Income Inequality Control

Curriculum Unit 18.01.06
by Aparna Shyam

This unit is designed to strengthen high school students’ understanding of statistical concepts related to linear regression. Many students report that in the past, they have not received enough opportunities in their mathematics classes to engage in peer-to-peer discourse about topics that are meaningful to them. Rather, their mathematics courses have focused primarily on execution of algorithms. By having students study linear regression in the context of income inequality, my aim is to show students that statistics can be open-ended. More specifically, it can be used to analyze real-world problems and come up with solutions accordingly. The main question students will investigate throughout the unit plan is, “Which factors contribute to the disparity in income among people of different income brackets and to what extent?”

Rationale

This interdisciplinary unit plan is intended for high school Statistics courses taught in New Haven Public Schools. Many of the students I teach have low skill retention and will benefit from a review of graphing skills covered in the prerequisite courses for Statistics (Algebra 1, Geometry, and Algebra 2). These graphing skills include providing a graph or chart with a descriptive title, labeling the axes of a coordinate plane with the appropriate scales and variables, plotting ordered pairs, using the slope and y-intercept of a linear function to graph a line on a coordinate plane, and determining the equation of a line in slope-intercept form given two points. Students enrolled in Statistics classes are in eleventh or twelfth grade and are likely to encounter problems that require application of the aforementioned skills on the SAT. In Statistics, students will also acquire new skills such as fitting a linear regression model to a scatterplot, using the correlation coefficient and coefficient of determination to assess the linearity of a scatterplot, and creating a plot of residuals to evaluate whether a linear model is a reasonable fit for the data. These tasks will be simplified with technological tools like the TI-84 Plus graphing calculator and Desmos.

While several of my students are capable of executing mathematical algorithms correctly, they often struggle to understand when and why we use them. They have difficulty tackling questions that require them to justify their solution steps or interpret their answer in the context of the data. This is why it might be helpful for them to learn linear regression concepts as they relate to a real-world issue – in this case, income inequality.
By facilitating discussions on the history of income inequality, the reasons for its existence, the factors that influence inequality levels in various countries, and ways to alleviate inequality, I will get to learn more about my students’ knowledge of topics beyond the realm of mathematics. Perhaps this will give students who lack self-confidence in mathematics and shine in other subjects more opportunities for oral participation. Income inequality is a topic that infuses statistics with history and social studies, two subjects that give students plenty of opportunities to share their personal opinions about world issues.

A large number of my students come from low-income backgrounds or live in poor neighborhoods. Their families, peers, or they themselves are likely to be affected by rising levels of income inequality. Many of my students face immense pressure to achieve greater financial or educational success than their parents. Therefore, it is expected that this unit will resonate with them, as the ultimate goal is for them to come up with policies to alleviate income inequality based on statistical findings. Of course, there is no solution to income inequality that is universally agreed upon. However, controversy can often generate discourse, which is typically a sign of engagement in the classroom.

**Teaching Strategies and Objectives**

By the end of the unit, students should be able to

- Interpret the slope of the linear regression model in the context of the data.
- Interpret the y-intercept of the linear regression model in the context of the data.
- Use the correlation coefficient for a scatterplot to describe the form, direction, and strength of the linear relationship between the explanatory and response variables.
- Use the coefficient of determination to determine the fraction of variation in the dependent variable that is explained by the independent variable.
- Understand that correlation does not imply causation.
- Create a residual plot for a bivariate data set and use the plot to determine whether a linear model is appropriate for the data.
- Use information from statistical analyses to come up with possible solutions to income inequality.

In this unit, students will have the opportunity to explore the extent to which variables of their choosing correlate with the distribution of income in a location, whether a state or country. They will acquire the data they need from resources like the World Inequality Database or the National Bureau of Labor Statistics. Once they have analyzed their data, they will share their results with their peers. By the end of the unit, they should be able to use their gathered data, research, and even personal convictions to formulate answers to the following questions:

1. Do you believe that income inequality should exist to a certain degree? Why or why not?
2. What are the ways in which income inequality can negatively impact our society?
3. Which variables most strongly correlate with income inequality and why?
4. Which variables do not seem to correlate strongly with income inequality?
5. What are the consequences of income inequality for earners in all brackets?
6. What are the public policies that have kept income inequality low in the countries or states with the lowest Gini coefficients?
7. How can the United States reform public policies to close the gap between high-income and low-income earners?

Content and Background Information

What is Income Inequality?

“Income” refers to money earned through employment, dividends, interest, capital gains, royalties, or transfer payments. This unit plan will introduce students to income inequality, defined as the extent to which income is distributed unevenly among a population. From 1947 to 1973, following World War II, the United States saw an increase in mean family income. During this 26-year period, the annual rate of income growth for the bottom 20% of earners was greater than that of any other income bracket. Since 1973, however, the share of income for individuals in the lowest quintile has decreased. Data from 2015 shows that individuals in the top 1% of the United States average about forty times as much income as those in the bottom decile. The top 1%’s share of income has more than doubled since the mid-twentieth century. A summary statistic called the Gini index or Gini coefficient, has alerted economists to the fact that the level of income inequality is particularly high in the United States. The Gini index is measured on a scale of 0 to 1, 0 being indicative of an egalitarian society and 1 implying that a single resident in a location has all the income. The United States has a Gini coefficient of .42, which is much higher than that of other developed nations including Sweden, Norway, and Australia.

Causes of Inequality

Globalization

But why has the Gini coefficient for the United States risen over the past few decades? One explanation is globalization, the process by which businesses or other organizations operate on an international scale. According to the theory of comparative advantage, countries export the goods and services that the majority of its workers are most efficient at producing. If a poor country specializes in a product or service that requires unskilled labor, then there will be a more of a demand for unskilled workers from this country in the global trade market. This might decrease inequality globally by raising wages for unskilled workers in the developing nation. However, it can also increase income inequality in developed nations like the United States, which export services that require highly-skilled professionals. On top of this, there are millions of educated, highly-skilled individuals in underdeveloped nations who would be willing to work for a fraction of what similar professionals make in the United States. This leads to outsourcing of jobs to foreign countries so that businesses in the United States can avoid heavy production costs. If a business does not have to allocate large sums of money towards employee salaries, then its executives are paid at exceptionally high rates. When U.S.-based companies outsource jobs to other nations, they can obtain goods and services from these nations for cheaper prices. The drawback of outsourcing is that it increases unemployment in the United States.

A Race Between Skill and Technology

Another explanation is that technological growth has reduced or eliminated the need for low-skilled workers.
Robots can perform tedious manufacturing tasks such as packaging, assembling, inspection, loading, and unloading with more efficiency than humans. In 2012, Amazon purchased a robotics company called Kiva Systems. Kiva’s robots can lift and transport boxes that weigh up to approximately 700 pounds. Amazon is also utilizing drones to deliver packages to customers. Even Uber Technologies, Inc., which employs millions of drivers worldwide, has teamed up with the German company Daimler to test automated vehicles. Since the mid-1940s, the percentage of Americans employed in the manufacturing industry has more than halved. At the same time, technological growth (particularly in robotics, machine learning, artificial intelligence, and big data) has incited a demand for workers with advanced programming and mathematical skills. In *Capital in the Twenty-First Century*, Piketty remarks that this “leads to the idea of a race between education and technology: If the supply of skills does not increase at the same pace as the needs of technology, then groups whose training is not sufficiently advanced will earn less and be relegated to devalued lines of work, and inequality with respect to labor will increase.” He states that to combat this increase in inequality, “the education system must increase its supply of new types of training and its output of new skills at a sufficiently rapid pace.” He further explains that the rate at which individuals acquire new skills should be higher for those in the lowest income bracket. He backs up his argument by referring to the mean wage increase throughout the 1900s in France. In spite of this increase and the democratic education system, the gap between the top 10% and bottom 10% of earners persisted because workers of different skill levels progressed at the same rate. Earners in the lowest decile ended up graduating from high school, while earners in the upper deciles achieved college or graduate degrees.

Declining Union Membership

Some researchers argue that the decline in labor union membership has contributed to rising inequality in the United States. In 1935, the National Labor Relations Act allowed people to use collective bargaining in order to negotiate their salaries, improve working conditions, and go on strike if their needs were not met. This led to a substantial decrease in income inequality at the time as union members were able to raise wages not only for themselves, but for workers not belonging to unions. From 1930 to 1970, the mean income for the bottom 90 percent of workers grew by approximately 243 percent. Labor union membership was at an all-time high in 1954. Data gathered by the United States Department of Labor in 2015 suggests that only 11.1% of workers belonged to unions (a huge drop from the 34.8% in 1954). Researchers from the Economic Policy Institute discovered that if the concentration of union members had remained at 34%, then the earnings of non-union male workers would be 9% higher (the equivalent of $3,172). Why are unionization rates lower than they were in the mid-twentieth century? One possibility is that the present generation of workers are said to exercise more independence at work, and labor unions were organized during a time when people did not feel as comfortable speaking up at work. The development of labor unions occurred during the Second Industrial Revolution after the Triangle Shirtwaist Factory Fire of 1911 in New York City killed 146 garment workers who were locked in a high-rise room with no means of escape. This catastrophic event led to a call for better working conditions in factories. As previously mentioned, technology has made certain professions less relevant, including manufacturing jobs. Manufacturing jobs only account for 8% of professions in the United States.

Parental Investment

Affluent parents tend to have more flexible work schedules or paid leave. These parents are considerably more involved in their children’s education than those who do not get paid time off. Currently, the United States is the only industrialized nation that does not guarantee women paid maternity leave. Only twelve
percent of private sector workers in the United States have paid family leave, and this percentage consists of individuals in managerial or upper-level positions. The 1993 Family and Medical Leave Act (FMLA) is the only piece of legislation that gives workers up to twelve weeks off to care for ailing family members or newborns. However, this time off is unpaid, and the FMLA only protects employment organizations with fifty plus workers. Legislation ensuring universal paid family leave would allow parents to more easily balance their domestic responsibilities with their occupational duties.  

The Harms Associated with Inequality

One might wonder why income inequality is considered to be an issue when humans are inherently different from each other in ways beyond their control. It seems obvious that a degree of inequality is necessary for a society to function. Some people might ask why we need to be concerned about the United States’ economy when the country has the highest gross domestic product (GDP) of every nation. Economist Joseph Stiglitz clarifies, “GDP is just the sum total of the output of the economy, it doesn't say how much of that is going into whose pocket.” According to a Pew Research Center survey, sixty percent of Americans feel that the distribution of wealth and income in the United States is unfair. A 2012 poll showed that 63% of Americans in the Top 1% believe that the gap between the rich and poor in the United States is too wide. In underdeveloped countries, residents view income inequality as a sign of social injustice. In Sub-Saharan Africa, 263 million children did not receive an education in 2016. This represents roughly twenty percent of the global population of youth and adolescents.

While it can be argued that income inequality has its advantages, the main reason it raises concerns is that it has a lot of adverse effects on our society. High levels of poverty can lead to increased crime rates and mass incarceration, both of which exhaust financial resources. Those in impoverished communities lack sufficient access to nutritious food and adequate healthcare, which causes disease, mortality, and obesity rates to soar. These consequences weaken the productivity of the work force. Moreover, economic growth is stifled when fewer individuals are able to invest money in education. Finally, high levels of income inequality threaten the country’s democracy, resulting in tax policies that largely benefit the wealthy. Individuals seeking political office spend a substantial amount of time soliciting donations from wealthy and business interest groups. To garner (and maintain) financial support from people in the top income bracket, these politicians end up serving the interests of the wealthy instead of catering to the needs of those in lower income brackets. Corporations allocate millions of dollars towards lobbying, the purpose of which is to influence politicians and get laws passed that benefit top earners. The amount of money large businesses spend on lobbying far exceeds that of consumer advocacy groups.

Proposals to Reduce Inequality

One should note that not all proposals to reduce income inequality necessarily decrease the number of people who are facing financial hardship. Measures to reduce inequality do not always lead to increased opportunities for citizens. For example, Communism was based on Karl Marx’s ideology that capitalism leads to underrepresentation of working class individuals in politics and perpetuates class divisions. The intention of Communism was to establish a fairer society by eliminating private ownership of capital, giving the citizens control of production resources, and allowing the central authority to control financial resources. In the process, the central authority rations goods and services, professions, and residences to the proletariat. One of the advantages of Communism is that when resources are allocated equally to members of society, there might be less crime and more social accord due to lack of competition. The disadvantage is that Communism does not entirely eliminate an imbalance of power; it bestows absolute power upon the government, while the
other citizens are treated as subordinates. Furthermore, when a government enforces stringent policies pertaining to how businesses operate, this leaves little room for innovation. 19

Other solutions have been proposed to alleviate income inequality – solutions that aim to address the needs of individuals more closely than Communism. Obviously, we cannot control changes driven by technology or globalization. Still, we can achieve lower levels of inequality through reformation of policies. In order to keep up with the technological revolution, it is important that all individuals living in the United States acquire the skills necessary to thrive in a competitive workforce. Long ago, getting a four-year college degree used to be a luxury. However, several high-paying jobs require applicants to have a graduate degree on top of a bachelor’s degree. As a result, colleges are raising tuition with the awareness that higher education is students’ ticket to upward mobility. In order to attract students, colleges use this tuition money to maintain facilities or invest in amenities and educational resources. Creating a less inequitable income distribution would require making colleges more accessible to individuals from low-income families. Ideally, these colleges should have high upward mobility rates for students whose parents fall at the bottom of the income distribution. 60 percent of students who attend Columbia University reach the top quintile, regardless of family income level. This is evidence that at even highly selective universities, students from low-income homes are capable of performing as well as their counterparts from wealthier families. 20 Yet, students who attend prestigious universities with low acceptances rates tend to be disproportionately wealthy. Piketty claims this is due to the fact that “admissions decisions clearly depend on the parents’ financial capacity to make donations to the universities.” University alumni tend to donate to their alma maters by the time their own children are ready to attend college. 21

Researchers for the Equality of Opportunity Project found that the top ten colleges for economic mobility were not “Ivy Plus” schools or universities with the highest tuitions, but rather schools that had a large population of low-income students and boasted high graduation rates. These schools, which included Cal State University, SUNY – Stony Brook, and Glendale Community College, found that a high percentage of their students who were from families in the bottom 20 percent ended up in the top 20 percent following their graduation. Several of the countries that have the lowest levels of income inequality (Sweden, Finland, and Iceland, to name a few) offer free college tuition to their residents. Other European countries that do not offer free college tuition like Germany, France, and Italy still charge fewer than 500 euros for attendance. 22 In a country like the United States, which is geographically large and has far more school options, it is a stretch to make tuition completely free when tax revenue is not sufficient to finance higher education. Piketty proposes giving universities “publicly financed incentives” to offset this issue. 23

To reduce income inequality, a proposed solution related to education has been to implement universal pre-K. Preschool gives children the advantage of three months to a year of more learning than their peers who begin school at the kindergarten level. Children enrolled in pre-K tend to have stronger vocabularies and an easier time identifying letters of the alphabet and reading words. The National Bureau of Economic Research has found that for each dollar allocated towards high-quality preschool, the rate of return to investment in human capital is seven to ten percent per child each year. 24

Raising the minimum wage would also eliminate some of the burdens faced by low-income families. From the 1950s to late-1960s, the minimum wage was consistent with economic growth and peaked in 1968 when it was $9.90 per hour. If the minimum wage had kept up with inflation rates and productivity in the years following, it would at present be close to $20 an hour. According to Forbes, in the United States, “You would have to earn $17.14 an hour, on average, to be able to afford a modest one-bedroom apartment in a safe area
without having to spend more than 30 percent of your income on housing. Make that $21.21 for a two-bedroom home — nearly three times the federal minimum wage of $7.25.”

In 2012 and 2013, fast food employees staged a walk-out to protest their wage of $7.25 per hour. They demanded that fast food chains increase their pay rates to $15 per hour. New York was the first state to pass legislation raising the fast food industry minimum wage to $15 per hour by 2018 for workers in New York City and 2021 statewide. Several large cities including Seattle, Portland, San Francisco, and Los Angeles have adopted measures to do so within a particular time frame as well.

The movement to raise the minimum wage has gained support from a majority (63%) of voters. Regardless, low wage rates persist for several Americans, particularly in southern states like Arkansas, Mississippi, and Kentucky. Data from the National Employment Law Project shows that 42% of workers in the United States are paid less than $15 per hour. Women, African Americans, and Latinos are disproportionately represented in this group, with 48.1% of women, 54.1% of African Americans, and close to 60% of Latinos making less than $15 an hour. Among the occupations that have a high concentration of workers paid below $15 per hour are cashiers, retail salespersons, servers, cooks, healthcare aides, custodians, customer service representatives, customer service representatives, and secretaries. The largest industry employing low wage workers is the food preparation industry, which has a median wage of only $9.00 per hour. Over 50% of families with workers in the fast food industry depend on public assistance in the form of food stamps or Medicaid.

Galbraith points out that immigrants are often blamed for the decrease in pay faced by low-skilled workers in the United States. When a growing number of immigrants are willing to work for low wages, then this “drives down the equilibrium wage for those segments of the labor market for which immigrants can supply qualified labor.” On the other hand, a business can profit from the willingness of an immigrant to work a job for very little pay – so much that businessowners might resort to employing illegal immigrants. Some businessowners automatically assume that illegal immigrants eager to take low-wage jobs will work diligently since improper conduct or questionable behavior could possibly lead to their deportation or detention. Galbraith contests that raising the minimum wage could decrease the incidence of illegal immigration and encourage natives to work in unskilled professions.

Another solution calls for making the tax code more progressive. A progressive tax system is one in which those in the top income distribution are taxed more heavily than those at the bottom of the income distribution. While the income tax system in the United States is considered to be progressive overall, certain tax policies within the system are not. For instance, capital gains, profits made from investments or selling property, are taxed at a lower rate than income. However, it does not make sense for investment income to be assigned more value than income earned through hours of labor. Instead, it seems logical to tax capital gains at a rate commensurate to that of work income.

The Earned Income Tax Credit (EITC) is a source of support for both low and middle-income workers in the United States. It has functioned as an incentive for people to leave welfare and enter the workforce. Research has shown that children of parents who receive the EITC are healthier as infants, perform better on standardized tests, enroll in college at higher rates, have higher salaries as adults, and work more hours once employed. In 2016, the EITC was able to assist 5.8 million people out of poverty. If the EITC did not exist, then the number of children in poverty would have been 25% higher. Workers receive the EITC starting with the first dollar of their earned income. The amount of credit one receives increases with earned income and remains constant up to a certain salary before it declines (this decline is referred to as a “phase-out”). The phase-in and phase-out rates depend on a filer’s marital status and number of children. The more children an
EITC-eligible taxpayer has, the more credit he or she will receive. In 2018, individuals with incomes ranging below $40,320 to $54,884 (again contingent upon marital status and number of children) received the EITC, and the maximum amount of money given to a beneficiary was $5,716. Although the EITC has eased financial burdens for millions of people, it currently favors families with children. Childless workers qualify for a maximum credit of only $503. Present circumstances prohibit a childless adult or noncustodial parent working full-time and consistently at the federal minimum wage from receiving the EITC. A parent working under the same conditions, on the other hand, would receive the EITC in full. Taxpayers have to be between twenty-five years old and sixty-four years old to qualify for the credit, but working parents do not have these age restrictions. One suggestion for reducing income inequality has been to expand the age ceiling for EITC eligibility to 67 years of age and allow people as young as 21-years-old to benefit from the credit. 

A large percentage of taxpayers receiving the EITC also benefit from the Child Tax Credit (CTC), which similarly operates on a “phase-in, phase-out” system. The CTC Policy was put into effect in 1997 to assist working parents. Through the policy, which was updated in December 2017 under the Tax Jobs and Cuts Act (TJCA), eligible parents can receive a maximum of $2,000 per child below seventeen years of age by the end of the fiscal year. Qualifying parents can deduct the credit from the amount of income taxes they have to pay. For instance, if eligible parents with two children owe $5,000 in income taxes, the amount of money they owe would be reduced to $1,000 since they would get $4,000 through the CTC. Another advantage of the CTC is that if the credit is greater than the amount of federal income taxes one owes, then a family will be refunded the difference. This is advantageous to even those whose low wages prevent them paying income taxes during a fiscal year. The tax code allows working parents to obtain a refund equivalent to fifteen percent of their earned income exceeding $2,500. Suppose a worker with children makes $15,000. Then, this worker could receive fifteen percent of $12,500 or $1,875. One of the limits of the CTC is that taxpayers with investment income exceeding $3,500 do not qualify. If our goal is to reduce income inequality, then this restriction is logical. However, the CTC does have its drawbacks. People with a household income of less than $2,500 do not qualify for the credit. In order to combat inequality, it might help to make the credit accessible to people who make under $2,500 a year.

Medicaid was introduced in 1965 to give individuals from low-income households health coverage. Medicaid cover prescription drugs, hospital stays, and hospital visits as well as a range of services like transportation to health centers. From 2010 to 2017, Medicaid costs increased by about 100 billion dollars. Even though Medicaid is intended to help people in lower income brackets, the richest states are given more aid by the government. This is because aid is awarded based on a state’s per capita income. “In 2016, federal Medicaid spending per poor person in the five poorest states averaged $8,661, whereas spending per poor person in the five wealthiest states averaged $13,594.” If Medicaid funds were awarded based on individual income rather than state per capita income, then poor families would benefit more from the program.

Reducing inequality means expanding the range of safety net or welfare programs for people from low-income families. An example of a safety net program is the Supplemental Nutritional Assistance Program (SNAP, previously referred to as “food stamps”), which gave over forty million Americans from low-income households affordable access to healthy foods in 2017. To be eligible, a family or an individual must have an income less than or equal to 130% of the federal poverty level. Through SNAP, qualifying receipts are given an electronic benefit transfer card that can be used to buy food at participating stores. In 2017, a single person received on average $134 per month, while the mean monthly benefit for a household with five people was $521. 93% of the 70 billion dollars in SNAP funding went towards assistance for recipients, while the remainder was used for operational costs.
In 1968, the Fair Housing Act prohibited property sellers from denying housing to people on the basis of race, ethnicity, or religion. Regardless, residential segregation persists because of land use or zoning regulations. Zoning regulations dictate where buildings can be constructed, the types of buildings that can constructed in specific locations, and the size of buildings. They pose a challenge in cities that are becoming more densely populated. It is worth mentioning that zoning regulations can serve positive purposes such as protecting people’s health or safety. “They can be used to preserve historic areas or to delineate between retail districts and residential ones.” However, when building space becomes scarce, the price of housing increases. Consequently, elevated housing prices discourage low-income individuals and families from moving to locations with a lot of career opportunities. Those whose incomes do not allow them to keep up with the price increases end up displaced. In 2013, house prices exceeded construction costs by 56 percent. One solution would be to remove restrictions on the height of a building, to create more living spaces. Wide plots of land are typically used to build houses for single families, when it would be more cost-effective to build apartments for multiple families. 35

Conclusion

The shooting that took place at Stoneman Douglas High School in Parkland, Florida earlier this year led to teenagers propelling the gun control movement. All across America, high school students held rallies demanding improved school security and more restrictive gun laws. The incident taught many of the students at the school where I teach that political apathy can be dangerous for our society. After the Parkland shooting, students became concerned with reforming gun laws because they realized that a similar incident could happen in their schools if actions were not taken. The reason why many teenagers do not follow politics is because they are not yet taxpayers and have not had to deal with the consequences of public policy directly. I hope that by learning about the issues exacerbated by income inequality and ideas to resolve them (including those not discussed in this paper), students will leave my course with a greater understanding of how the country’s public policies can potentially affect them.

Lesson Plans

Lesson 1: What is Income Inequality?

An introductory lesson to the curriculum unit will have statistics students examining three different pie charts on Kahoot, a phone application that enables teachers to poll or quiz their students. Teachers can then view the results. The first pie chart would represent an even income distribution, with 20% belonging to each of the top quintile, second quintile, third quintile, fourth quintile, and bottom quintile. The second pie chart would represent an income distribution that is a little less equitable, and the third pie chart would represent a far more inequitable income distribution. The students will be instructed to select the pie chart that they believe reflects the distribution of income in the United States. Following their selection, they will need to justify on paper which chart they picked and why. They will be invited to share their justifications out loud after.

Next, students will be presented with graphs that illustrate the uneven distribution of income in the United States. Students can use the graph in Figure 1 to answer the following questions. These questions are meant to assess their prerequisite knowledge of how to gather information from a bar graph, describe its shape, and
draw conclusions accordingly. Students will be asked to volunteer their answers out loud after.

- The bar graph explores the relationship between which categorical variable and which quantitative variable?
- How would you describe the shape of the data distribution? Is it left-skewed, right-skewed, unimodal, symmetric, or random? Does the distribution favor any group of people?
- About how many times greater was the Top 0.1%’s average income than the Bottom 90%’s average income in 2015?
- Based on the bar graph, does income inequality exist for people at the top of the income distribution? Justify your reasoning mathematically.
- Which types of jobs would you expect people in the Top 0.1% to have?
- Which types of jobs would you expect people in the Bottom 90% to have?
- Why do think a graph like this is useful to us in the real world?

Figure 1: Average Income For Various Income Brackets in the U.S. in 2015 (36)

Figure 2 will expose students to a line graph. Students will use the graph to answer the questions during a whole-group discussion:

- How has the Top 1%’s share of total U.S. income changed from 1913 and 2015?
- During what year has the Top 1%’s share of total U.S. income peaked? Why do you think this is?
- The Top 1%’s share of total U.S. income seems to have declined until about the 1970s. Why do you think this is?
- The Top 1%’s share of total U.S. income seems to increase overall from the 1970s onward. Why do you think this is?
After examining both the bar graph and line graph, student will revisit the pie charts presented to them at the beginning of the lesson. They will be asked if they would change their initial choice of the chart that reflects the income distribution in the United States. Finally, they will be exposed to the correct chart (the pie chart that is most inequitable) via a PBS News Hour video called “Land of the Free, Home of the Poor,” which might surprise them.  Once the video is finished, students will be exposed to the class results for the pie chart poll. They will then need to answer the following questions for a post-video discussion.

- Why do you think the survey participants underestimated the level of income inequality in the United States?
- The low-income workers in the video were able to accurately identify the pie chart that represented the distribution of income in the United States. Why do you think they were able to make the correct choice?
- Do you think it’s disconcerting that so many Americans are unaware of the extent to which income inequality exists in the United States?
- Why do you think income inequality might be considered a problem?

Lesson 2: Using Linear Regression to Determine Which Factors Correlate with Income Inequality

Students will be alerted to the fact that income inequality is a global concern as well as a domestic issue. As a warm-up activity, students will be split into small groups. Each group will be required to list as many possible causes of income inequality as they can. Among the causes they list might be historical disenfranchisement of minorities, differences in education levels, and unfair compensation for labor. The teacher might also need to use guiding questions to prompt students for the less obvious answers like technological growth, declining unionization rates, tax systems, or globalization. Once students share their listings aloud, the teacher will need to explain that before we can alleviate income inequality, we need to know which factors contribute to
the problem. One of several ways to do this is to see which causes have a linear relationship with income inequality. The method we will use to do this is called linear regression.

A linear regression model allows us to predict output values for different input values of our independent variable. It is in the form \( y = mx + b \), which is slope-intercept form. We use the caret mark on top of the \( \hat{y} \) for a linear regression model to indicate predicted output value as opposed to a theoretical output value. The slope, \( m \), tells us how much the dependent variable increases or decreases for each one-unit increase in the independent variable, \( x \). The y-intercept, \( b \), tells us the value of the dependent variable when the independent variable takes on a value of zero. Students can attempt to fit a trend line to a scatterplot using a ruler and determine the equation of this line by hand, but this unnecessary because the TI-84 Plus graphing calculator allows us to perform these tasks with more ease.

The correlation coefficient, denoted by the letter \( r \), gives us information about the strength and direction of the linear relationship between the independent and dependent variables. The correlation coefficient is always a value from -1 to 1. If \( r \) is negative, this indicates that as the value of the independent variable increases, the value of the dependent variable decreases. If \( r \) is positive, this indicates that as the value of the independent variable increases, so does the value of the dependent variable. A correlation coefficient of zero indicates that there is no linear relationship between the independent and dependent variables. A correlation coefficient of -1 or 1 indicates a perfectly linear relationship between the independent and dependent variables. However, it is important to note that just because two variables are highly correlated, this does not mean that one causes the other. There are outside factors called confounding variables that can influence the linear relationship between two variables. The coefficient of determination, given by \( r^2 \), is simply the correlation coefficient multiplied by itself. It gives us the percentage of variation in the dependent variable explained by the independent variable. The coefficient of determination is always a value from 0 to 1.

The teacher will need to walk the students through an example of a problem involving linear regression. Suppose we want to come up with a model that can predict the average SAT math score of a state given the Gini coefficient of that state. Suppose further that we want to know whether or not there exists a strong linear relationship between the Gini coefficient of a state and the average SAT math score within that state. The teacher will first have to give students an overview of what a Gini coefficient tells us about the income inequality level of a specific location. After, students will examine the data table below.

**Table of Gini Coefficients and Average SAT Math Scores of Twenty-Two States**

<table>
<thead>
<tr>
<th>State</th>
<th>Gini Coefficient</th>
<th>Average SAT Math Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utah</td>
<td>.419</td>
<td>614</td>
</tr>
<tr>
<td>Alaska</td>
<td>.422</td>
<td>533</td>
</tr>
<tr>
<td>New Hampshire</td>
<td>.425</td>
<td>520</td>
</tr>
<tr>
<td>Wisconsin</td>
<td>.430</td>
<td>649</td>
</tr>
<tr>
<td>Idaho</td>
<td>.433</td>
<td>493</td>
</tr>
<tr>
<td>Maine</td>
<td>.437</td>
<td>499</td>
</tr>
<tr>
<td>Minnesota</td>
<td>.440</td>
<td>651</td>
</tr>
<tr>
<td>Washington</td>
<td>.441</td>
<td>534</td>
</tr>
<tr>
<td>Vermont</td>
<td>.444</td>
<td>551</td>
</tr>
<tr>
<td>Oregon</td>
<td>.449</td>
<td>548</td>
</tr>
</tbody>
</table>
Students will be required to use the TI-84 Plus to determine the linear regression model, correlation coefficient, and coefficient of determination for the data. They will be supplied with the following instructions:

- After turning on the TI-84 Plus, hit [STAT][EDIT][1][ENTER].
- Enter the data for the Gini coefficient (the independent variable) into L1 and the data for the average SAT math score (the dependent variable) into L2.
- Hit [STAT] [CALC][4].
- Next to Xlist, input L1. Next to Ylist, input L2. Then scroll over the word “Calculate” and press [ENTER].
- You should see the values of a, b, $r^2$, and r. To determine the linear regression model for the data, plug in the values of a and b into the equation $y = ax + b$.

When one follows the above procedure, one will obtain the linear regression model for the data: $y = -709.60x + 86.76$. Students at this point should provide answers to the questions below.

- How does the sign of the correlation coefficient relate to the sign of the slope?
- Interpret the slope of the model in the context of the data.
- Interpret the y-intercept of the model in the context of the data.
- What is the value of the correlation coefficient, and what does this tell us about the form, direction, and strength of the linear relationship between a state’s Gini coefficient and average SAT math score?
- Use the linear regression model to predict the SAT math score of a state with a Gini index of .468.
- Use the linear regression model to determine the Gini coefficient of a state with an average math SAT score of 650. What do you notice happens? Why is this a problem?
- Does this model seem practical to you in a real-world context? Why or why not?

Next, students can create a scatterplot of the data by following the instructions below.

- Hit [2nd][STAT PLOT][1].
- Make sure Plot1 is the only plot that is turned on.
- For Type, select the first option, which is the scatterplot.
- Next to Xlist, input L1.
- Next to Ylist, input L2.
- Next to Mark, select the first option (which displays each ordered pair as a point).
- Hit [ZOOM][0].
Some questions to ask the students about the scatterplot are listed below.

- Looking at the scatterplot, does the association between the Gini coefficient of a state and the state’s average SAT math score appear to be positive, negative, or completely random?
- Does the scatterplot appear to be linear to you?
- Which states appear to have particularly high SAT math scores? Why do you think this is the case?

The scatterplot for this data allows the teacher to address that in the real world, most data is not perfectly linear. The correlation coefficient for the data is -.34, which indicates that a linear relationship exists between the Gini index and the average SAT math score of a state. This linear relationship is weak, since the correlation coefficient is closer to 0 than it is to -1. The regression equation tells us that for every one-unit increase in the Gini coefficient, the SAT math score of a state decreases by 709.60 points. This is not exactly useful information considering the Gini coefficient cannot be lower than 0 or higher than 1. The data equips the teacher with an opportunity to discuss extrapolation, the process of estimating the value of a variable or function outside an observed range. If one tries to use the regression model to estimate the Gini coefficient of a state with an average SAT score of 650, one will discover that the estimated Gini coefficient is negative. However, a Gini coefficient can never be negative. The moral of the story here is that the usability of a linear model can be limited. This data allows room for a discussion of confounding variables. Some states (like Wisconsin or Minnesota) might have unusually high average SAT math scores because the SAT is not a required test for their students. These states require the ACT instead, and those who take the SAT tend to be the most ambitious students. However, when the correlation coefficient of a scatterplot indicates a strong linear relationship between an independent and dependent variable, the linear regression model can help us effectively make predictions given specific input values.

Another tool that allows us to assess whether a linear model is an appropriate fit for a data set is a plot of residuals. A residual is the difference between the predicted y-value (ŷ) and the actual y-value (y). The residual plot is a scatterplot of each (x-value, residual) ordered pair. A plot of residuals can be created quickly using the TI-84 Plus graphing calculator. The steps, which will be reviewed with the students for the SAT example, are outlined below.

- Press [Y=] to deselect the stat plots and functions.
- Press [2nd][Y=][2] to access Stat Plot 2 and enter L1 next to Xlist.
- Enter the Ylist by hitting [2nd][STAT] and using the up and down cursor keys to scroll to RESID.
- Press [ENTER] to insert the RESID list.
- Press [ZOOM][9] to access the plot of the residuals.

If all the points look randomly dispersed around the x-axis, then this tells us that a linear model is appropriate for our data. Otherwise, another type of model (i.e. exponential, logarithmic, etc.) is a better fit. The residual plot for the data is somewhat random, but does not resemble a cloud of points evenly scattered around the horizontal axis. This plot is consistent with the correlation coefficient, which suggests that a linear relationship exists between the average SAT math score and Gini coefficient of a state. However, this relationship is far from perfect.

After students are introduced to the graphing calculator functions associated with linear regression, they will work on a project that will have them picking a variable that potentially influences income inequality. They will have to find data that quantifies this variable using a website such as the World Economic Forum or World
Inequality Database. They have the freedom to use data relating to states within the country or several different nations. Their goal will be to find out if a linear relationship exists between the variable of their choosing and the Gini coefficient of a state or nation. They will need to do the following in the form of a written report.

- Specify the independent variable and dependent variable in your study.
- Create a scatterplot of your (x, y) ordered pairs by hand or using a program like Microsoft Excel. Title the scatterplot and label the axes with the appropriate units and variables.
- Use the TI-84 Plus to determine the equation of the linear regression model, in slope-intercept form, for the scatterplot.
- Interpret the slope and the y-intercept of the linear regression model in the context of the data.
- Find and interpret the correlation coefficient and coefficient of determination in the context of the data.
- Create a plot of residuals for the data by hand or using a program like Microsoft Excel. Does this plot suggest that a linear model is appropriate for the data? Why or why not?
- Are there any political policies that might have caused the value of your variable to be higher or lower for certain countries? Include any interesting statistics related to these policies. Also include and interpret any graphs or charts that might allow us to understand the impact of these policies.
- For countries that are adversely affected by the variable, what policies do you think should be put into place to alleviate income inequality?

There will also be an oral presentation component to this project that should be no longer than eight minutes. Students will be required to use some type of visual to summarize their answers to the focus questions. This can be a PowerPoint presentation, video, poster board, or any other approved visual. While students are presenting, their classmates will be required to fill out questionnaires describing any new insights or facts they learned from the presentation and explaining why they agree or disagree with the presenters’ proposals for alleviating inequality.

As an example of a project, students might decide to see if a linear relationship exists between a country’s minimum wage and the Gini coefficient of that country. The group would be advised to select forty to fifty countries with varying levels of income inequality. Then, they would have to research the minimum wages of these countries in United States dollars using a website such as the World Economic Forum to gather the data that they will analyze for their written report. They would need to use the TI-84 Plus or statistical software to come up with the scatterplot, residual plot, regression model, correlation coefficient, and coefficient for the data. They would be required to research policies or factors that have influenced the minimum wage in various countries. They might decide to examine a graph of how the minimum wage in the United States has changed over time and compare to a similar times series graph for another nation. Based on factors or policies that have affected the minimum wage in different nations, they would need to come up with their own proposal for increasing the minimum wage in countries where it is relatively low.

Notes

3. Atkinson, 22.

   https://www.dissentmagazine.org/online_articles/our-inequality-an-introduction


   https://cepr.net/blogs/cepr-blog/union-membership-and-income-inequality


18. Galbraith, 6.

19. “Businesses Under Communist Systems,” Lumen Learning, 
   https://courses.lumenlearning.com/boundless-business/chapter/businesses-under-communist-systems/


23. Piketty, 620.

24. Emily Badger, “The Best Thing We Could Do About Inequality is Universal Preschool,” City Lab, June 17, 2013. 


28. Galbraith, 78.

29. Galbraith, 79.
30. Galbraith, 143.
37. Ibid

Bibliography


Additional Reading Materials

The following textbooks will be used to assign homework relating to the unit plan objectives and give students additional practice with course concepts in the classroom.


Implementing District Standards

The Common Core State Standards addressed in this unit are listed below.

CCSS.MATH.CONTENT.HSS.ID.B.6 Represent data on two quantitative variables on a scatterplot, and describe how the variables are related.

In this unit, students will learn to draw a scatterplot by hand and using the TI-84 Plus graphing calculator. Their scatterplots will depict the relationship between the Gini coefficient of a location and another measurable variable of interest. The scatterplots will allow them to visualize the extent to which the points in their data set form a linear pattern. Finally, the scatterplots will alert them to outliers, points that deviate from the overall pattern of the data.

CCSS.MATH.CONTENT.HSS.ID.B.6.A Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

CCSS.MATH.CONTENT.HSS.ID.B.6.C Fit a linear function for a scatterplot that suggests a linear association.

CCSS.MATH.CONTENT.HSS.ID.C.7 Interpret the slope and the intercept of a linear model in the context of the data.

Students will meet the above three standards with the help of technology. They will use the TI-84 Plus to produce a linear regression function that describes the relationship between the Gini coefficient and their variable of interest. They will utilize their linear regression function to make predictions. Furthermore, they
will use the slope of their regression model to find out what happens to their variable of interest when the Gini coefficient increases or decreases.

**CCSS.MATH.CONTENT.HSS.ID.B.6.B** Informally assess the fit of a function by plotting and analyzing residuals.

Students will learn to create a residual plot to assess whether a linear model appropriately describes the relationship between the Gini coefficient of a location and their variable of interest. By examining how the points of their residual plot are dispersed, students can judge how well a linear model fits their data.

**CCSS.MATH.CONTENT.HSS.ID.C.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.

Students will use the TI-84 Plus to determine the correlation coefficient for the relationship between the Gini index and a measurable variable of interest. They will use the correlation coefficient to assess whether a linear model is appropriate for their data and describe the strength of the linear relationship between their independent and dependent variable. The correlation will tell them whether their independent and dependent variables are positively associated, negatively associated, or neither.

**CCSS.MATH.CONTENT.HSS.ID.C.9** Distinguish between correlation and causation.

This unit will enable students to understand that although a correlation coefficient may indicate a linear relationship between the Gini coefficient and their variable of interest, this does not imply that one variable is responsible for the other. For example, we cannot conclude that lack of mathematics education causes a nation to have a high level of income inequality, even if a correlation coefficient suggests a strong linear relationship between the Gini coefficient and the average years of math education in a country. There might be other factors that contribute to income inequality.