Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
2014 Volume I: Picture Writing

Fractions: Seeing the Whole Picture in Many Parts

Curriculum Unit 14.01.10
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Introduction

"What strategies can you use to solve math equations and word problems?" I ask my class of 27 curious learners. I wait 30 seconds to discover a few hands raised while other students have a quizzical look on their faces with their hands half raised. I ask the question again, and students are unsure of how to answer my question. I see about two brave students raise their hands again, and one says, "By drawing tally marks," and the other student shouts out, "I can count on my fingers." I inquire, "Anyone else?" I notice the faces with blank stares. I proceed to draw pictures of happy faces, apples, squares, among other shapes, and houses with paths connecting to measure distance. The reactions from students are "oh yeah," and "I knew that." Students begin to shout out other objects and ways to draw pictures to solve problems.

Although my students realize you can use pictures to solve mathematical equations and word problems, they haven't figured out how to connect these pictures to represent the numbers in equations and the words in the word problems. It can get difficult to transfer a tangible object such as a 10-rod (base10 block) into a picture of a 10-rod. For example, students are able to count the number of parts on a 10-rod, but will fail at drawing a 10-rod with ten equal parts. Students may also find it challenging to draw a picture to illustrate the fraction 3/4. Students will have a difficult time solving problems with fractions if the drawing has unequal parts. At times, fraction parts are of unequal size. Students will attempt to compare two images of fractions that are not of equal size or contain unequal parts. Pictures of fractions must been drawn with accuracy and precision in order to interpret and solve equations and word problems.

It is my goal to teach children how important and useful pictures can be when they study math. They are a tool and strategy they can use to figure out difficult math problems. Pictures can be drawn to tell a math problem to complement words or equations. The pictorial stage in teaching and learning mathematics is used for all students, but can appeal even more to the visual learners, special needs students, and English Language Learners. For these students, math problems come to "life," and they are more understandable. Pictures maybe easier to solve than words and equations. I need all students to learn to appreciate drawing pictures as a strategy to use when they get stuck on solving problems. Words, numbers and symbols are important, but pictures can substitute for written communication to make math more engaging and easier to learn. In this unit, I focus on fractions because they are a real challenge for my students in 3 rd grade.

Rationale

As an adult learner in a teacher preparatory program, I was taught to teach math in three steps. The first step is to teach math concepts concretely. Students should use manipulatives and hands-on materials to solve math problems. Examples of concrete materials include base ten-blocks, fraction pieces, counting bears, chips, and connecting cubes just to name a few. After students are able to represent and understand how to solve math problems concretely, then they can move on to the second step, representing math in pictorial form. At this stage, students are required to transfer their concrete learning into representing math problems as pictures. Students are now required to represent and solve math problem by using only pictures. Pictorial representation can include pictures of base ten blocks, fraction circles, and graphs just to name a few. Once students have mastered the concept of solving math problems concretely and pictorially, the final step is to solve problems abstractly. Solving problems in the abstract includes number math equations (4x4, 13-4, 2+2, 54/2, etc.) and word problems. At this stage students should be able to apply concrete materials and pictures to solve equations and word problem. They are required to make the connection of solving a math problem concretely, transferring their thinking into a picture, to finally solving a word problem or number equation.

At times, my students have difficulty with representing math problems in pictures, once they've mastered representing equations concretely. I discovered this issue when teaching fractions. Students were eager to learn how pieces of various manipulatives were added together to make a whole. My class was able to show different fractions with fraction pieces, manipulate the fraction pieces to compare and order them from least to greatest or greatest to least, and to add or subtract fractions. The challenge came in to students applying this knowledge and solving fraction problems pictorially. Most of them had a hard time drawing fractions and comparing fractions. This made it even more difficult to introduce fractions in the abstract form of word problems and fractions in number form, such as 1/2, 3/4, 2/5. I plan to help students successfully move from the concrete stage into the pictorial stage. Once students have the capability to draw pictures and solve problems, then the abstract stage will be much easier to master and comprehend.

There are three common fraction models that are typically discussed in elementary and middle school textbooks: linear model, area model, and discrete model. Even though several other fraction models are taught, the three models mentioned are more widely found in textbooks. Teachers and students often represent fractions with concrete objects as well as static drawings. Moreover, there are many ways to represent fractions, and it is up to the individual to find which model strategy and hands-on materials work best to understand and manipulate fractions. [1]

Some students do not realize that drawing a picture can help with counting, adding, subtracting, multiplying, and dividing. Pictures can also help with drawing shapes and figures in geometry and with fractions. In this unit I illustrate how pictures and figures can make solving math problems and word problems easier. Students normally draw tally marks, sticks, and circles. They need to understand that these are not the only pictures they can draw. For an example, students can solve the math equation of 4+4= by drawing four apples and then another four apples to get the answer. Students can draw numerous objects to represent numbers to solve math problems. Here I focus on using pictures to assist students to understand fractions.

Content Objectives

Students will be able to develop an understanding of fractions as numbers.

Students will be able to name fractions represented pictorially.

Students will be able to order and compare fractions.

Students will be able to draw fractions.

Students will be able to show equivalent fractions by drawing pictures.

Students will be able to create math journals to illustrate and explain fractions, key concepts, and equations (problem solving).

Teaching Strategies

Connecting Math and Literacy

Picture books are wonderful resources and teaching tools to utilize across all academic areas, but they are particularly useful in teaching fractions. Teachers can read picture books at the beginning of a lesson to capture students' attention and gain enthusiasm for the lesson topic. Picture books are great in providing vivid illustrations with minimal text to explain vocabulary and challenging concepts. Students can use picture books to reinforce skills they find difficult or review skills they haven't practiced in a while. Additionally, children may find pleasure in reading a picture book for enjoyment. There are a plethora of books about fractions and how to teach the topic. I included a comprehensive list of picture books that connect literacy to learning fractions. Teachers are encouraged to build a math library with the recommended titles as well as with other titles not mentioned. The books I've included will help teach fractions, define key terms used in studying fractions and explain how to manipulate fractions within the constructs discussed later in the unit.

Teachers must first understand what they are teaching. Like students, teachers must rely on schema but push aside any negative memories, if any, about learning fractions. Elementary teachers are taught basic concepts and strategies for teaching fractions. Unlike high school teachers who are taught in-depth content and procedures, elementary teachers have to independently research challenging subject matter to become familiar and comfortable with teaching difficult concepts.

"Children learn mathematics through the use of language." [2] Vocabulary and key words are introduced orally through discourse during math lessons and with other students during practice. Through the use of language students begin to understand operations, mathematical processes, and the names of figures, shapes, and symbols used in everyday math. In order to enhance comprehension and mastery, students must also have a pictorial representation to explain and justify their work. It is true for elementary grades that there is a natural connection between children's literature and mathematical concepts. [3]

Children's literature infused during math time is critical in developing "math sense" in students. Shatzer cites

Whiten and Wilde stating, "Literature motivates students to learn, provides a meaningful context for math, celebrates math as a language, demonstrates that math develops out of human experience, fosters the development of number sense, and integrates math into other curriculum areas." [4] In essence, literature is required in all subject areas, and picture books complement and supplement lessons to develop students' thinking and learning. It is essential that teachers build a library not only for literacy blocks but for all content area subjects. High-quality books are necessary for development.

Children's literature is a driving force for giving students a "meaningful context for learning mathematics as it helps learners value mathematics, encourages [them] to become mathematical problem solvers. . . [and] to communicate mathematically, supports . . . reasoning mathematically, and explores a variety of mathematical topics." [5] Picture books engage student and provides visualizations of math concepts in illustrations. Children are better at explaining their reasoning and strategies which resulted in math becoming more enjoyable. Students are more persistent with difficult tasks and increased in internalizing more about what they learned to experience success. Teachers must link literature and numeracy in an interactive and meaningful way in order for students to understand math concepts and sustain the knowledge. Research supports higher math scores when strategies are combined with literature. [6]

Understanding Fractions

Fractions have always been considered as a daunting challenge for students. One explanation of why fractions are not well understood by young learners is because there is a lot to know about them, from part-whole to ratios to division and other mathematical operations. It is critical to connect visual representations with procedures (adding, subtracting, multiplying, dividing, etc.) in order to process and master fractions. Students seem to prefer to draw circles to represent fractions rather than squares, rectangles or any other shape. [7] Moreover, teachers need to provide multiple opportunities and various strategies to work with fractions.

In learning about fractions, the part-whole construct of rational numbers and the process of partitioning are the fundamental developments of understanding the four subordinate constructs of fractions. Priority has been given to learning the concept of part-whole in understanding fractions, and has been the traditional base to introducing the other constructs in lower elementary grades. [8] Secondly, the ratio construct has been noted as the most "natural" step to promote the concept of equivalence and subsequently the process of distinguishing equivalent fractions. [9] The third construct, operator, is helpful for developing an understanding of the multiplicative operations of fractions. The quotient measure is used for division. The fifth construct, measure, is considered a necessity for the development of additive operations of fractions. [10] Ultimately, the perquisite for solving problems in the domain of fractions is a sound understanding of all five constructs.

Charalambous and Pitta-Pantazi, along with Van de Walle, thoroughly define the five fraction constructs, and I provide a pictorial representation to explain each construct.

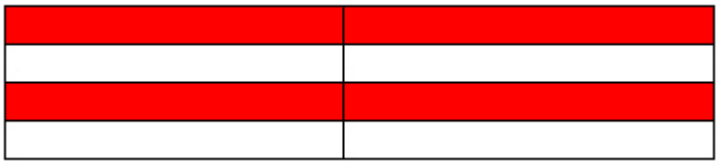
The Part-Whole Construct

"Part-whole construct is a continuous quantity or a set of discrete objects split into parts, of equal size." [11] Part-whole can extend into "part of a group" [12] (ex. part of a class) or part of a length (running 2 ½ miles). The part-whole fraction "represents a comparison between the number of parts in the split unit to the total number of parts in the entire unit. As an end result, the numerator of the fraction is less than or equal to the denominator." [13] "A circle model is most effective in displaying the part whole relationship. Part-whole is much

more than shading an area of a circle." [14]

Figure 1. shows two examples of part-whole

There are four parts to the whole, and two parts are shaded red, two parts, white.



Two-fourths are shaded red. Each row is considered one part. There are four rows. There are two rows shaded red.

The Ratio Construct

The ratio construct is represented as "part-part or part-whole." [15] A ratio compares two quantities and is considered as "a comparative index rather than just a number." [16] Additionally, a ratio can mean probability, the chance or likelihood of an event occurring. For example: ¼ means 1 out of 4 girls like baseball or whole probability. It could translate into comparing girls who like baseball (part) to those who do not like baseball (part); or girls who like baseball to those in (part) the class (whole). Students must pay attention to context in order to understand how to successfully represent this fraction in picture. [17]

Figure 2. illustrates ratios

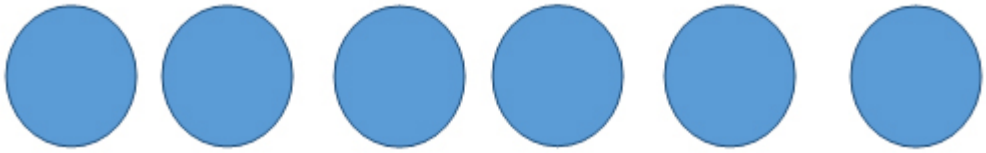
For every moon there are 3 stars. The ratio is 1:3. In fractional terms ¼ is moon and ¾ are stars. If there are 2 moons, there are 6 stars. The ratio is 2:6. In fractional terms 2/8 are moons and 6/8 are stars.

The Operator Construct

The operator construct indicates there is an operation to perform when manipulating a fraction. For example ½ of 10 sq. ft. or 1/3 of the class reading silently. These two examples indicate a fraction of a whole and "students may be able to use mental math to determine the answer." This construct receives some emphasis, but not enough. And "knowing how to represent fractions doesn't" equate to students "knowing how to operate with fractions." [18]

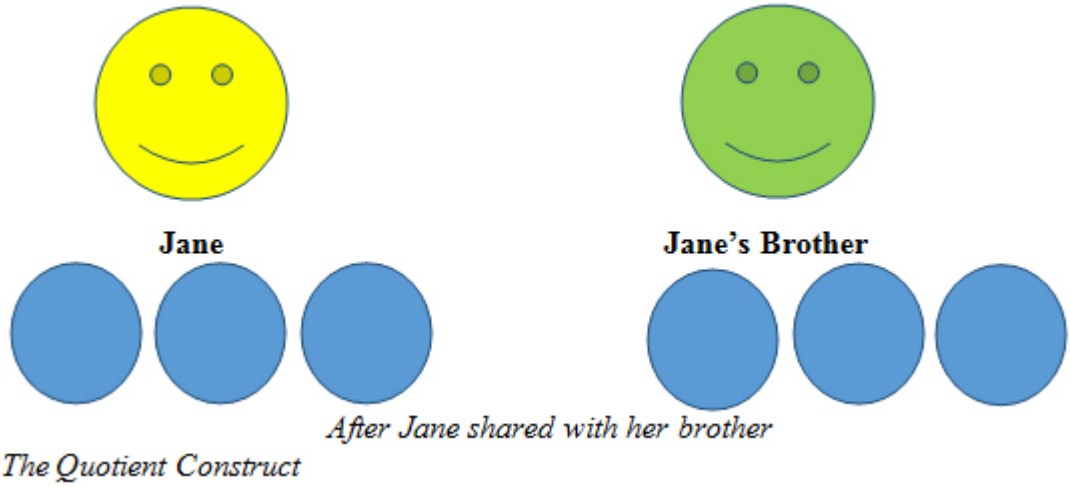
Figure 3. shows the operation of either subtracting or dividing. Jane is either subtracting from her pile or dividing between her brother and herself.

Jane's marbles



Jane had 6 marbles. She gave ½ to her little brother. How many does Jane have left?

Since there are 2 people, Jane and her brother, and 6 marbles, both will get equal amounts. Jane will have three left.



The Quotient Construct

The quotient construct is synonymous with the term division construct. This construct is not a part-whole situation, but relates to the fraction in term of dividing. For example, $30 is shared amongst 4 people. Each person receives ¼ of the money, or $7 ½ dollars. Unfortunately division is rarely connected to fractions. Students should have a sense and "feel comfortable" with fractions represented in division. [19]

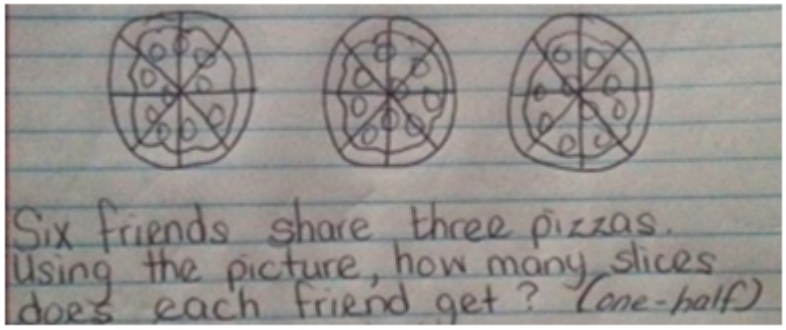


Figure 4. demonstrates how six friends share three pizzas. Each friend gets 4 slices which is ½ of a whole pizza.

The Measure Construct

The measure construct "focuses on how much rather than how many parts." A length is identified "as a measurement piece to determine the length of an object." For example: in the fraction 4/7, the unit fraction 1/7 is the selected length to "count or measure to show that it takes 4 of those (1/7) to reach 4/7." The measure construct is most associated with using the number line or other measuring tools (rulers) to determine the length from one point to another. [20]

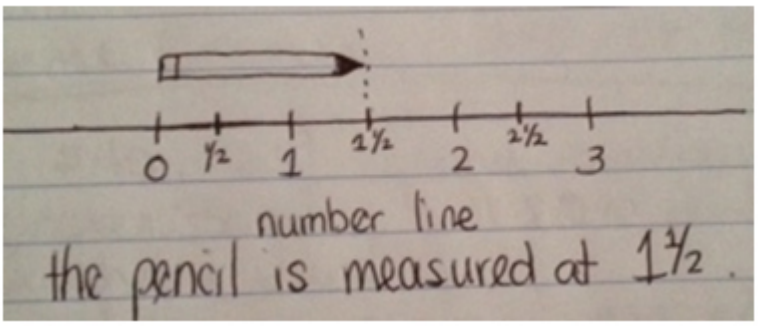


Figure 5. illustrates a number line with fractions

Pictures and math problems are used as strategies to gain students' attention. Pictures are used to engage students in the lesson and maintain their attention during the lesson. After capturing their attention, students can begin to perceive and explain the details seen in the pictures. Problems in math generate thinking, therefore eliciting students' attention to solve an equation. [21]

Fractions and Elementary Students

Teaching Fractions

The concrete-representational-abstract strategy (CRA) is used during instruction with individual students and a small or whole group. The CRA strategy is implemented in three steps in which the first step concentrates on modeling and teaching math skills with concrete materials. Base-ten blocks, cubes, pattern blocks, colorful bears, chips, geometric shapes, fraction bars/circles are examples of concrete materials students can manipulate to learn math concepts. In the representational stage, math skills are demonstrated through drawing pictures. Drawings can consist of replicas of the concrete objects, tally marks, shapes, or other small objects. The last step is abstract in which signs, symbols, and numbers are introduced and used to solve equations and word problems. Students should learn the math skills at each stage before progressing to the next. Students are encouraged to continue use of concrete objects and pictures in the abstract stage to enhance mastery. The key to the CRA strategy is to ensure students learn how to visually represent math skills using manipulatives and pictures to explain their thought processes.

One possible solution to aid students with learning fractions is creating and using math journals. A math journal will organize students' thinking, provide space to link vocabulary with pictorial representation, and give students a visual representation to explain challenging math concepts, rules, procedures and operations. Math journals are great supplements to help visual learners to understand and learn math through images. Several educational resources such as Pearson Education, teachersvision.com, etc. encourage teachers to utilize journals during math. Students can create a math journal to record thoughts, observations, and experiences. Math journals become tools students rely on to capture what they're learning. Math journals are not only beneficial to visual learners, but can also motivate low achievers and reluctant learners. Low achievers and those learners who are reluctant to participate, may find it beneficial to document their learning through drawing, using symbols, and keywords (very minimal) in a journal. [22]

The key to using math journals is taking a few minutes for students to illustrate their learning rather than rushing a lesson and moving on to another when understanding of the present concept(s) has yet to be achieved. Teachers must model how to create and use math strategy journals to solve math problems by

drawing pictures, symbols and creating charts, graphs, and tables. [23] Gunderson and Gunderson suggest to write out the fraction words rather than in standard notation at the commencement of fraction instruction. [24] For example, teachers should write out three-fourths instead of writing the number fraction ¾. This is consistent with teaching students the abstract stage last. After writing out the words *three-fourths* , a simple fraction picture can accompany the fraction word for a more clear understanding.

Practicing Theory

The theory of looking versus seeing is a powerful tool that teachers must utilize when teaching students how to focus on and analyze fraction images. The idea of looking at an image is naming basic details and attributes visible to the eyes. Looking is an act of choice to gain information about an image's physical and surface appearance, and seeing entails a deeper thought process than looking. The concept of seeing involves meaning and interpretation. Seeing requires an analysis of the image to fully capture what the image is about. [25] In order to gain a deeper understanding of fractions, students must learn the constructs and begin to see the relationship of fraction images to other fraction images and fraction words. Students must begin to see that fractions are divided into equal parts and apply this concept to finding equivalent fractions and comparing fractions. If the parts are unequal, then it becomes impossible to determine if fractions are equal to each other. The importance of seeing the relationships between fractions and their parts determines how successful students are in solving problems of equivalency and comparison.

Classroom Activities

Pre-Activity

Before any formal lessons are taught, give students plenty of time to "play" with fraction pieces, base 10 blocks, connecting cubes, and any other manipulatives used to introduce and create fractions. For the purpose of this activity, I prefer to use fraction pieces that are both rectangular and circular. Allow the children to mix and match pieces, divide them into groups, and explore the different sizes and lengths of the fraction pieces.

Activity 1 can last a typical class period 30-45 minutes. Students may work independently or in small groups. As a whole group, ask students to describe the fraction pieces and reflect on how they manipulated the pieces. Student responses are recorded by the teacher on chart paper.

Activity 1

Concrete Stage

Objectives: Students will be able to name fractions.

Students will be able to create fraction pieces to use as a concrete material (manipulative).

For the purpose of this activity students will create the fractions one whole, two-halves, three-thirds, four-fourths, five-fifths, six-sixths.

Each student will need 6 paper plates. The first paper plate can remain blank or students may color the plate one solid color and on the back write the label one whole. Plate 2 is folded in half and students will color both halves the same color. Students will cut the plate at the fold and label each half 'one-half' on the back of the plate. Plate 3 is divided into thirds by using a ruler to segment each piece. Students will repeat the same process as in plate 2, except one-third is written on the back of each piece. Plates 4-6 are divided into their respective pieces, colored, and labeled (one-fourth, one-fifth, one-sixth). Each plate has its own color and each piece is properly labeled in words, not numbers. Teacher may opt to draw the segment lines for students and have them color and cut the pieces.

Fraction circles can be stored in a large sandwich bag for later use and preservation.

Students are given ample time to manipulate the fraction pieces individually or in small groups.

Teacher will model how to use fraction plates by naming fractions shown with the pieces. Example: teacher will say "this is one-half," and show the fraction plate one-half, "this is three-fourths," and show three-fourths. Teacher will continue calling out fractions and students will demonstrate knowledge by showing and naming the fraction with the fraction plates.

This activity can be extended into math centers with students working together independent of teacher.

Activity 2

Once students feel comfortable with handling and manipulating fraction pieces, students should begin to illustrate fractions they make with the fraction pieces. Begin to introduce only the images of fractions and have students verbally name each fraction. Upon mastery, students can begin to label the fraction by representing the fraction name in words. Activity 2 can last over the course of 2 class periods. Repetition is critical at this stage because students have the opportunity to recall and store information in the long-term memory. For further lessons, students will rely on their schema to progress to the abstract level of solving fraction problems.

Representational Stage

Objective: Students will be able to create a math journal and draw fractions.

Students can make math journals using a binder with loose leaf paper, a spiral notebook, composition book, or several pages of copy paper stapled together.

Using the fraction pieces students created in Activity 1 students will transfer fractions made concretely to pictorial representations in their journal. Teacher will repeat activity in Activity 1 by calling out fraction and first have students represent it concretely, then draw the fraction in their journals. Students may label the fraction with words only. Teacher must check each student's journal for accurate pictorial representation.

Activity 3

The Operator Construct

For the purpose of this unit, the operator construct is represented concretely and in pictorial form. Students will continue to use their journals to record their drawings and add fractions.

Teachers will model how to add one-half + one-half. Teacher will demonstrate adding the two fractions by holding up one-half and connecting it to the other half. Teacher will ask students about what they saw. Teacher will illustrate adding the two halves by drawing one circle half + one circle half = one whole. Students will document this example in their journals. Teacher will continue the process with adding simple fractions such as one-third plus one-third, etc. Note: the fractions must be like fractions in order to complete this activity. Students are not expected to add one-third plus two-fifths. This activity is done until students mastered drawing fractions to show the addition operation. This activity can be replaced with subtracting like fractions as well. Multiplication and division are operations introduced in later grades.

In their journal under the pictures, students may write the steps to the operation performed. This can extend into a writing lesson for informational text.

Materials

Concrete materials used during math: fraction bar, fraction circles, pattern blocks, base-ten blocks, connecting cubes, rules, hand-held number lines. This list is not limited to the manipulatives teachers can use.

Chart paper and markers should be used to record observations during activities and posted as anchor charts for reference.

Small or large plates (6 per student), gallon-sized plastic bags (1 per student), scissors, rulers, and at least 6 different crayon colors for the fraction plate activity.

Note books, composition books, binder and loose-leaf paper, or copy paper are suitable materials for a journal.

Student Reading List

This reading list is for teachers to create a math library for students to enjoy during the literacy and math blocks.

The Doorbell Rang by Pat Hutchins illustrates measurement and fractions by using a family sharing cookies.

Eating Fractions by Bruce McMillian is a simple picture book illustrating food items as a whole, divided into parts.

If You Were a Fraction by Trista Speed Saskan contains fraction facts.

Fraction Action by Loreen Leedy is a fictional book illustrating how a class of animal characters learn fractions.

My Half Day by Doris Fisher is about a little boy who experiences his daily activities happening in fractions.

Full House: An Invitation to Fractions by Dayle Ann Dodds is a rhyming book focusing on a house filling up with guests with the rooms representing the denominator and the guests as the numerator. The numerator changes as guests fill the rooms.

The Wishing Club: A Story about Fractions by Donna Jo Napoli is a story about four siblings who each make a wish but only receives a

fraction of their wish. They must figure out how to combine their wishes to receive a whole pet.

Hershey's Milk Chocolate Bar Fractions Book by Jerry Pallotta teaches fractions through the story of eating chocolate. This book covers the topics of illustrating parts of a whole, adding fractions, finding equivalent fractions, and mixed fractions.

Working with Fractions by David Adler is a picture book for upper elementary students.

Discovering Math: Fractions by David Stienecker contains activities, games and puzzles for students who are advanced in learning fractions.

Mighty Math by Sara Pistoia has pictures of food divided into parts. This books introduces equal parts and is a resource for reviewing fractions.

Inchworm and A Half by Elinor J. Pinczes teaches fractions to the inchworm with woodcut illustrations.

Multiplying Menace: The Revenge of Rumpelstiltskin by Pam Culvert is a story about Rumpelstiltskin using his multiplying wand to multiply fractions.

Pizza Counting by Christina Dobson is a picture book giving an introduction to fractions.

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Appendix A Implementing Common Core State Standards

The Common Core State Standards have shifted the emphasis to students explaining their thinking rather than assessing their learning through paper and pencil testing. The standards hold students accountable for rational thinking and reasoning. Moreover, it's important to understand how students arrived at an answer, rather than merely students getting the correct answer. Students need to understand and explain the "why" and "how" behind an answer. In this unit, students will use visual representation to understand, represent, and compare fractions. Students will explain their thinking and reasoning pictorially.

In accordance with the Common Core State Standards for Mathematics Grade 3, this unit addresses the following standards:

CCSS.MATH.CONTENT.3.NF.A.1 Understand a fraction 1/ *b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a* / *b* as the quantity formed by *a* parts of size 1/ *b* .

CCSS.MATH.CONTENT.3.NF.A.2

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

CCSS.MATH.CONTENT.3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Notes

[1] Ted Watanabe, "Representations in Teaching and Learning Fractions," 457.

[2] Patricia Moyer, "Communicating Mathematically: Children's Literature as a Natural Connection," 246.

[3] Patricia Moyer, "Communicating Mathematically: Children's Literature as a Natural Connection," Ibid., 247.

[4] Joyce Shatzer, "Picture Book Power: Connecting Children's Literature and Mathematics," 649.

[5] Joyce Shatzer, "Picture Book Power: Connecting Children's Literature and Mathematics," Ibid., 650.

[6] Joyce Shatzer, "Picture Book Power: Connecting Children's Literature and Mathematics," Ibid., 650.

[7] John Van de Walle, *Elementary and Middle School Mathematics* , 313.

[8] John Van de Walle, *Elementary and Middle School Mathematics* , 287.

[9] Charalombos Charalambous and Demetra Pitta-Pantazi, "Drawing on a Theoretical Model to Study Students' Understanding of Fractions," 295.

[10] John Van de Walle, *Elementary and Middle School Mathematics* , 287.

[11] Charalombos Charalambous and Demetra Pitta-Pantazi, "Drawing on a Theoretical Model to Study Students' Understanding of Fractions," 296.

[12] John Van de Walle, *Elementary and Middle School Mathematics* , 287.

[13] John Van de Walle, *Elementary and Middle School Mathematics* , 287.

[14] Charalombos Charalambous and Demetra Pitta-Pantazi, "Drawing on a Theoretical Model to Study Students' Understanding of Fractions," 296.

[15] John Van de Walle, *Elementary and Middle School Mathematics* , 287.

[16] Charalombos Charalambous and Demetra Pitta-Pantazi, "Drawing on a Theoretical Model to Study Students' Understanding of Fractions," 297.

[17] John Van de Walle, *Elementary and Middle School Mathematics* , 287.

[18] John Van de Walle, *Elementary and Middle School Mathematics* , 287.

[19] John Van de Walle, *Elementary and Middle School Mathematics* , 287.

[20] John Van de Walle, *Elementary and Middle School Mathematics* , 287.

[21] Louise Soares, *Education* , 80.

[22] Kathleen Kostos, "Using Math Journals to Enhance Second Graders' Communication of Mathematical Thinking," 226.

[23] Kathleen Kostos, "Using Math Journals to Enhance Second Graders' Communication of Mathematical Thinking," 226.

[24] Ted Watanabe, "Representations in Teaching and Learning Fractions," 462.

[25] Peggy Albers, "Theorizing Visual Representation in Children's Literature," 168-171.