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The Mathematics of Convection: Nature's Model for Energy Production

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Introduction

Global warming burst into the American consciousness in the early 1990's, as the scientific community publicized that a significant correlation between temperature fluctuations and Carbon Dioxide (CO_2) levels in the atmosphere over the past 160,000 years was discovered in ice core samples. The "Greenhouse Effect" immediately took its place among the wars and diseases of the 20th century as a potentially inevitable apocalyptic threat. Suddenly, the industrial gains of the 20th century which built and sustained our communication, transportation, urban infrastructure and residential communities seemed to be eroding the foundation of human existence, planet earth.

The chilling explanation is simple: a delicate balance of gases form a barrier in earth's atmosphere that allows some solar radiation to be reflected away from the earth, and some radiation to be retained on the earth's surface. Our dependence upon fossil fuels and our propensity to steamroll nature has increased dramatically over the past century, producing an over-abundance of greenhouse gases such as methane, CO_2 and nitrous oxide and a reduction in the amount of carbon-consuming vegetation on our planet. This process upsets the delicate balance of nature's carbon cycle and threatens to change our climate in ways that threaten human existence.

There are the technological means available to counter this threat that fall into two main categories of the energy regime: supply and demand. Supply-side measures focus on the extensive use of renewable resources; the main targets in curbing demand are buildings and transport. [1] This unit is meant to equip students with analytical knowledge pertaining to availability and feasibility of specific alternative and renewable energy sources.

As the world's best and brightest scientists attempt to harness the diverse and abundant sources of renewable energy that exist on earth, humans continue to struggle with political and economic barriers to progress in developing renewable energy sources. This unit focuses on convection and its potential role in a future without fossil fuel dependence. Important relationships between convection currents and solar energy will be discussed in the unit. One model that uses solar energy to create convection currents is the solar chimney model, which is discussed in detail in this unit.

In the United States, our addiction to consumption has thrust us to the forefront of global responsibility on climate change issues. As more forest and prairie lands give way to suburban houses and strip malls, the ecological balance of our nation's exurban and rural areas becomes increasingly precarious. [2]

We are in desperate need of a multifaceted but focused approach to our fossil fuel problem. This unit deals with the mathematics involved in the socio-economic and political barriers that stand in the way of large-scale movements towards renewable energy sources as primary energy sources for humanity.

Furthermore, this unit brings the purpose of mathematics and science curriculum into focus for students, and creates built-in motivation for students to perform well when exploring and describing mathematical models, comparing the sizes of numbers and performing unit conversions. The unit allows students to explore the concepts of and relationships between volume, mass and density in real-world situations, providing rich opportunities for mathematical and scientific discourse.

This unit is designed to cover approximately 2 weeks of instruction for students in an intermediate to advanced Algebra 1 course. This unit could also be used as an interdisciplinary review sequence for students in a Geometry class to prepare students for standardized tests such as the Connecticut Academic Performance Test (CAPT). Specifically, the unit addresses the mathematics and science portions of the CAPT and aims to bolster students' problem solving skills. The unit could be modified to accommodate students in Algebra 2 or above, but care must be taken to develop appropriate grade-level and content-level materials and activities or to modify the ideas contained within the unit to serve the appropriate grade level.

During the unit, students will discover the answers to basic, yet scientifically critical questions such as: Why does warm air rise? Why does a 2-ton boat float while a 1-ounce rock sinks? What are the properties of water in liquid, gas or solid form? How does a hot air balloon fly? The answers to these questions will provide a foundation for students to advance their understanding of natural phenomena associated with convection. During the unit, student discourse will play a critical role in cooperative learning situations as students discuss and debate the results of experiments and calculations.

The unit takes an experimental approach and develops mathematical models to represent real-world phenomena. Valuable connections are made between scientific and mathematical concepts. Students learn about the complex relationships between buoyancy, density, temperature and volume. While exploring these relationships, students describe, compare and contrast the relationships using the properties of linear and quadratic functions. Students make predictions and draw conclusions about scientific experiments using mathematical models. Through this process, they discover critical scientific concepts and use mathematics to make conjectures and develop evidence to support their conclusions.

During the unit, students perform simple scientific experiments and collect their own data. Students formulate educated opinions and debate the viability of their findings using research, mathematical calculations and physical observations as evidence. Students discover the importance of mathematics and science and the role they play in solving the worlds' most pressing problems. It is my hope as an educator that students develop a deeper sense of the importance of science, technology and mathematics careers. This unit should provide hope to students in a variety of ways. With careful analysis and a problem-solving approach, many problems can be solved or averted. Today's high-school students will become leaders of a world in crisis. Through this unit, students will be exposed to science and mathematics principles that they can apply directly. They will obtain perspective on the availability of meaningful and rewarding careers.

As we advance into the 21st century, it becomes more and more apparent that our civilization must discover a healthy equilibrium with our ecosystem. In the United States, our addiction to consumption has thrust us to the forefront of global responsibility on climate change issues. As more forest and prairie lands give way to suburban houses and strip malls, the ecological balance of our nation's exurban and rural areas becomes increasingly precarious. [2]

Students' ideas about climate change, sustainable living, renewable energy and global warming will be significantly developed following this unit. Students will be able to analyze and discuss the consequences of continued reliance on fossil fuels as an energy source. Finally, students will learn about the efficiency and energy potential of natural phenomena such as convection and its implications for research and development of new renewable energy sources. Students will develop a respect for nature's methods for maintaining equilibrium and harmony in earth's ecosystem, and they will learn that humans are looking to nature as a primary blueprint for the development of renewable energy.

Unit Objectives

My goal for this unit is to build the knowledge base of today's high school student. I want to motivate students to learn about the challenges facing our planet. Specifically, at the conclusion of the unit, I want students to be able to discuss global warming and climate change from an informed perspective. Students should be able to articulate what they have learned in scientific and mathematical terms.

Since one primary role for educators of all disciplines is to provide our students with the ability to filter and analyze information gleaned from a multitude of sources, this unit aims to provide students with healthy skepticism and keen radar for propagandized information. Finally, students should relate the concepts and facts presented in the classroom to the real world. Students must be aware of the significance of the subject matter provided in this unit and its implications for their future career opportunities and the future of the planet.

Overall Scientific Objectives:

Students will be able to:

1. Define density and buoyancy.
2. Explain what makes objects and substances sink or float.
3. Describe in detail the relationship between mass, volume, density and buoyant force.
4. Use knowledge of density and buoyancy to explain what happens to the volume of certain gases and liquids when temperature changes.
5. Relate the processes of convection and thermal expansion to the search for renewable energy sources that reduce reliance on fossil fuels.

Overall Mathematical Objectives:

Students will be able to:

1. Use mathematical models to make predictions about real-world phenomena.
2. Construct graphs of real-world data and analyze data using graphs.
3. Find the equation of a line in slope intercept form and describe the meaning of the slope and y-intercept.
4. Compare and order numbers and use proportional reasoning to solve problems.
5. Construct appropriate graphical and/or symbolic mathematical models to fit real world data.
6. Calculate buoyant force, mass, volume, density and temperature using symbolic and graphical mathematical models.
7. Find the equation of a quadratic function given its tabular and/or graphical model.

Global Climate Change & Sea Level Rise

The Intergovernmental Panel on Climate Change (IPCC) calls attention to the real and documented threat of sea level rise. The IPCC's 2007 report on Oceanic Climate Change and Sea Level states that global sea level rose by about 120 m during the several millennia that followed the end of the last ice age (approximately 21,000 years ago). Sea level indicators suggest that global sea levels did not change significantly from then until the late 19th century. The instrumental record of modern sea level change shows evidence for onset of sea level rise during the 19th century. Estimates for the 20th century show that global average sea level rose at a rate of about 1.7 mm yr^{-1}. [3]

The United States Environmental Protection Agency (USEPA) predicts in great detail the environmental, economic, social and political tolls sea level rise will have on humanity. A 2008 report details the potential devastation to low-lying land and shoreline ecosystems, not to mention the potential horrors faced by human civilization as people struggle to obtain limited and dwindling resources necessary for survival. There are two categories to consider: the impact on humans and the impact on the environment. The human impacts include flood damages, land structures lost to the sea, costs of protecting land and structures from the sea, the indirect economic and human toll from the migration necessary by the entire loss of a community, and the costs of shifting to alternative water supplies when the original water supply becomes saline. [4]

The mathematics and science behind the potential rise in sea level mostly escapes the knowledge base of the general public. A tremendous teaching opportunity exists in the explanation for potential sea level rise due to global warming. Students have the opportunity to apply some of the most basic concepts of physics and mathematics, such as the states of matter, Archimedes Principle, density, buoyant force, and the concepts of volume and direct and inverse variation. Viewing these concepts and principles through the lens of today's energy and environmental crises provides built-in motivation for students as they seek the truth. Students thirst for reasons why they are learning certain concepts and skills.

Thermal Expansion

According to the IPCC, the average temperature of the global ocean has increased to depths of 3000 meters, and the ocean has been absorbing over 80 percent of the additional heat. The warming of the ocean causes seawater to expand, increasing the volume of the water and contributing rising sea levels. [5] In addition ice tends to reflect sunlight while sea water absorbs sunlight. There has been a measurable reduction in mountain glaciers and snow cover in both hemispheres. These widespread decreases in glaciers and ice caps have contributed to sea level rise due to the reduction in ice cover. [6]

One common misconception is the perception of a direct cause and effect relationship between sea level rise and the melting of glaciers. Students may think that melting massive amounts of ice simply increases the amount of water in the ocean. Although land-based ice sheets would add volume to the earth's oceans if they melted completely, floating polar ice sheets would not add volume if they melted. Another significant threat to current sea levels lies in the principle of thermal expansion. Finally, the addition of massive amounts of fresh water to our salt water oceans is a concern, since the density of salt water is greater than the density of fresh water.

A current central issue in ocean climate theory is to understand the possible links between the variability of temperature and salinity anomalies and that of weather and climate fluctuations on the intraseasonal to decadal time scales. [7]

The reasons for thermal expansion lie in the properties of almost all substances, in liquid, gas or solid states. When substances experience changes in temperature, their properties change in various ways. Electrical properties and sizes can change, and these changes can be used to measure the change in the temperature of the substance. Nearly all materials expand when their temperature is raised and shrink when their temperature is lowered. [8]

When a substance is heated, the temperature change activates its particles causing them to move farther apart. Simply put, this change in particle behavior is obviously accompanied by an increase in volume. This is Thermal Expansion. Therefore, although the salinity of seawater changes with time due to the changes of freshwater flux into the ocean, its effect on sea level change may have been underestimated by the current model setup. Consequently, the sea level calculation is dominated by thermal expansion of the water column. [9]

Buoyancy and Archimedes' Principle for Solids and Gases

The discussion of Thermal Expansion and its relationship to density leads naturally into the concept of buoyancy. When an object is immersed in fluid, it either sinks, is neutral, or it floats. In the classroom, simple experiments will lead students to draw important conclusions about the relationships between density, volume and mass. Since density is defined mathematically as mass per unit volume, students can easily use numerical and proportional reasoning to predict whether or not an object will sink or float.

Since New Haven is an urban center with beaches and an active port, students are familiar with barges, boats and other types of sea-faring vessels. Discourse and investigations about the reasons why tremendously heavy ships and barges float on water while much lighter pebbles and rocks sink immediately bring the concept of buoyancy into focus for students. A ship, although it has an extremely large mass, will float since its large volume contains a large proportion of air. The ship is simply less dense than the salt water it travels upon; therefore, it floats.

A broader discussion of buoyancy, neutral buoyancy, and natural and synthetic mechanisms to control buoyancy might deepen students appreciation for the concepts covered in this unit. Buoyant force lifts an object in a fluid and essentially makes it float. Buoyant force is described in the context of the works of the Greek philosopher and scientist Archimedes. Archimedes discovered the following: "If a solid lighter than a fluid be forcibly immersed in it, the solid will be driven upwards by a force equal to the difference between its weight and the weight of the fluid displaced." [10]

Students can easily calculate buoyant force, since buoyant force is equal to the weight of the fluid displaced by an object. Simply put, Weight on earth equals Mass multiplied by earth's gravity. Calculating the mass of the fluid displaced by an object is simple since the object immersed in fluid displaces its own volume in fluid. Using the fluid's density and the volume of the fluid displaced, finding the Weight of the fluid displaced (which is equal to the buoyant force) is the solution to a simple linear equation. Whether or not the object will float, sink or remain in place is decided with a simple comparison between the Weight of the object and the Buoyant Force. Although the relationship between density, mass and volume is readily seen and sometimes more accessible through experiments involving matter in a liquid state, the same principles can be applied to gases, such as air, since both liquids and gases are classified as fluids.

Students might find the example of the hot air balloon both interesting and exciting when exploring the concepts related to buoyant force. The process by which a hot air balloon stays afloat relies on thermal expansion. The introduction of heat into the balloon excites the particles inside the balloon, increasing the space between them and creating

air pressure. Air is forced out of the bottom of the balloon as a result, causing the density of air inside the balloon to decrease, and the balloon flies. The discussion of the function of hot air balloons dovetails nicely with the reasons why hot air rises. Students may have noticed that wind tunnels can be created in urban environments with clusters of man-made structures. The buildings generate heat that warms the air. The hot air rises and does so consistently, causing a continuous flow of wind on city streets.

Natural Ventilation through Convection and Solar Power

According to the United States Environmental Protection Agency (USEPA), the energy used in the average household is responsible for twice as many greenhouse gas emissions as the average car. [11] The vast majority of homes throughout the United States are powered by fossil-fuel burning power plants which contribute to the release of greenhouse gases into the atmosphere. The United States Government has gone so far as to offer individual homeowners rebates and tax credits for taking steps to reduce energy consumption. Since energy demand will only rise with the growing world population, our reliance on fossil fuels must end, and a multi-faceted approach is necessary.

Many natural ventilation systems have been developed and tested over the past 50 years. These systems employ the natural process of convection through thermal expansion of air to create buoyant force. One example of a natural ventilation system that has been developed for residential structures, large building structures and power plants is the Solar Chimney.

Solar Chimney Design and Operation

One of the more innovative but simple solar power systems is the solar chimney, a passive ventilation system or power generation system that is powered by the sun and buoyant force. A simple schematic for a solar chimney power system was proposed by Schlaich in 1978, and the system proved successful through the operation of a 50 kW pilot plant in the early 1980s. [12] The simplicity of the design and the physical principles of the system's operation are based on the process of convection.

The solar chimney structure is constructed with three main components: structures to gather air and heat water, the cylindrical or prism shaped "chimney" and a turbine. The three components work together like a greenhouse. The sun heats the water and air, in essence jumpstarting the process of thermal expansion. The warmer air is collected inside the chimney which accelerates the process of convection, and airflow is created. This airflow turns turbines. Its working principle is that direct and diffuse solar radiation heats a large body of air in the collector, which is then forced by the laws of physics (warm air rises and creates a convective flow) to move up the chimney as warm wind, driving the turbines to generate electricity. [13] Of course, hotter air escapes through the mouth of the chimney.

This amazingly simple, clean and renewable system modeled after natural phenomenon converts solar energy to heat energy, then heat energy to kinetic energy, and finally kinetic energy to electric power. The surplus of kinetic and heat energy provides the gravitational buoyant force that completes the cycle.

A prototype of a solar chimney built in Manzaranares, Spain is 195 meters tall. The solar collector for the chimney is 240 meters in diameter. Measurements show the solar chimney warms the air by approximately 17 degrees and creates airflow through the chimney at a speed of 12 m per second. The system puts out 50 kilowatts of electricity. [14]

Teaching Strategy

This unit will employ experiential strategies to motivate and captivate the interest of students. Students will manipulate matter and devices that exist in the real world. They will conduct experiments and construct mathematical models of real-world data. These models will lead to discourse and deeper meaning and understanding of scientific and mathematical concepts.

Please note that these example lessons are only suggested sequences, and in some cases, depending on the level of the class, these lessons could take 2-4 days of instruction. Teachers should strategize where to expand the lessons and/or reteach prerequisite knowledge when necessary.

Example Lessons

Lesson 1: Density

Scientific Objectives

1. Describe the relationship between mass, volume and density.
2. Explain how an object's density determines whether it will sink or float.

Mathematical Objectives

3. Construct graphical models of real-world data.
4. Find and describe the equation of a line in slope intercept form.
5. Compare and order numbers and use proportional reasoning to solve problems.

Materials Needed

For this lesson, students will need a 100 mL graduated cylinder, 50 mL of water, 50 mL of Vegetable Oil and 50 mL of Alcohol. Students will also need a graphing calculator, graph paper, pencils, rulers, and balance scale (preferably a digital scale). The teacher will need a large, transparent container such as a fish tank filled with water in the front of the room where all students can see it. The teacher will also need one can of diet cola and one can of regular cola, and a computer with an internet connection.

Suggested Tasks/Strategies

Students will begin the lesson in pairs. Pose the following question to the class as a whole: What determines whether or not an object will sink or float? Let students discuss this question in their pairs, and inform them that they will share their answers with the whole class in 5 minutes. Most students will have the popular misconception that mass is the deciding factor in whether or not an object will sink or float. Display students' answers to the opening question, and show an excerpt of the YouTube video: "Buoyancy and Density," from Science Online. To access the video entitled Buoyancy and Density, go to http://hilaroad.com/video/sink_float.html and select "low resolution preview."

Show from the 1:14 mark to the 7:50 mark.

There are various concepts covered in the video that should be reiterated to ensure student understanding. The following questions can promote productive discourse and classroom discussion: How can we calculate the density of an object? Why does a cruise ship float while a pebble sinks? In general, what are the most important things to consider in determining whether or not an object will sink or float? Since the density of water is calculated in the video, it may be unnecessary to have the class calculate the density of water. However, the teacher may choose to not show that portion of the video and have students find the density of water by calculating mass and volume using a graduated cylinder and a scale.

Ask for a pair of volunteers and provide one of them with a can of cola and the other with a can of diet cola. Ask this group to predict what will happen if both cans are immersed in the same tub of water. Will they both sink? Will they both float? After students make their predictions, have the volunteers drop the cans into the clear water receptacle. As a whole class activity, calculate the densities of the two cans and discuss why the regular cola sinks, while the diet cola floats.

Following this discussion, students will work in pairs. They will begin an experiment to find the densities of three fluids, for example: vegetable oil, salt water and alcohol. They will do so by calculating the densities of the fluids. Students will construct a scatter-plot where mass is the dependent variable and volume is the independent variable for each of the fluids. For each fluid, students will collect five mass measurements that correspond to volumes 0 mL, 10 mL, 20 mL, 30 mL, 40 mL and 50 mL to generate ordered pairs and plot the ordered pairs on their scatter-plot. Be sure to model finding the mass of the fluids by subtracting the mass of the empty cylinder (this is demonstrated in the video).

Students will calculate the densities of the fluids by drawing a line of best fit on their scatter-plot. They will find the equations of each line of best fit. The slope of the line of best fit will be the density of the fluid in question. At the teacher's discretion, the graphing calculator may be used to create the scatter-plots and a linear regression to find the equations of the lines.

Students should be assessed based on their ability to solve word problems and answer conceptual questions related to density. Students should be adept at using the symbolic formula for density and should be able to find the density of fluids and objects using their mass to volume ratios.

Lesson 2: Buoyant Force

Scientific Objectives

1. Define buoyant force and explain Archimedes Principle.

Mathematical Objectives

2. Construct appropriate symbolic mathematical models to fit real world data.
3. Calculate buoyant force, mass, volume, and density using symbolic models.

Materials Needed

Two liters milk, a two liters of soda (not diet) and two liters of fresh water. (The amount and type of fluid needed for the experiments may vary, depending on the preference of the teacher and the number of students in the class). Each group will need three dense solids that will sink in all three liquids and can be hooked up to a spring scale. Suggested dense solids include one right cylinder, one sphere and one right rectangular prism. In addition each group will need and one right rectangular prism of solid wood and a centimeter ruler. Each group will need a spring scale and a wide-mouthed graduated cylinder tall enough for each solid to be completely submerged.

Suggested Tasks/Strategies

Students will work in pairs for this lesson. Ask students to place their block of wood into the fresh water. One student should push down on the block of wood. To reinforce previous material, have the class briefly discuss why the block of wood does not sink in the water. Ask the class to recall the density of fresh water, and as a whole class, calculate the density of the wood. Find the mass of the block using the spring scale and calculate its volume using the following equation, where m = mass, d = density and V = volume: d = m/v.

Ask students to place the block of wood in the water again. Ask students to push down on the block of wood until it is completely submerged. Students may notice that the block is "pushing against" their hand as they try to keep it submerged. Tell students that today, they will learn about this force that is pushing against them to keep the block of wood afloat. Show an excerpt of the YouTube video: "Buoyancy and Density," from Science Online. Show from the 7:50 mark to the 9:29 mark. To access the video entitled Buoyancy and Density, go to http://hilaroad.com/video/sink_float.html and select "low resolution preview."

During the video, display Archimedes' Principle in front of the class. Following the video, discuss the role Archimedes' Principle plays in keeping the wood block afloat in the water.

It should be noted that following the video, students may also need a refresher and some examples on using the formulas for volume of solids, specifically the formula for spheres: $V = 4/3\ \varphi\ r^3$, and Right Cylinders: $V = \varphi\ r^2 h$. Students may also need an example to remind them how to calculate the weight of an object on earth. For W = weight on earth, m = mass and g = the gravitational constant: W = mg.

Significant time should be spent explaining and discussing relationships that lead to an understanding of buoyant force. Since buoyant force is the weight of the water that is displaced by a submerged object, students need to be able to calculate the volume of their solids in cubic centimeters. The following symbolic understanding of buoyant force is necessary to conduct the experiment:

Given the following:

m_a is the mass of the object in air

m_f is the mass of the object submerged in the fluid

m_{df} is the mass of the displaced fluid

F_b is buoyant force exerted on the submerged object

g is the gravitational constant

d_f is the density of the fluid

The mass of the displaced fluid can be found by subtracting the mass of the submerged object from the mass of the object in air, and $m_{df} = m_a m_f$ (This reinforces the fact that buoyant force is actually a net force, and could lead into a discussion of neutral buoyancy, where the weight of the object is equal to the weight of the fluid displaced).

The relationships listed above yield $F_b = m_{df} g = (m_a m_f)g$. Since the volume of the object is the same as the volume of the displaced fluid when the object is fully submerged,

$d_f = m_{df}/V$ and $d_f V = m_{df}$. Using substitution, this yields $d_f = (m_a m_f) / V$

The teacher can now design an experiment where students can use the spring scale to calculate each object's mass when submerged in fluid and when suspended in air. The ability to measure the dimensions of each object with a centimeter ruler allows students to calculate the volume of spheres, cylinders and rectangular prisms using their formulas. With this information, students can calculate the density of each fluid.

Following the experiment, students should spend time solving word problems and answering discussion questions which reinforce the concept of buoyant force and bolster their comfort level with the symbolic representations and formulas. CAPT-like questions including grid-in and open-ended examples are suggested.

Lesson 3: Sea Level Rise and the Relationship between Temperature and Volume

Scientific Objectives

1. Explain the relationship between changes in temperature and changes in volume and density.

Mathematical Objectives

2. Use mathematical models to make predictions about real-world phenomena.
3. Construct graphs of real-world data and analyze data using graphs.
4. Find the equation of a quadratic function given its tabular and/or graphical model.

Materials Needed

A classroom set of glass graduated cylinders with the ability to measure volume in 1/10 mL increments, markers, ice cubes, and a large tub or jug full of fresh water. Graphing Calculators and computer with an internet connection.

Suggested tasks/strategies

At the opening of the lesson, students should be provided with a graduated cylinder or beaker filled approximately halfway with water. Students should place 3-4 ice cubes in their cylinders, and mark the water level on the beaker. The teacher should ask students what they think will happen to the water level when the ice melts. Classroom discussion should center on the central question: will the water level rise? How much will it rise? How much water is displaced by the ice and can it be calculated? The ice cubes will be left to melt at room temperature. Have students record their predictions on a worksheet about the water level.

At this time, students should view the PBS video "Extreme Ice." This video can be found online at http://video.pbs.org/video/1108763899/. The video takes approximately 50 minutes. Following the video, students should observe how the water level changed when the ice melted in the beakers. As a class, discourse should center on how the results would have been different if the ice were melted separately from the water, and then the melt water were added to the beaker. Students should make the connection from the experiment to the difference in impact on sea levels between sea based melting--ice and land--based melting ice.

Some critical questions:

If it melted, which ice would have a more significant impact on global sea levels, land-based ice or sea-based ice?

How does the reduction in snow and ice cover on land and on the sea affect the amount of sunlight that is absorbed by the oceans and the land?

How do particles create "dirty snow or ice" and how does this phenomenon affect the impact of the sun on melting ice?

Following this sequence, ask students to consider what would happen if the oceans absorbed more and more sunlight. Is there another factor contributing to sea level rise? Is there some reaction that might take place if the oceans were absorbing more heat from the sun?

Students will be provided with a table of data [15] (see the table below). The data will contain values for temperature and volume changes for 1 mL of fresh water.

Table of Data:

Temperature (Celsius)	0	4	8	12	16	20
Volume (mL)	1.0002	1.0000	1.0002	1.0006	1.0011	1.0016

Students should use this data to find a quadratic equation that models volume of 1 mL of water as a function of temperature. Students may use the quadratic regression feature of the graphing calculator to find the quadratic equation. Students should then use their equations to predict how much the volume of the water will increase as the temperature increases to 24 and 28 degrees Celsius. The accuracy of this quadratic approximation can be discussed. Teachers may wish to ask students if they believe this quadratic function will become a more or less accurate predictor of volume as temperature increases. Students could calculate the volume at 100 degrees Celsius using the function and compare it to the measured value of the specific volume of water at 100 degrees Celsius. They will discover that the function is not accurate at high temperatures.

Lesson 4: Thermal Expansion and Convection

Objectives

1. Design a hot air balloon that uses convection as an engine.
2. Research renewable energy sources that employ convection currents

Materials Needed

Dry cleaner bags, flexible straws, thin birthday candles, aluminum foil, a lighter, scotch tape, exacto knives and scissors

Suggested tasks/Strategies

The lesson will begin with students viewing the final segment of YouTube video: "Buoyancy and Density," from Science Online. To access the video entitled Buoyancy and Density, go to http://hilaroad.com/video/sink_float.html and select "low resolution preview."

Students should discuss the following important questions before creating their hot air balloons:

Some critical Questions

How does a hot air balloon fly? What role does thermal expansion play in the engine of a hot air balloon?

Why does a helium balloon float and how does the density of the air inside of a hot air balloon change as it is heated up?

What happens to the mass of the air inside the balloon? How about the volume?

For this culminating activity, students will be divided into groups of three. Each group will design a homemade hot air balloon. The instructions on creating a hot air balloon are located in the resources section. Teachers can decide for themselves how to use the exciting process of creating a hot air balloon. Some suggestions might include varying the size of the bags used, and the number of candles in the engine. Unless the hot air balloons are teathered, it is suggested to fly them indoors to prevent runaway ballons that can cause fires and other problems.

Teachers could extend the lesson by having students use the Ideal Gas Law to calculate the density of air. This could be used to compare the density of hot air in the hot air balloon with room temperature air outside the balloon. Rearranging PV = nRT gives: density = m/V = MP/RT, where m = mass of the air in a volume V and M is the molar mass of air (roughly 28 gm/mol since 80% of air is nitrogen gas which has M = 28 gm/mol). Using 300 K as room temperature, 1 atm as the pressure and R = 82.05 cm3 atm/mol K, one obtains the density of air as approximately 1.22e--3 gm/cm3. By using the temperature of hot air, you could also calculate the density of hot air, and from this the buoyant force for a hot air balloon.

For a culminating assignment, students will research solar chimneys. Students will write a paper detailing the history of solar chimneys and describe how they work. In their research papers, students should connect what they have learned about buoyancy, density and thermal expansion to the operations of a solar chimney.

Vocabulary List

Mass: The measure of the quantity of matter in an object.

Volume: The measure of space, such as the capacity of a container.

Density: The mass per unit volume of a substance.

Buoyancy: The force with which a more dense fluid pushes a less dense substance upward.

Weight: The force with which gravity pulls on a quantity of matter.

Convection: The transfer of energy by the movement of fluids with different temperatures.

Convection Current: The flow of a fluid due to thermal expansion followed by cooling and contraction.

Calorie: The amount of heat required to change the temperature of 1 gram of water by 1 degree Celsius.

Archimedes: The Greek philosopher who discovered the relationship buoyancy and displaced liquid.

Gravity: The force of attraction between two particles of matter because of their mass.

Global Warming: An increase in earth's temperature due to an increase in greenhouse gases.

Fossil Fuels: Any fuels formed from the remains of ancient plant and animal life.

Heat: The transfer of energy from the particles of one object to those of another object due to a temperature difference.

Appendix A

The 2005 Connecticut Mathematics Curriculum Framework provides standards for this unit to follow.

Students should be able to utilize linear and non-linear functions in all of their representations to illustrate real-world situations and predict real-world phenomena.

1.1 Understand and describe patterns and functional relationships.

9-12 core: Describe relationships and make generalizations about patterns and functions.

9-12 extended: Model real-world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Represent and analyze quantitative relationships in a variety of ways.

9-12 core: Represent and analyze linear and nonlinear functions and relations symbolically and with tables and graphs.

9-12 extended: Relate the behavior of functions and relations to specific parameters and determine functions to model real-world situations.

This unit gives students the opportunity to use their understanding on proportions, proportional reasoning to solve real world problems.

2.2 Use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

9-12 core: Develop strategies for computation and estimation using properties of number systems to solve problems and solve proportional reasoning problems.

Students are given the opportunity to explore the concepts of volume and use various formulas to solve problems related to the volume of solids. Students are asked to manipulate formulas and understand complicated vocabulary related to formulas, such as the meaning of a constant or complicated symbolic manipulation involved in calculating buoyant force.

3.3 Develop and apply units, systems, formulas and appropriate tools to estimate and measure.

9-12 core: Solve a variety of problems involving 1--, 2-- and 3--dimensional measurements using geometric relationships and trigonometric ratios.

9-12 extended: Approximate measurements that cannot be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Students are asked to model real-world data in various ways. They are asked to make predictions based on their models.

4.1 Collect, organize and display data using appropriate statistical and graphical methods.

9-12 core: Create the appropriate visual or graphical representation of real data.

9-12 extended: Model real data graphically using appropriate tools, technologies and strategies.

Appendix B

Instructions for Creating and Launching one Homemade Hot Air Balloon

Materials Needed:

2 drinking straws (approximately 10-12 inches in length)

One exacto knife

Scotch tape

One Dry Cleaning Bag (or a garbage bag that is less than 6 microns thick)

5 or more cylindrical birthday candles (preferably 3/16 inches in diameter or less)

A permanent marker

A lighter

A partner

A piece of aluminum foil

Steps to follow:

Measure the length of one drinking straw to find its exact midpoint. Turn the straw so it is horizontal and cut a slit with the knife that is also horizontal (the slit should be parallel to the sides of the straw). The slit should be approximately the same length as the diameter of one of the straws.

Find the midpoint of the 2 nd straw. Carefully feed the 2 nd straw through the slit until the two straws form a plus sign. The two straws should be pretty close to, if not exactly, perpendicular.

You and your partner should spread open the mouth of the bag until your hands form the 4 corners of a square. Use the permanent marker to mark the 4 corners of the square.

Next, fasten these 4 locations to the ends of the straws with scotch tape. Use as little tape as possible. One small piece about 1/2 inch long will do fine. Remember, balloons will not fly if they are too heavy!

Now that the frame of your balloon is complete, you can get to work on the balloon's engine. Use your scissors to cut out a square piece of aluminum foil that is 2 inches in length and 2 inches in width.

Work together with your partner to melt the ends of 5 candles and fasten them to the aluminum foil using their own wax. Be careful here, you might get burned! Try to evenly space out your candles on the foil. When complete, looking down at your engine should be like looking at the "5" side of a 6-sided die.

Roll a small piece of tape so both sides are adhesive. Place one side of the tape at the intersection point of the straws on the frame, on the side facing the inner part of the bag.

Place the engine, candles pointing up into the bag, on the other adhesive side of the tape. Your partner should stand and hold the bottom corners of the bag while you light the candles. Once the candles are lit, it is only a matter of time until you are in flight! Be patient, it can take awhile sometimes.

Enjoy!

Endnotes

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