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The Mathematics of a Warming Arctic

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Introduction

The significance of human actions upon observed global warming patterns is the topic of much debate in the media, politics, and popular culture. In the past sixty years, average winter temperatures in the Arctic region have increased by nearly seven degrees Fahrenheit. Significant change, both qualitative and quantitative, has been observed in the structure of Arctic ice sheets. Some climate models even predict that by 2013 the Arctic region will be entirely free of ice in the summer months. Land that has been frozen solid year-round in the past has now been freed for agriculture. The changes occurring in the Arctic region are stunning and, if climate models are to be believed, likely to continue and perhaps even accelerate in the coming years.

The reasons for these changes remain the subject of intense debate. Some scientists and world leaders are skeptical that human beings can have any real effect on global climate, while others are of the mind that global behavior and attitudes must be immediately changed if catastrophic and irreversible change to our planet is to be avoided. In its attempts to shed light on this question, the scientific community is in general agreement that human society does at least some role in shaping the overall character and future of the Arctic environment. Regardless of the cause the rise in Arctic temperatures is undeniable and significant. Thus while continued investigation into the cause of rising temperatures is critically important to the planning of long-term policies and strategies, it remains only one aspect of a broad spectrum of new issues. Aside from understanding why changes to the Arctic are occurring, attention must be devoted to how to best handle and adapt to the concerns, questions, and possibilities that arise from a new, warmer reality.

The young people of today will inevitably be drawn into the process of adapting to this reality. Global warming, and mankind's response to it, will truly be one of the defining issues of their lives. It is therefore important that young people are exposed to the types of opportunities and challenges that will emerge in the coming years. With climate models predicting continued increases in temperature, the question of how humanity as a whole is to come to terms with a warming world will only become more salient with the passage of time. Far from being confined to any one region, this question will come to dominate all levels of human society in all areas of the globe. Individuals, governments, and global institutions will all have to play a role in adapting - economically, politically, and socially - to an uncertain and precarious future.

Nowhere is new reality more evident than in the Arctic regions. There a population in the millions must witness and grapple daily with the changes brought about by warmer temperatures. Melting permafrost, disruptions in transportation routes, and shifts in animal- and plant-life are only a few indicators of the new reality imposing itself upon traditional modes of existence in the Arctic. As these changes are occurring right now in dramatic fashion, the Arctic offers a focused context in which students can discuss and think critically about some of the pressing dilemmas of our time. Furthermore, an investigation of warming-induced changes in the Arctic can provide students with a meaningful example of how mathematics is used to understand and address real concerns that have an impact on individuals, governments, and environment. In linking mathematics to a problem as vast and complex as global warming, students will be able to understand how mathematics and the many facets of human existence are intertwined. They will come to see mathematics and science as tools to clarifying the future of their generation and the world they will inherit.

This unit will focus on the three major dimensions of global warming as it pertains to the Arctic region: the environmental, the economic, and the geopolitical. Though these three will be considered separately, connections between the three will be reinforced throughout the unit. Breaking up the unit in this way is essential to giving students a solid understanding of the myriad issues that have arisen from global warming. The unit itself will be geared towards geometry students, though elements of the content will likely lend themselves to use in my precalculus classes as well.

Students will be involved in three engineering-related investigations. These activities are meant to strike a balance between reinforcing mathematical concepts and furthering students' understandings of the environmental, economic, and geopolitical issues which underlie this unit. Students will, for example, examine how melting sea ice will impact the routes taken by supertankers on journeys across the world. They will also analyze the mathematics of pumping crude oil through pipes. Concepts utilized in the three-lesson sequence (each lesson will take two 45-minute periods or one 90-minute block) include area, volume, distance/rate/time problems, direct measurement using rulers, analyzing the scale of a map, and converting between units. More detail can be found in the instructional design strategy below.

Care has been taken to ensure that the unit's overall purpose and specific learning objectives align with new mathematics guidelines established as part of the Common Core curriculum overhaul. In keeping with the Common Core emphasis on depth rather than depth, this unit is designed with only the truly core areas of geometric content in mind. Thus emphasis is placed on volume, area, and direct measurement as opposed to the large quantity of small properties and factoids which characterizes so much of the study of geometry. While geometric properties are indeed important, this unit is intended to help students grasp how the main ideas of geometry can be used to visualize, explain, and understand a profoundly important real-world problem.

Environmental Change in the Arctic

Recent climate models have forecast significant temperature increases for the Arctic region over the course of the 21st century. In fact, the Arctic region seems to be particularly affected by warming brought about by increased concentrations of atmospheric greenhouse gases. While a recent climate model predicts that global average temperatures will have risen by 1.5 degrees Celsius by the end of the 21st century, temperatures in the Arctic region will have increased by between 5 and 7 degrees Celsius in the same time span [1].

This increase in temperatures, more than twice that of the global mean, is modelled as being accompanied by a substantial reduction in sea-ice levels. The ice of the Arctic is in a constant state of flux, as the warmer summer months naturally lead to a reduced amount of sea-ice. In the previous several decades, however, scientists have noted that there have been substantial reductions in the amount of ice that exists year-round in the Arctic. This type of ice, termed multi-year ice, can reveal broader changes that analysis of seasonal ice would not consider. Satellite analysis indicates that multi-year sea-ice decreased in area by an average of 7% per decade between the years 1978 and 2003 [1] . The depth of sea-ice has also been undergoing significant change. It is thought that while average sea-ice depth in the Arctic is currently 2.5 to 3 meters during the summer months, this could change to perhaps only 1 meter by the end of this century. The melting of sea ice directly impacts the consequences and rate of global warming: since ice is more reflective than is water, the melting of the ice leads to more of the sun's energy being absorbed by the ocean. A warmer ocean then leads to a warmer atmosphere.

The warming of the Arctic not only has implications for the physical landscape of the Arctic, but also for the flora and fauna which inhabit the region. Arctic warming changes the structure of the region's permafrost, the layer of soil that normally remains frozen throughout the year. As temperatures rise and permafrost melts, the vast forests which border the northernmost Arctic regions are undergoing rapid changes. Some forests are advancing northwards into previously inhospitable terrain, while more southern regions are experiencing a drop in vegetation levels as plants struggle to adapt to new environmental conditions. The Arctic Climate Impact Assessment (ACIA), a key piece of research into the issues confronting the Arctic today and in the years ahead, notes that while warming temperatures will serve to increase overall vegetation levels - and thus increase the Arctic's ability to moderate rising greenhouse gas levels - this benefit will be mitigated by the de-icing of other regions. Without the reflectiveness of ice, the region will likely end up causing further warming [2]

As the forests undergo changes, the ecosystems they support will shift as well. The ACIA notes that warmer forests will likely lead to an influx of non-native species. As with any such occurrence, this will lead to a period of chaos in which native and non-native species interact in an unnatural and destructive way. This non-native invasion, coupled with the increased risk of forest fires and devastating insect swarms, will translate to radical change by which the fauna of the Arctic forests may be permanently altered. The animals that inhabit the region north of the Arctic forests will face similar existential threats. Much scrutiny in the popular media has been directed towards the plight of polar bears whose migration habits have been altered by the gradual disappearance of sea ice. The ACIA claims that caribou, seals, whales, and other Arctic animals will be similarly stressed by the effects of rising temperatures.

All climate models carry with them a considerable amount of uncertainty. This is especially true when the models attempt to explain climate behavior in a region as unpredictable and mysterious as the Arctic. Yet even given the inherent variability in modelling long-term climate change, the data accumulated by the global scientific community suggests that the Arctic of the coming decades will be substantially different than the one we know today.

Economic Change in the Arctic

The environmental changes that will likely be taking hold in the Arctic in the coming years and decades will lead to a host of new economic challenges and opportunities. As sea ice recedes, vegetation creeps northwards, and temperatures drift upwards, humanity will have new ways of exploiting the Arctic region for economic gain. At the same time, communities of indigenous Arctic peoples will undoubtedly be forced to change their ways of life. Traditional modes of survival, such as hunting and herding, may become unfeasible should the environmental changes discussed above indeed come to pass. If opportunity exists for economic gain, there will undoubtedly be many willing to take whatever risks are needed to realize it. This fact necessitates that careful consideration is given to the impacts of such economic ventures. The economic opportunities of greatest significance are likely those involving oil and natural gas, agriculture, and shipping. These are also the ones which might lead to the greatest damage of the Arctic region.

Large fields of oil and natural gas in the Arctic region have been discovered in the Arctic region. In 2008, the United States Geological Survey (USGS) published a report that put the recoverable resources in the Arctic at 90 billions barrels of oil, 1,669 trillion cubic feet of natural gas, and 44 billion barrels of natural gas liquids [3] . These quantities are substantial; if the statistics are accurate, it would mean that the Arctic regions contain 13% of the world's undiscovered oil and nearly 30% of its natural gas [4] . As these vast resources are found primarily offshore, it has until recently been economically untenable to invest in the infrastructure required to extract the resources. Added to this difficulty is the complexity of transporting the resources to processing sites. Pipelines are costly to build and maintain and pressure from environmental groups makes construction a lengthy process to even begin.

Rising temperatures have sparked resurgent interest extracting Arctic oil and natural gas resources. Receding sea ice has made it more practical to construct offshore oil drilling platforms. Furthermore, the state of global affairs, especially the turbulence in the Middle East, has pushed Canada, the US, Russia, and the EU to more vigorously investigate petroleum deposits that lie in more secure environs. The Arctic thus stands on the verge of an economic revolution. Countries and corporations stand to reap massive economic gains in oil export revenue. The construction and maintenance of extraction infrastructure could lead to an explosion of growth in cities and towns in Iceland, Greenland, northern Canada, and Russia. Greenland's economy, which today is based mostly upon agriculture, fishing, and mining could be transformed radically.

Although oil has the potential to radically change the economic character of the Arctic region, it is unknown if deposits will be extractable anytime soon. Research is being done daily to determine if the investments required to extract the oil would even be practical. And if extraction of oil does make economic sense, it remains to be seen how governments would deal with the possibility of catastrophe, such as a pipeline breach or a drilling site leak oil into environment. Natural gas offers considerable less risk for investors and, given natural gas' rise in popularity and use in recent years, perhaps offers a more concrete path towards economic revolution in the Arctic. Agriculture and shipping will also play a role in this process.

Agriculture in the Arctic has a long history. Farmers have cultivated cold-weather crops such as grains and potatoes throughout much of the broader Arctic region for centuries. Warming temperatures would have a clear effect on agriculture in the region. While a warmer climate may prove beneficial for increasing crop yields in some of the more southern Arctic regions, it would negatively impact how crops would be grown further north. Warmer temperatures "can speed crop development and thereby reduce the amount of time organic matter (dry measure) is accumulated" [5] . This would lead to significant structural changes in the region's agriculture industry and would perhaps be most impactful for indigenous communities. The Arctic Climate Impact Assessment (ACIA) also notes another disturbing consequence of warming temperatures: the amount of water that would be available for farmers - and Arctic communities in general - would decrease. A decrease in the water supply would lead to smaller crop yields and economic turmoil for communities across the region.

While agriculture faces clear threats from Arctic warming, it is expected that the positive effects would ultimately have more influence than the negative. As the Arctic warms, more land will be available for agricultural use. At the same time, rising temperatures would allow the population of the region to increase, boosting demand for agricultural products. Furthermore, melting sea ice would allow for cheap and quick transport of Arctic crops across the globe. While agriculture in the region will be forced to adapt to dramatic changes in the environment, it is expected that Arctic warming will also bring with it a number of opportunities that will be beneficial to the growth, development, and distribution of crop yields.

The potential dangers and opportunities of warming temperatures on the extraction of natural resources and Arctic agriculture are strongly linked to the third economic change to be considered here: the reduction in shipping distances as a result of melting sea-ice. As the amount of ice near the north pole shrinks, new shipping routes are being revealed. Ships that traverse the waters of the Arctic are presently limited by the sea-ice that is a year-round hazard for ocean-going vessels that have not been specially protected. This ice forces ships to take longer routes, thereby increasing both travel time and the cost of each voyage. Sea-ice's negative effects on regional shipping routes will be lessened by rising temperatures. It is thought that there will eventually be periods during which travel directly across the north pole will become possible. Shortened shipping routes will dramatically increase the flow of goods into and out of the Arctic region. Larger population centers will become sustainable and many forms of economic activities, especially resource extraction, would receive a significant boost. In all, an increase in the efficiency of Arctic shipping routes would result in a region that would be economically and socially different than today.

Geopolitical Change in the Arctic

It has been noted above that the prospects for significant resource extraction in the Arctic, while generally considered quite sound, are by no means fully clear. The physical environment of region makes any attempt at developing extraction infrastructure a risky and costly affair. This fact remains true even if the likely changes brought about by rising temperatures are taken into account. It is therefore possible that extraction of much of the Arctic's resources will not increase appreciably for years, perhaps even decades.

The state of Arctic geopolitics can, however, be strongly influenced by hypotheticals and promises of future wealth. For this reason, there is a concern among many political, environmental, and economic groups that the warming of the Arctic will be accompanied by a litany of disagreements and conflicts between major Arctic powers. Nations that border the Arctic region, such as the Nordic states, Russia, Canada, and the US, have obvious reasons for safeguarding their sovereignty over resources and transportation routes. These governments have made their territorial intentions clear in a number of ways. In early 2009, the US government issued a National Security Directive that made clear the government's intentions to, among other things, "meet national security needs relevant to the Arctic region, ensure that resource management is environmentally sustainable, [...] and strengthen institutions for cooperation among the Arctic nations." [6] In 2007, Russia made a more flamboyant show of their attitude towards the Arctic by using a nuclear submarine to plant a Russian flag on the sea-bed of the North Pole.

While the actions of the governments of the Arctic states may at first glance make it appear that the Arctic is a free-for-all for those with the capacity to carve out their own territorial claims, it should be noted that a large body of treaty has been established to govern how states interact in the region. The UN Convention on the Law of the Seas, a treaty established in the 1980s to clarify maritime commerce and territorial claims, has been a stabilizing force in delineating how member states interact in the Arctic. The Law of the Seas makes clear, for example, that a nation's territory extends 200 miles off its coasts. Furthermore, it provides guidelines for assessing exactly how maritime borders are to be measured and drawn. Yet the Law of the Seas has not completely eliminated the possibility of conflict over territory or resources: there have been disagreements over where each nation's territory ends. If territorial claims are not resolved, it might be the case in the future that several nations could claim hegemony over the same regions in the most northern regions of the Arctic. At present, however, the fact that so little is known about resources in this area will make any type of open conflict extremely unlikely in the immediate future. Furthermore, while many countries in the region have made investments in military and scientific equipment in the region, all have expressed their willingness to resolve disputes in through the UN.

Two more pressing geopolitical issues confront today's Arctic. The first is the possibility that the expanded economic opportunities brought about by rising temperatures will alter the status of indigenous communities. Greenland, now a territory of Denmark, could be persuaded to petition for more autonomy if the government could be assured that Arctic resources would allow for the economic independence required for full political independence. Secondly, the melting sea-ice will require nations to work together to manage the increased flow of shipping traffic across the polar region. Shipping has already led to squabbles over territory. Canada maintains, for example, that the Northwest Passage, a shipping route which snakes its way through islands belonging to Canada, should be classified as Canadian territory. The US and Europe, however, rebut this claim and currently use the passage without petitioning Canada's permission. The US and Russia have also debated whether certain shipping routes lying to the north of Siberia should be considered international waterways. As sea-ice continues to melt and Arctic shipping continues to grow, it is likely that these debates will only increase in frequency. Moreover, the rise of new global powers, especially China, may yield further disagreements over ownership of lucrative shipping lanes.

Teaching Strategy

This unit will be a way for students to understand and become actively engaged in one of the most pressing issues of our time. By analyzing the changes occurring in the Arctic, students will expand their view of mathematics and see how it can be used to make sense of an extremely unpredictable and uncertain context. Math, rather than be confined to the analysis of isolated problems, will become a means of understanding broad environmental, economic, and geopolitical concerns. Mathematics will become a means of understanding changes that will influence for years to come the way humans in the Arctic interact with and survive in their environment.

The unit will primarily be geared towards students in geometry and will therefore be structured to make use of geometric properties and basic algebraic skills. It is hoped that the unit will also, through simple adjustments, be scalable to algebra II or precalculus classes. The unit will serve to reinforce, deepen, and extend mathematical concepts that students have already seen in a more focused and skill-based context. The goal is not to teach new mathematical content, but rather to get students thinking about and working with mathematics in a broader and more meaningful setting. For this reason, the teacher's role will resemble that of a guide, facilitator, and helper. Each lesson will have a clear task and a concrete set of deliverables (including assessments), but there will also be ample room for students to take initiative and exercise a sizeable amount of independence in how they approach the learning activities.

A guiding principle in this unit is the idea of mathematics as a stabilizing and clarifying force. The world we live in - and the Arctic in particular - is saturated with complexities that defy simple and complete explanations. Uncertainties are a natural and inevitable part of daily life. It is a goal of mathematics to clarify some of these questions and allow individuals and social groups to make informed decisions about the present and the future. Only by looking at data and using basic mathematical principles can some degree of order be constructed from the constant chaos around us. This guiding principle will help students grapple with the question any math teacher is sure to get "when are we ever going to use this stuff?

Learning Plan

The unit is broken down into three lessons. While it is possible to complete the three lessons on three successive 90-minute periods, it may be useful to space out the unit over the course of weeks or even several months. This is because of the geometric content with which students will be expected to be familiar. Specifically, students will be asked to understand both area and volume. Area is usually covered several weeks before volume is addressed, so it might behoove the teacher to use the part of the units involving area first and then, once volume has been covered to a suitable degree, to return to the unit and add on the missing volume piece.

This pacing style might have the benefit of making the unit more of an overarching theme in the class rather than just an intellectual excursion that, while possibly interesting for the few days in which it is being covered, will quickly be forgotten once the class moves on to new content. As this unit is intended to help foster in students a new perspective on the potency of mathematics to describe the world around them, this would not be the ideal outcome.

Additionally, it should be noted that the three lessons need not be done in strict sequence. Each one is almost entirely independent of the others, so the teacher should feel free to pick and choose what is most useful to him or her at the time. It is encouraged, however, that students are provided some background information before they begin work on the learning activities. This will allow them to contextualize the mathematical work they are being asked to do.

Lesson 1:

Purpose:

This lesson has students examining some of the environmental impacts of the warming Arctic by performing some relevant calculations of temperature and the melting of sea-ice.

Objectives:

Students will use algebraic and geometric skills to calculate Arctic warming. Students will also write down their own ideas about how mathematics could be used to make sense of a variety of real-world challenges brought about by rising temperatures.

Relevant Mathematical Skills:

1. Calculation of percentage increase and decrease
2. Estimation of future data points through extrapolation of a trend.
3. Graphical analysis of student-generated data
4. Analysis of area and volume

Background:

a. The Arctic is undergoing a number of dramatic changes. Warming temperatures have been observed for the past few decades and, while some temperature variation is a natural occurrence in regional climates, it is generally accepted by the scientific community that human beings are having an accelerating - perhaps even causative - effect on this process.
b. One of the most visible consequences of this increase in temperature is the melting of Arctic sea-ice. It is thought that within a decade, sizeable portions of the Arctic Ocean will be entirely ice-free during the summer months. Since the 1980's Sea-ice has generally been decreasing in area by 7% per decade. Average depth of sea-ice could change from roughly 3 meters today to around 1 meter by the end of the 21st century.

Lesson Design:

1. Following an overview of the current state of the Arctic, students will be asked to consider each and hypothesize about what geometric/general mathematical concepts could be useful in describing/analyzing it. This will be a whole-class discussion.
2. Next, students will analyze the area of the Arctic. Using the observation that sea-ice has been decreasing by 7% in area per decade, students will create a graph comparing time and sea-ice area. Students will be responsible for labeling the axis and then answering the following questions on the back of their graph:

a. In what year will the area of sea-ice be half of what it is today?

b. In what year will the area be one quarter of what it is today?
c. Compared to the sea-ice area today, what percent will there be in 20 years?
3. Students will then complete a very rough calculation of the volume of sea-ice at present and in the future by using the facts about the average depth of sea-ice.

Lesson 2:

Purpose:

The second lesson of this unit is designed to get students acquainted with relevant background and to lay the groundwork for the connection between the real-world situation and the mathematics. Students will develop an initial sense of what the current state of the Arctic is, one of the economic changes occurring in the region, and some of the challenges and opportunities associated with this change.

Objective:

a. Students will summarize the key economic changes brought about by a warming Arctic
b. Students will use principles of trigonometry and basic measurement to map out the path of a large freighter through the Arctic region

Relevant Mathematical Skills:

1. Direct measurement of distances with a ruler
2. Calculation of real-world distances using a map scale
3. Calculations involving rate, time, and/or distance

Background Information:

a. While it is difficult to see on a normal 2D map, passage through the Arctic region is often the quickest route between two points on the globe.
b. Melting sea-ice is making it possible for more ships to travel directly across the Arctic ocean rather than make lengthy and costly detours.
c. This type of shipping is expected to become more and more common in the future and may lead to some international disputes about who owns the shipping lanes, how they should be used, and whether they harm the environment in some way.

Lesson Design:

1. Students will be placed in groups of two. Each group will be assigned a ruler, protractor, and map of the Arctic region (with scale). Additionally, students will be assigned a shipping route. Each group will use the tools provided to measure the shipping route's distance (kilometers) if the ship is a) allowed to pass unimpeded through the Arctic region and b) required to avoid the Arctic circle.
2. Students will draw the shipping routes on the map and calculate distances.
3. This will be followed by a whole-class discussion in which students share results, discuss which shipping route would be most shortened by passage through the Arctic circle, and what the potential consequences of such travel could be.
4. Lastly, students will create a "report" for their shipping route in which they summarize the results of their mathematical analysis. Specifically, they will address:

a. The difference in total distance between the two calculated shipping routes

b. The difference in total travel time between the two calculated shipping routes (students will assume that a tanker travelling their route will have a constant speed of 17 mph)

c. The difference in total cost between the two calculated shipping routes. Students will use the fact that an average supertanker travelling at 17 mph will require 41 gallons of fuel to travel its own length. The price of fuel can be set by the teacher.

Lesson 3

Purpose:

This lesson continues students' investigation of the economic opportunities and challenges brought about by a warming Arctic. Students will analyze the extraction of liquid and gaseous natural gas

Objective:

Students will use principles of volume, area, and unit conversions to analyze the collection and transport of oil and natural gas (both liquid and gaseous).

Relevant Mathematical Skills:

1. Calculations involving the area of circles and the volume of cylinders.

Background Information:

a. The Arctic is thought to have a large supply of natural resources, especially natural gas. Extracting these resources would have a large economic impact on the region and even on the world in general.
b. Extracting resources is not easy; it requires the construction and maintenance of a huge amount of complex infrastructure, as well as the employment of a large number of people and machines.
c. Geometry can be used to understand how these resources are extracted and transported.

Lesson Design:

1. Working again in pairs, students will analyze the flow of oil in a pipe and its storage aboard a tanker. Each pair will receive the dimensions of a pipe and information about the flow rate. Using this information, they will be responsible for determining how much oil is released into an awaiting tanker per unit of time. For the sake of simplicity, the pipe will be treated as a cylinder.

Guiding/extension questions:

i. What would happen if more oil were pumped through the pipe over the same period of time?
ii. What real-world consequences might there be for trying to pump oil more quickly through the pipe? This will be an intuitive/conceptual discussion, as most geometry students will not be at the point to study work and its dependence on flow rate / pipe diameter. The teacher is encouraged to ask the following:
What effect will making the pipe more narrow have on the oil's flow speed? (imagine a garden hose that gets squeezed. What happens to the water's speed?)
iii. What consequences - for both the environment and the flow of oil in the pipe - if there were a leak in the pipe?

2. After students have completed their analysis of the oil flow, they will turn their attention to the transport ship. They will determine how long it would take to fill the tankers (whose tanks will be treated as cylinders - students will be given the measurements of these tanks). Next, using data about the density of oil, they will calculate the amount by which the ship has increased in weight.

3. Lastly, students will calculate the market value of the collected crude using current data about world oil prices. Students will also use data about the cost of oil transport by tanker to generate some cursory calculations about the cost of transporting the crude oil to various refineries (e.g. those in Alaska, Washington State, and Russian cities along the Pacific coast).

Appendix: Connecting to the Mathematics Standards

This unit adheres to the Common Core Curriculum framework for geometry. The following standards are especially applicable:

1) (G-GMD.1) Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
2) (G-GMD.4) Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
3) (G-MG.1) Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
4) (G-MG.2) Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
5) (G-MG.3) Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

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Endnotes

[1] Johannessen et. al, "Arctic Climate Change," 328.
[2] Arctic Climate Impact Assessment, Executive Summary p.10, 2004
[3] Circum-Arctic Resource Appraisal Fact Sheet, 2008
[4] Arctic Gold Rush, p.63, 2009
[5] ACIA Report, chapter 13
[6] National Security Presidential Directive 66, January 2009.