvet as a nutritional supplement are at risk of exposure to the prions [9] .

Skin and Soft Tissue Infection (Patera Foot) in Immigrants to Spain

An unusual skin and soft tissue infection of the lower limbs has been observed in immigrants from sub-Saharan Africa who cross the Atlantic Ocean crowded on small fishing boats called pateras. These immigrants are trying to reach the Canary Islands. The boats are over crowded with between 40 -50 persons. They have minimal food and water for a journey that lasts several days. During the journey the passengers are exposed to inclement weather, deficient hygienic conditions and are sitting for prolong periods of time in the same position. In instances their feet are immersed in the sea water for long periods. The sea water is contaminated by traces of feces, urine, decaying food and fuel emulsion. These refugees arrive with severe skin and soft tissue infections, which consist of cellulites, deep tissue abscesses and tissue necrosis. Microbiologists suggest that shewanalla algae could be the agent of this infection [10] .

Cholera

Cholera is caused by the virus vibrio cholerae. The disease is spread primarily by bacteria- contaminated sewage that has mixed with water used for drinking. The disease can also be transmitted by contact with soiled bedding or clothes of an infected person.

The virus can survive for some days on damped clothing. If the contaminated sewage is used as fertilizer, it can be transmitted by domestic animals that can carry the virus, but are not affected.

The virus enters the body through the mouth, and then passes through the stomach to the small intestines where it multiplies. The infection period can take a few hours to two or three days. The virus produces a toxin which affects the membrane of the small intestines. This causes a loss of fluid and essential minerals. This loss of fluids is very severe and rapid. The patient loses fluids also from vomiting.

The patient suffers muscular cramps and extreme thirst. The blood thickens, and the blood pressure falls. If the patient survives, recovery is very quick, leaving no evidence of damage to the body. An attack of cholera does not provide any immunity to the disease.

Malaria

Malaria is caused by Plasmodium, a microscopic protozoan. The organism enters the human body through the feeding bite of a female mosquito. The mosquito takes blood in order to obtain protein for egg production.

The life cycle of the protozoon is very complex:

1. The mosquito bites the human. Its saliva introduces the organism into the human body. At this stage the protozoon is in its sporozoite form.
2. In the human body the organism collects in the liver. There they invade the cells and multiply as merozoites. These merozoites are released into the blood stream. In the blood stream they use the red blood cells to multiply and produce gametocytes which are sexed. The gametocytes are carried in the blood stream until an uninfected mosquito feeds from the host.
3. In the mosquito digestive system the sexed gametocytes mate to form a zygote and enter the wall of the insect's stomach where sporozoites are produced.

The disease presents itself with fever, chills, and sweats as the merozoites are released from the liver. Nausea, vomiting and severe headache also accompany the reaction. In the process certain organs are damaged, and the destruction of the red blood cells causes the urine to turn either dark red or black.

Immunity

The term immunity refers to the stage where an individual if infected by a disease organism will not suffer life threatening consequences. There are three types of immunity.

1. Short term immunity: When the disease organism enters the body, the body responds with a huge increase in the production of white blood cells. These white blood cells attack the invading organisms. If the counter attack is successful in destroying the disease carrying organisms, the patient recovers, and the body's defenses are de- mobilized and the white blood cells return to their normal levels.

   In this stage high levels of overall resistance can build up within the human body. If all persons in a community respond in a similar way, then an epidemic will be averted.
2. Long Term Immunity: the body's defense system retains a memory of the infection by the diseases organisms. If it returns, the process of immune system counter attack begins again with greater strength.
3. Natural Immunity: The immunity for a disease can be passed from parent to offspring across generations. Examples are:
   a) certain blood types which hinders nourishment for the organism
   b) chemical constituency within the body's defense system

The immune system has evolved to deal with an enormous number and varieties of foreign substances. The response is graded to the level of the invasion. The body provides a vigorous response when needed. For example the arrival of a new organism on the skin does not call for a major response, since the normal flora of the skin does not invade the inner cells. Antigens are the proteins in viruses and bacteria that trigger an immune response. A satisfactory immune response gives protection from repeated disease caused by a specific virus or bacteria.

When an invading pathogen enters and replicate in the host, the viral antigen is manufactured. This elicits an immune response by the host. The host's immune system must discriminate between foreign antigens that are not produced by the host and the body's own proteins (i.e. hormones such insulin, and cell proteins that make up muscle)

After an infection by a virus or bacteria, the body's immune system competes with the virus or bacteria that are replicating rapidly, to limit the replication and then to clear them from the body. The host uses a variety of weapons to fight the attack. If, the host's immune system succeeds to defeat the pathogen, then pathogens are destroyed, and the host develops immunity to that virus or bacteria. If the pathogen is not defeated and expelled from the body, then the infection ends with the death of the host, or the host develops a chronic persistent infection.

Vaccination is the medical strategy for stimulating the immune system to protect against a specific disease agent before exposure [11] . The provoking an immune response before a natural viral attack, acts as a blueprint immunologic memory, so that cells involved in making the potential anti-viral immune response are primed and alert. When confronted with a full strength infectious virus these primed cells react quickly and with greater intensity than the unprimed cells. The host has the ability to better able to successfully combat and control the infection.

There are many different routes for the development of vaccines. Three strategies are listed below.

1. The use of live virus. These are prepared in the laboratory animal and tissue culture. The ability of the virus to cause an infection is reduced. This process is called attenuation. A form of virus is developed with just enough potency to cause an immune response, but not enough to cause a disease.
2. The virus is inactivated, that is killed by the use of a chemical. The killed virus is then tested for its capacity to cause an immune response.
3. The preparation of a sub-unit, recombinant or DNA vaccine.

The purpose of vaccines

The purpose of vaccination is to limit the impact of an infectious disease. The only disease that was eliminated by vaccination is the smallpox disease. This was possible because smallpox virus is a strict human pathogen, the only host is man, and it has no animal reservoir. For childhood diseases such as measles, vaccination seeks to protect the community as well as the individual. Vaccination is also used to protect individuals at high risk of acquiring a specific infection, for example people in certain high risk professions.

Vaccination is valuable to the whole community. If the majority of people are vaccinated against an infection, the rest of the population is also protected. This is called the herd immunity principle. Herd Immunity provides protection of the few unvaccinated by the immunity of the bulk of the population.

The Mathematical Connections

The time taken for each of the bacterial cell in a population to divide is normally distributed therefore the time taken for the average cell in the population to divide is the population's the mean generation time. The time taken for the population to double in size is called doubling time. The systematic increase in bacterial numbers can be described by the exponential growth model. This model is applicable because the rate of increase in bacterial numbers is proportional to the number of bacteria at the given time. This means that the more bacteria present, the greater the increase in number.

The number e is used to describe the population growth, because the growth is a continuous process, and indicate that the population is growing without bound. The number of bacteria that will be present after n generation can be expressed as $2^n \times N_0$. The rate of increase of bacteria can also be expressed as a geometric progression. It is sometimes convenient and easier to measure the weight or density of the population of the bacteria.

If different organisms have different growth rates under unrestricted conditions, then the rate of increase in cell number can be described numerically. This is called specific growth rate and is defined by μ. μ represents the instantaneous rate of increase of a single cell, and not the increase measured at fixed intervals, and is different from doubling time. If bacterial mass is used to measure exponential growth, the increase at any point in time is represented by μ and is given as $dn/dt = \mu n$. The expression dn/dt represents the rate of increase in the number of bacteria over time. The constant of proportionality μ, represents the number of doublings per unit of time (per hr) and describes how fast the bacteria grow.

When bacteria are grown in the lab environment two assumptions called the Most Probable number are made, these are:

1. The organisms are randomly distributed in the sample being studied
2. Any viable organism in the test sample will grow.

The statistics that underline the Most Probable Number is the Poisson distribution. The Poisson distribution is a derivative of the binomial theorem. The assumption is that if a large number of samples are examined, then the likelihood of finding a positive result is small. The Poisson distribution is appropriate to describe the pattern of results that are likely to occur. The following conditions must exist to apply the Poisson distribution.

1. There are whole number observations
2. Each number is a small fraction of the total number
3. Each observation are independent of other observations
4. The probability of each outcome does not vary from one observation to the next.

The Poisson distribution is given as $P(x) = (m^x e^{-x}) / x$ where P is the probability, m = the average number of organisms per sample, and x = the actual number of organisms per sample [12].

Chemical disinfectants and antibiotics are used to control the multiplication of bacteria. Two words are used to explain the decay rate or the killing of bacteria. The term bactericidal describes the act of reducing the number of bacteria, and the term bacteriostatic refers to the act of inhibiting the spread or growth of bacteria. That is, bacteriostatic antibiotics prevent cell replication, but do not kill the bacteria.

If the bacteria are exposed to a bactericidal agent over a period of time, the number will decline progressively, rather than instantaneously. The rate of killing is constant. A proportion of the total will be killed per unit of time. This can be described mathematically as $N / N_0 = e^{-kt}$. The expression N / N_0 is the number of survivors at any given time divided by the number at the beginning, t is the time and k is the constant that describes the relationship between time and bacterial numbers.

An infectious disease that is transmitted randomly between infectious and susceptible persons is modeled by N = BSI. The rate of transmission N will be proportional to the product of the number of susceptible persons S, times the number of Infected persons I. B is the constant of proportionality, and represents the probability of effective transmission. The value of B is dependent on the micro-organism and their route of transmission.

The number of people directly infected by someone who is infected by a particular micro organism in an entirely susceptible population is labeled R_0 (the basic reproductive rate). $R_0 = S B D$ where S is the number of susceptible persons, B the probability of effective transmission and D the period of time each person is infectious. If S, B, and D are increased then R_0 is also increased. If the number of persons who are immune is not calculated in the equation then R_0 gives the number of new cases of the disease in the population.

Very infectious diseases have high R_0 values. Low infectious diseases have lower values close to one. R_0 values of infectious diseases have three limits [13].

1. R_0 1: The infection declines and disappears from the community
2. $R_0 = 1$: The infection will persist and remains endemic
3. $R_0 > 1$: The infection will increase in incidence and spreads as an epidemic.

Application to Geometry

Viruses are described as having the following shapes helical, icosahedral, or complex.

Helical: The nucleic acid is contained in a cylinder made of protein arranged in a helical stack.

Icosahedral viruses: The majority of viruses are icosahedral. An icosahedron is a regular polyhedron. Viral icosahedra are constructed from 20 equilateral triangles and have 20 flat surfaces 12 corners (apexes) and 30 edges. An icosahedron has three lines of symmetry. The icosahedron houses and protects the viral nucleic acid.

Complex: Some viruses are made up of a combination of icosahedra and helices [14].

Lesson Plan I

Topic: The Growth of bacteria. Exponential growth

Purpose: Using the exponential function to explain:

a) How bacteria multiply
b) How the population of bacteria grow.

Objectives: The students will be able to distinguish between how bacteria multiply versus how the increase in mass.

Background Information:

1. A bacteria cell increases in size and then divides in two (growth)
2. Growth in bacterial population is taken to mean the increase in mass and therefore the increase in numbers through replication (multiplication)
3. The time taken for the bacteria to double is called doubling time and is given as $f(n) = 2^n N_0$

4. The growth rate of bacteria can be expressed as $P(x) = e^x$

Problems:

1. Suppose that a bacteria population starts with 500 bacteria that double every hour.
   a) What is the population after 3 hours? After 4 hours?
   b) Estimate the rate of increase of the bacteria after 6 hours.
2. A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 3 hours there are 10, 000 bacteria. How many bacteria were present initially?
3. Suppose that the cholera bacteria in a colony grow unchecked according to the law of Exponential change. The colony starts with 1 bacterium and doubles in number every half hour.

a) How many bacteria will the colony contain at the end of 24 hours?

4. The bacteria population in a lab at time t has the size $P(t) = 1{,}000e^{0.35t}$. After how many hours will there be 5,000 bacteria?

5. A certain bacteria population obeys the exponential growth law $P(t) = 3{,}000e^{1.3t}$ (t in hours).

a) How many bacteria are present initially?
b) At what time will there be 10, 000 bacteria?

Lesson II

Topic: The Decay rate

Purpose: Investigate the killing of bacteria

The Essential question: Can we reduce the growth rate of bacteria?

Can we kill bacteria?

Background Information:

1. Bacteria are defined as dead when they cannot be grown. They replicate themselves by dividing once they reach a certain size. If in the process of duplication of the DNA lethal mutations are made, then the bacteria stop multiplying and die. Bacteria will also die if the conditions for growth are compromised. For example if the nutrients needed for growth are exhausted.
2. The half life of bacteria is the time required for half of the bacteria present in an experimental sample to be killed.

Problems:

1. Suppose the half life of a certain strain of bacteria in a Petri dish is 20 seconds and there were 10, 000 bacteria present initially. Find the time when there will be 1,000 bacteria remaining.
2. Bacteria cannot continue to grow exponentially forever. They are limited to the size of the Petri dish. Suppose the growth is modeled by

$P(t) = 800 / (1 + 49e^{-0.2t})$ a) find the initial number of bacteria in the dish
b) When will the population reach 20,000?

Lesson Plan III

Topic: The spread of diseases

Purpose: To use mathematical models to determine the spread of diseases.

Background Information:

The spread of infectious diseases randomly through a community can be modeled by N = BSI, where S is the number of susceptible people.

I is the number of infected people

N is the product of the rate of mixing and the concentration of susceptible and infected persons, and B is the transmission coefficient (the probability of transmission). The probability of transmission will vary between micro organisms and their route of transmission.

Problems:

1. Assume that the relative spread of a disease is constant. Suppose that in the course of any given year the number of cases of the disease is reduced by 20%.
   a) If there are 10,000 cases today, how many years will it take to reduce the number to 100,000?
   b) How long will it take to eradicate the disease that is to reduce the number of cases to less than 1?
2. The spread of the number of people infected by malaria in a certain Africa country is given by $P(t) = 200 / (1 + e^{5.3-t})$
3. The spread of the swine flu is modeled by the equation $P(t) = 100 / (1 + e^{3-t})$ where P (t) is the total number of students infected after t days after their trip to Mexico.
   a) Estimate the initial number of students infected with the flu.
   b) Haw fast is the flu spreading after 3 days?
   c) When will the flu spread at its maximum rate? What is the maximum rate?
4. The measles virus is a pathogen that is spread from people to people. The table shows the percentage of students infected from an outbreak among children in a daycare center.

Days	0	10	20	30	40	50	60
Percentage	0	18	56	82	91	96	98

a) Compute the average rates of change of the infection over the intervals [0, 12], [20, 32], [40, 52] and [30, 50].

b) Draw the tangent line at t = 40 and estimate the slope. Explain what that means.

c) Discuss the rate of infection over the following intervals [0,12], [20,32] and [40, 52].

5. An epidemiologist finds that the percentage N(t) of susceptible children who were infected on day t during the first three weeks of a measles outbreak is given to a reasonable approximation by the function $N(t) = (100t^2) / (t^3 + t^2 - 100t + 380)$

Graph the function

a) Draw the secant line and use it to approximate the average rate of increase between i) [4, 6] ii [12, 14].

b) Find the average rate of change on day 12.

Lesson Plan IV

Topic: Probability

Purpose: To calculate the likelihood of cultivating a specific species of bacteria.

Objectives: Students will be able to apply the Poisson distribution to growth of bacteria.

Background Information:

It is assumed that a large number of samples of bacteria are present in the drinking water, then the Poisson distribution can be used to describe the pattern of results that is likely to occur.

Problem:

In a lab experiment to grow bacteria in a Petri dish, the bacteria will grow at a rate of 100 per second.

a) Find the probability of 100 that fewer bacteria will be found in a 1- second period.
b) Find the probability that at least 100 bacteria will be in the dish in a 30 second period.

Glossary of Terms

1. Attachment; first step in viral infection, the virus binds to a specific receptor on the host's cell.
2. Bacteria: prokaryotic microbes with rigid cell walls
3. Binary fission: division process in which each cell divides into two cells. This makes the cell population increase exponentially.
4. Emerging infectious diseases: These are diseases that have appeared in a population, and or diseases that have existed in the past, but are rapidly increasing.
5. Envelope: layer of protein surrounding some viruses
6. Genome: refers to the entire collection of genes within an individual. In bacteria, these genes can be located in the bacterium's chromosomes as well as in genetic elements such as plasmids that are inside the cell cytoplasm.
7. Generation time: the amount of time it takes bacteria or viral genome to divide
8. Parenteral transmission: Transmission by needle
9. Pathogen: organism capable of causing diseases
10. Plasmids: are autonomous lengths of DNA that replicate independently from bacteria chromosomes.
11. Ribosome: is the collection of proteins and different ribosomal RNA molecules that form the site of protein syntheses.
12. Systemic: affecting the whole system
13. Virion: an entire virus particle
14. Virus: microorganism that must replicate by using the biosynthetic machinery of free- living organisms.

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Appendix: Connecting to the Mathematics Standards

The NCTM standards in mathematics state that students should be given the opportunity to connect mathematics to other subjects and content areas.

Standard 8: Communication. This standard states that students should be given the opportunity to:

a) organize and consolidate their understanding of mathematics through communication
b) Communicate their mathematical thinking coherently and clearly
c) Use the language of mathematics to express mathematical ideas.

Standard 9: Connections. This standard states that:

a) Students should be given the opportunity to recognize and use connections among mathematical ideas
b) Recognize and use mathematics in contexts outside of mathematics
Understand how mathematical ideas build on each other to produce a coherent whole.

Endnotes

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