Curriculum Units by Fellows of the Yale-New Haven Teachers Institute

Revealing Truth: An introductory Unit for Algebra I
Curriculum Unit 22.03.02
by Annie Lerew

Introduction

“For all of us, knowledge of the ideas of others can enlarge our view of what is mathematical and add a more humanistic and global perspective to the history of mathematics. This enlarged view, in which mathematical ideas are seen to play a vital role in diverse human endeavors, provides us with a richer and fuller picture of mathematics and it’s past.”

This ten-lesson unit aims to help 9th grade Algebra 1 students reframe their thinking about studying mathematics through two simultaneous and equally important strategies. First, students will spend time reflecting on their own mathematical stories and heritages culminating in a project where students write personal math narratives, which they will submit at the end of the unit. Second, the unit focuses on the history and very nature of mathematics. Students will spend time writing and thinking about the infinitesimal and infinite, patterns, irrationality, and the history of humanity’s interactions with these concepts. Through individual writing and thinking as well as small group and full class discussions, students will unpack their own mathematical histories and rebuild their frameworks around the discipline in ways that will serve them throughout their high school mathematics endeavors. The role of the educator throughout this unit is to intentionally reflect on and present to students the ways that racism has intertwined itself into the mathematics classroom. As Battey states, “Naming white institutional spaces, as well as identifying the mechanisms that oppress and privilege students, can give those who work in the field of mathematics education specific ideas of how to better combat racist structures.”

Many American children are struggling in the mathematics classroom. A recent international exam given to teenagers ranked the USA 31st in math literacy out of 79 countries. Often a student will enroll in their first course of Algebra lacking the arithmetic skills to be successful in the course. They can logically determine what they need to do to solve an equation and they can think abstractly as one needs to do in Algebra. However, they are not sure how to add, or subtract, or they forget what happens when a negative is divided by a positive. Additionally, students lack mathematical enthusiasm and general anxiety about studying mathematical topics often has negative impacts on their ability to learn. “Classes often focus on formulas and procedures rather than teacher students to think creatively about solving complex problems.” One goal of Ethnic Studies classrooms is to remove the teacher from the center and replace it with student voice and
empowerment. Thus, progressive mathematics pedagogy and Ethnic Studies are a perfect fit and complement each other as they both have the goal of making student voice the center of the curriculum.

Low standardized test scores and unhealthy relationships with mathematics are common amongst many learners. Black, Latinx, and Indigenous students are struggling to excel in the math classroom, particularly when their teachers are White. This dire situation is not new, in fact, it has been the case for more years as noted by David Stinson in his writings on equity and justice in mathematics education: “too often policy and reform efforts do not address the needs of marginalized learners but rather reinforce the economic, technological, and social interests of the powerful.”

The issues of the American Mathematics classroom have been exacerbated by the disruptions and inequity of the pandemic school years, which as of 2022, number three years of disrupted learning. It is more imperative now more than ever for students to see that thinking mathematically is intimately tied to being human. Frances Su states “To do mathematics means more than just learning the facts of mathematics—it means seeing oneself as a capable mathematical learner who has the confidence and the habits of mind to tackle new problems.” Algebra is often referred to as the gatekeeper for the study of higher-level mathematics. If students can succeed in Algebra they are promoted onto geometry, calculus, statistics and beyond. However, if Algebra is a struggle, they are often taken down a path of remedial math classes. These courses earn students’ credits towards their High School diploma, but do not open the doors to the beautiful world of higher-level mathematics.

A student's arithmetic abilities often correlate to their ability to thrive in the elementary and middle school classrooms. Arithmetic skills work in conjunction with number sense to develop a learner who is truly ready for the abstract world of equations and algebra. If a student arrives in high school classrooms deficient in these areas, it can become daunting to recover. These high stakes are only aggravated by the fact that many classrooms are organized such that there is only one narrow path that is considered the correct way to learn. This idea is summarized well by Frances Su as follows, “We often signal to others that there’s only one way to be successful in mathematics—by forcing kids to do math quickly, or rushing students into calculus in high school, or telling professionals that they aren’t “real mathematicians” if they don’t do research. There are multiple ways to be successful. Mathematical achievement is not one dimensional, and we must stop treating it like it is.”

All too often mathematics is considered a neutral discipline set apart from the other areas of study when it comes to looking at curriculum with a critical social lens. Math classrooms and math teachers are given a pass on being culturally responsive because their subject matter is considered to have equal access to all. However, “Schools and mathematics classrooms are not exempt from the ubiquitous impact of racism. Both racism and mathematics have an omnipresence.” The general argument for a culturally neutral math classroom is that if a student shows up to class, pays attention and works hard, then success should be easy to obtain. During the Spring of 2022 the Department of Education for the state of Florida rejected math textbooks that included lessons on critical race theory. The Governor of Florida, Ron DeSantis ordered text be sent back to publishers with the command to “take the nonsense out of the math books.” This rejection of historical facts and top-down push for a color-blind mathematics classroom hurts all students, including White students.

American students, and particularly Black, Latinx, and Indigenous students, are not thriving in their math classrooms. Berry explains tension between reform and the color-blind mathematics classroom as follows: “This brief review of policies and reforms in mathematics education suggests that economic, technological,
and security interests were, and continue to be, drivers of many policies and reforms. These policies and reforms situated mathematics education in a nationalistic position of being color-blind, in a context where race, racism, conditions, and contexts do not matter. This positions schools and communities as neutral sites rather than cultural and political sites.”

Arthur Powell and Marilyn Frankenstein make the following argument against a Eurocentric math classroom in their writings on Ethnomathematics: “Institutionalized Eurocentric curricula constantly reinforce the racial and sexual inferiority complexes among people of color and women. The dominant curriculum in use today throughout the United States is explicit in asserting that mathematics originated among men.”

Ethnic Studies refers to course content and an approach to teaching and learning that is collaborative and builds relationships. Teaching math with an Ethnic Studies lens requires the educator to adjust their presentation by revealing the histories that have been in the curriculum the whole time. “Some have argued that social justice should be a primary goal in mathematics education.” Math educators should develop an antiracist stance that recognizes historical biases and focuses on helping every student succeed in math. “Too often policy and reform efforts do not address the needs of marginalized learners but rather reinforce the economic, technological, and social interests of the powerful.” By making math more relevant, powerful, and exciting to students educators can bring those who have been marginalized into the fold of successful high school mathematicians.

Whose mathematics is taught and what mathematics is ignored is political and has never been neutral. Educators should be asking if their Mathematics classroom is being used to examine the social world and make it more just or to replicate the current unjust social order. “The mathematics Black students engage in must help them understand how issues of race, and racism impact them, their families, Black communities, and the masses of Black people locally, nationally, and internationally. The goal must be the collective betterment of Black adults and children’s lived realities and education, especially in mathematics.” This can include making space for students to explore why solutions work and making connections between the elementary and high school curriculum. “We don’t realize what we might gain by having diverse people, new expertise, fresh ideas to draw from. The field of mathematics is itself poorer because of the voices that are not present.” The math education professor Rochelle Gutiérrez reminds us that math needs a diversity of people in order to grow in new ways, not just that people need math: “The assumption is that certain people will gain from having mathematics in their lives, as opposed to the field of mathematics will gain from having these people in its field.”

**Background**

I am a white, cis-gendered female, social justice oriented high school math teacher who believes that love is love and that Black Lives Matter. Currently, I work at an arts magnet school in New Haven, Connecticut where I serve approximately 100 students. I am currently finishing my first year of teaching in New Haven although it is my 14th year of teaching overall. I have had previous experience at the high school level in The Bronx and at the middle school level in Philadelphia. This unit is intended for any educator interested in weaving number
sense from a historical context into an Algebra I classroom using socially just pedagogy. As a member of a
seven-person math department I have colleagues who teach similar sections of each of the courses I teach.
Cooperative Arts and Humanities High School (CO-OP) is in downtown New Haven, Connecticut. The total
9th-12th grade student population is 630 with a 13:1 student to teacher ratio. More than 84% of the student
body is part of the global majority. 66% of students are from low-income families and qualify for the Free and
Reduced-Price Meals program. While at CO-OP students participate in one of six arts majors. Students spend
1.5 hours per day studying their major and have their other classes on a every other day block schedule.
There is a 94% graduation rate. 34% of students who take AP exams pass and the school has a 24.3% on the
College readiness index.\textsuperscript{16}

**Revealing truth for Algebra 1 students**

Revealing Truth is an introductory unit for 9th graders enrolled in Algebra I that will simultaneously cover
remedial topics in arithmetic, specifically the four basic operations with integers, while also recognizing the
vast array of ways that humans have thought about and interacted with numbers throughout history. “Among
those who study and write about the history of mathematics there has been a growing understanding that
what is generally referred to as modern mathematics is, itself, built upon contributions from people in many
cultures. There is now greater acknowledgement of mathematical developments in China, India, and the
Arabic world. We may find that some ideas we have taken to be universal are not while other ideas we
believed to be exclusively our own, are, in fact, shared by others.\textsuperscript{17} Situating mathematical concepts inside
their stories of origin will enable students to connect with them on a new, more personal, level. Students will
explore the origins of the algorithms they were exposed to in elementary school as well as some algorithms
that have been ignored by the Eurocentric version of algorithms and number sense. This ten-lesson unit will
include an introduction to the field of Ethnomathematics, which is the study of the relationship between
mathematics and culture. “The ideas of non-Westerners belong, as do ours, in the global and ongoing history
of mathematics, always keeping in mind that there is no single linear ordering and no necessary route that all
must follow. At the very least, ethnomathematics can lead to an appreciation of the intellectual endeavors of
others.”\textsuperscript{18}

This unit intends to spark a love of mathematics and lead towards higher levels of student success in Algebra
and high school mathematics. “Mathematics and the natural sciences are the only areas of study presented
with little or no historical, cultural, or political references. This ahistorical approach is essential for what
Alkalimat identifies as a process of “indoctrinating an elite with the metaphysical myth of eternal Eurocentric
domination of the world.” This pedagogical approach reinforces the institutionalization of Eurocentrism, class
elitism, and sexism. European names such as Pythagoras, Euclid, Cauchy-Rieman, Fourier, and Newton are
tossed about sans flesh, bones, and personalities; then they are attached to various levels of abstractions as if
they always existed.”\textsuperscript{19} S.E. Anderson is a mathematics educator who has dedicated their work to helping
students of color change a sense of alienation towards mathematics with an attitude that math is
“intellectually stimulating.”

Anderson outlines four components of her typical introductory lecture as follows:

a. People of color or were the original founders and innovators of mathematics and science.
b. Europe was never isolated from Third World mathematical and scientific achievements.

c. European capitalism developed because of Europe's incorporation of the mathematical and scientific ideas and techniques of the First World into their capitalist superstructure.

d. Europe dominated, enslaved, and colonized Africa, Asia, and the Americas and thereby stopped and / or reversed most, but not all, forms of First World intellectual, mathematical, scientific, and technological activity.

Anderson explains their key points as follows: "Through this brief historical survey, I attempt to put mathematics within a human context. Although I limit term papers to my calculus and "math as a human endeavor" classes, many of my students become very interested in the historical development of mathematics. Often, they wind up doing more reading and writing term papers in the field of mathematics and science history for other classes."

The Number Devil by Hans Magnus Enzensberger will be used as an anchoring point for the mathematical content in each lesson. One chapter will be read allowed by the class each day with the use of a document camera during the unit. These chapters will take approximately 10 minutes to read. At the conclusion of each chapter students will do reflective writing, which they will share in small groups or with the whole class. Each lesson will also contain a warmup problem from Which One Doesn't Belong collection of images. These images are considered "low floor/high ceiling" exercises. Meaning there are multiple entry points into the problem and students can go as in depth as they'd like to.

**Pedagogy & Strategies**

"Mathematics makes the mind its playground. Doing math properly is engaging in a kind of play: having fun with ideas that emerge when you explore patterns and cultivating wonder about how things work. Math is not about memorizing procedures or formulas, or at least that’s not where you start." Let’s begin by looking at what Anderson calls the six disasters in mathematical pedagogy. Separate arithmetic from algebra; Teach mathematics without any historical references; Use textbooks that are elitist and cryptic; Do work and be tested as an individual as opposed to working and being tested as study groups; Accept the myth that mathematics is pure abstraction and, therefore, antithetical to one’s cultural and working environment; and Memorize, memorize, memorize. This unit will try, whenever possible, to avoid these “disasters” as outlined by Anderson. As Berry states, “one main idea of “new math” was to reduce focus on the drill and practice approach to teaching mathematics and increase focus on approaches where students could develop conceptual understanding of mathematics. These pedagogical approaches included the use of manipulatives, guided discovery learning, teaching practices, and the spiral curriculum. This unit will include the following Pedagogy:

**Holding space for student voice in the classroom**

Bob Moses famously said, “We don’t listen to kids enough. Really listen. It is a difficult thing for grown-ups to do, listen and actually pay serious attention to what young people are saying.” This unit will involve various opportunities for student voice in each lesson. Turn and talks, see/think/wonder, and think/pair/share are just some examples of how space will be made for student voices in the lesson plans. In *Rough Draft Math*
Amanda Jansen makes an argument for prioritizing student voice in the classroom as follows: “Listening to our students is not only productive for their learning, but also powerful for developing their identities as learners. Communicating about thinking helps understanding develop and the distinction between listening to students rather than for an answer is useful for teachers to consider when promoting rough draft thinking.”

Revision policy on all assignments

Allowing students to revise their work is a productive way to cultivate a mathematically positive classroom culture. By allowing students to revise homework assignments, teachers alleviate stress and pressure that can hinder the learning process. As Su states, “In the math education community, the term productive struggle describes the state of actively wrestling with a problem, persistently trying out various strategies, being willing to take risks, being unafraid of mistakes, and progressing incrementally in understanding the underlying ideas. This wrestling produces a certain kind of endurance, which enables us to be comfortable with the struggle. This endurance produces an unflappable character that benefits us in addressing life problems—calming us with the knowledge that it’s okay if we don’t solve a problem right away. We appreciate that not solving a problem can be just as important as solving it—that, as Simone Weil suggested, the effort to grasp truth is itself worthwhile, for increasing our aptitude, even if it produces no visible fruit.” Jensen elaborates on this idea further by identifying three key features of rough draft math: Imperfect but precise. Unfinished and unsure. Involves revisiting and revising. (#8 p.8)

Include time for thinking and writing

All too often students see math class as a place to be right or wrong. When it is far more nuanced than that. The mathematics classroom should be a space for thinking and collaborating, not for judgment. Therefore, each lesson will include time for reflective thinking and writing. Students will write in response to poetry and quotes as well as in response to the daily chapter of The Number Devil.

The unit will culminate in a final project that aligns with the students’ Art major at COOP. Students will create a personal mathematics narrative, which they will submit on the final day of the unit. For instance, the theater majors will write and perform a monologue based on their mathematical heritage. The visual arts majors can create a piece of work that illustrates their personal narrative. This unit will be an exciting way for 9th graders to begin their high school careers by both honoring the arts and the strong academic tradition that is embedded at CO-OP. For educators who don’t teach at an arts themed high school it is encouraged to have students write their narrative as an essay and adjust the rubric accordingly.

Below is the unit project description that students will be given on the first day of class:

Welcome to Revealing Truth, the first unit of this Algebra I course. At the end of this unit, in approximately four weeks, you will be asked to submit your personal mathematics narrative that connects your major here at COOP to the topics we are studying in Algebra. The initial rubric for this project is below, we will spend time at the end of our first-class making changings to this rubric together until we are satisfied with its language.

Personal Mathematics Narrative Rubric

Name: Major: Project title:

| 3 Exemplary | 2 Proficient | 1 Adequate | 0 No submission |

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| Writing/Thinking | Student includes a thoughtful reflective writing as part of their project that explains their process and how it connects to the unit | Student includes a thoughtful reflective writing as part of their project that explains their process | Student includes a reflection on their process | No reflection is submitted |
| Content | Project has clear connections to the unit that are creative and varied | Project has one connection to the unit | Project has vague connections to the unit | No project is submitted |
| Presentation/Display | Presentation or display is engaging and demonstrates project buy-in | Presentation is sufficient | Presentation lacks effort | No project is submitted |

### Lesson Activities

The following are summaries and details for ten lesson activities that are suggested to be used during this unit. The summaries contain quotes that are recommended for the instructor to take time and reflect on before implementing the activity. The activities involve students working on their personal math narratives, guiding quotes, and a chapter from *The Number Devil* by Hans Enzensberger. Activities also include suggested songs to play while students write as well as a StoryCorps episode that aims to inspire students to tell their own stories. Each lesson begins with a low floor, high ceiling problem from the collection of images from Which One Doesn’t Belong (https://WODB.ca). These are helpful warm activities because there are several correct answers and students can engage with the material at whatever level they enter the lesson at.

### Activity 1 Summary

In the first activity students will explore number lines and the bases used by different peoples throughout history as well as look at sets of numbers (the integers, the rational, etc.). Students will be introduced to the unit project in this lesson and will brainstorm changes to the rubric for the project. While preparing for the lesson it is recommended that the instructor review and reflect on the following quotes:

- “But as everyone began counting by using their ten fingers, most numbering systems that were invented used base 10. All the same, some groups chose base 12. The Mayans, Aztecs, Celts, and Basques, looked down at their feet and realized that their toes could be counted like fingers, so they chose base 20. The Sumerians and Babylonians, however, chose to count on base 60, for reasons that remain mysterious.”
- “The basic arithmetic operations of elementary school, multiplying and dividing, appear to have derived from extremely early economic needs. 2500BC clay tablet found near Baghdad which concerns the problem of sharing.”
- “The peoples of West Africa and Middle America, as well as the Inuit and other Eskimo peoples of the far north, group by twenties. In some languages, such as Mende of Sierra Leone, the word for twenty means “a whole person”—all the fingers and toes.”
- “Children can learn about numeration systems by examining the construction of larger numbers. In the
Yoruba (Nigeria) language, for example, the name for forty-five means "take five and ten from three twenties," using the operations of multiplication and subtraction, rather than multiplication and addition, as in most European languages. Different solutions to the same problem, one just as good as the other.”

**Activity 1 Details**

**Essential question:** Some say mathematics was invented and others say it was discovered, what are your thoughts?

**Sound track for the day:** “A change is gonna come” by Sam Cooke

**Materials:** Math notebook, The Number Devil, calculators, rulers,

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<tr>
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<tbody>
<tr>
<td>Warm up</td>
<td>WODB for the day: <a href="https://wodb.ca">https://wodb.ca</a></td>
<td>- Students use think, pair, share strategy to decide which one of the four images doesn’t belong.</td>
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<td>- Brainstorm topics to include</td>
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<td>- Brainstorm types of presentations</td>
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<tr>
<td>Classwork</td>
<td>- Storycorps: <a href="https://storycorps.org/animation/schools-out/">https://storycorps.org/animation/schools-out/</a></td>
<td>- Students write reflectively in response to the podcast and to the poem</td>
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<td>- Poem/quote for the day: Keeping Quiet by Pablo Neruda (<a href="https://www.youtube.com/watch?v=k5kjfqbt-FA">https://www.youtube.com/watch?v=k5kjfqbt-FA</a> )</td>
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<tr>
<td>Writing &amp;</td>
<td>Chapter 1 of The Number Devil(^{33})</td>
<td>- Students use think/write, pair, share to respond to the chapter</td>
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<td>Thinking</td>
<td>An introduction</td>
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<td>Conclusion</td>
<td>Discuss/Revise unit project rubric</td>
<td>- Brainstorm revisions to the project rubric</td>
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**Activity 2 Summary**

In the second activity students will read about Zero and how its addition to the number line resulted in the place value system and our Arabic numerals (which are from India!). Additionally, students will read and then reflect individually and in small groups on a comparison reading from two mathematical perspectives. One reading argues that the natural order of the world is straight while the other provides evidence that circles are the true nature of the universe. While preparing for the lesson it is recommended that the instructor review and reflect on the following quotes:

- “Al-Khowarizmi wrote several important books, like Al-jabr walmuqabala, a book of several equations. Al-jabr roughly translates to “completion” and, gave us the term for Algebra. Algorithm is a corruption of the authors name as well.”\(^{34}\)
- “A numeration system can be additive, like the Roman number system, or it can be positional, as is ours today. It can involve one or more bases and it may or may not use zero.”\(^{35}\)
## Activity 2 Details

**Essential question:** What is important in mathematics?

**Sound track for the day:** “Feelin Good” by Nina Simone

**Materials:** Math notebook, The Number Devil, calculators, rulers,

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<td>Introduction</td>
<td>Circle vs line readings (see below)</td>
<td>· Personal mathematics narrative check-in</td>
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<tr>
<td></td>
<td></td>
<td>· Group reading &amp; reflective writing in response to the Circle vs Line readings</td>
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<tr>
<td>Classwork</td>
<td>· Storycorps: <a href="https://storycorps.org/animation/lessons-from-lourdes/">https://storycorps.org/animation/lessons-from-lourdes/</a></td>
<td>· Students write reflectively in response to the podcast and to the quotes</td>
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<td>· Poem/Quotes for the day</td>
<td>· These quotes are all centered on the idea of zero, which is also the focus of the Number Devil chapter for the day.</td>
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<td>“In the history of culture the discovery of zero will always stand out as one of the greatest single achievements of the human race.” -Charles Seife</td>
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<td>“So India, as a society that actively explored the void and the infinite, accepted zero” -Charles Seife</td>
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<td>“In the earliest age of the gods, existence was born from non-existence.” -The Rig Veda</td>
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<td>“It was in India and China that negative numbers first appeared. Brahmagupta, 7th century mathematician wrote: “Positive divided by positive, or negative divided by negative, is affirmative.” “positive divided by negative is negative. Negative divided by affirmative is negative.” -Charles Seife</td>
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<td>“Does Man forget that We created him out of the void?” -The Koran</td>
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<td>“Zero” comes from the Hindu word Sunrya meaning “empty”</td>
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<td>1596 Descartes puts zero on the European number line</td>
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<tr>
<td>Writing &amp; Thinking</td>
<td>Chapter 2 of <em>The Number Devil</em> Zero</td>
<td>· Students use think/write, pair, share to respond to the chapter</td>
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### Circle Vs Line Readings

Just before he died in the 1930s, Black Elk, an Oglala Sioux, spoke about his life and thoughts. His statement about the circle (below, right) is presented in contrast to a statement about the line (below, left) which appeared in a highly lauded work by two American professors of mathematics. While they differ on the geometric form, the writers share their degree of conviction in the rightness of their ideas and support their view with nature, God, achievement of goals, and proper human development. Black Elk and the Sioux, however, were forcibly made to realize that their view was not shared by other cultures.
In every human culture that we will ever discover, it is important to go from one place to another, to fetch water or dig roots. Thus, human beings were forced to discover—not once, but over and over again, in each new human life—the concept of the straight line, the shortest path from here to there, the activity of going directly towards something. In raw nature, untouched by human activity, one sees straight lines in primitive form. The blades of grass or stalks of corn stand erect, the rock falls down straight, objects along a common line of sight are located rectilinearly. But nearly all the straight lines we see around us are human artifacts put there by human labor. The ceiling meets the wall in a straight line, the doors and windowpanes and tabletops are all bounded by straight lines. Out the window one sees rooftops whose gables and corners meet in straight lines, whose shingles are layered in rows and rows, all straight. The world, so it would seem has compelled us to create the straight line so as to optimize our activity, not only by the problem of getting from here to there as quickly and easily as possible but by other problems as well. For example, when one goes to build a house of adobe blocks, one finds quickly enough that if they are to fit together nicely, their sides must be straight. Thus, the idea of a straight line is intuitively rooted in the kinesthetic and the visual imaginations. We feel in our muscles what it is to go straight toward our goal, we can see with our eyes whether someone else is going straight. The interplay of these two sense intuitions gives the notion of straight line a solidity that enables us to handle it mentally as if it were a real physical object that we handle by hand. By the time a child has grown up to become a philosopher, the concept of a straight line has become so intrinsic and fundamental a part of his thinking that he may imagine it as an Eternal Form, part of the Heavenly Host of Ideals which he recalls from before birth. Or, if his name be not Plato but Aristotle, he imagines that the straight line is an aspect of Nature, an abstraction of a common quality he has observed in the world of physical objects.

Activity 3 Summary

In the third activity students will create a sieve of Eratosthenes to examine the prime numbers under 100. Additionally, students will explore the world of mathematical play by engaging in the game of cycles, a variation of tic-tac-toe. While preparing for the lesson it is recommended that the instructor review and reflect on the following quote:

- “The Ahmes Papyrus (c. 1650 B.C.) and the Moscow Papyrus (c. 1850 B.C.), while not deductively axiomatic, were and still are valid proofs. As Joseph (1987) notes: “Egyptian proofs are rigorous without
being symbolic, so that typical values of a variable are used and generalizations to any other value are immediate."\(^3\)

### Activity 3 Details

**Essential question:** Prime numbers are often called “building blocks”, why is that? Sound track for the day: “Glory” by John Legend & Common  
**Materials:** Math notebook, The Number Devil, calculators, rulers, small wipe off boards, dry erase markers, paper with 100 blank cells

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| Introduction | The game of cycles from Frances Su (see below) | - Explain the rules of the game and then play a few rounds as a group.  
- Students then play 1-1 using wipe off boards and dry erase markers.  
- Emphasize the importance of play in mathematics. |
| Classwork |  
- Poem/Quotes for the day  
- “Like the crest of a peacock, like the gem on the head of a snake, so is mathematics at the head of all knowledge.” -Vedanga Jyotisa (c 500 bce) | - Students write reflectively in response to the podcast and to the quotes |
| Writing & Thinking | Chapter 3 of *The Number Devil*\(^4\) Primes | - Students use think/write, pair, share to respond to the chapter |
Activity 4 Summary

In the fourth activity students will discuss the relationship between sides of a right triangle - usually called the Pythagorean theorem. Students will determine a more accurate name for this theorem as it was used in China and India over 1000 years before Pythagoras was born. This class will also address the square root of 2 and how it presented a problem in the Vedas, which gave precise directions on how to construct a Hindu altar. While preparing for the lesson it is recommended that the instructor review and reflect on the following quote:

- “It is no surprise, then, that math explorers can be found in every society throughout history. This is most readily apparent in the games that people play, especially games of strategy, which generate interesting mathematical questions. Achi is a game played by the Ashanti people of Ghana in West Africa.”

Activity 4 Details

Essential question: When you multiply a number by itself it is often called “squaring it”, why is that so?

Sound track for the day: “Lost ones” By Lauryn Hill

Materials: Math notebook, The Number Devil, calculators, rulers, Wipe off boards, dry erase markers

<table>
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<td>Group demonstration of the rules of the Achi. Rather than pegs students will play with wipe off boards and dry erase markers.</td>
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The Game of Circles

"Draw this starting diagram (below) of dots and edges, which divides a triangular region into three smaller triangles, called cells. Players take turns, marking a single arrow along an edge of the diagram, obeying these rules: only one arrow may be placed on each edge, and no dot can be a sink or a source. Some edges may become unmarkable during the course of play. The object of the game is to produce a cycle cell—a cell bordered by arrows that cycle in one direction, clockwise or counterclockwise. The player who creates a cycle cell or makes the last possible move is the winner. After you play this game for a while, see if you can figure out whether the first or the second player has a winning strategy. Then explore the game of cycles using non-triangle game boards.

I just created this game, and at the time of writing I’m unaware of whether it has been invented or studied before. So there are lots of open questions, and I’m playfully exploring the game.”

Activity 4 Summary

In the fourth activity students will discuss the relationship between sides of a right triangle - usually called the Pythagorean theorem. Students will determine a more accurate name for this theorem as it was used in China and India over 1000 years before Pythagoras was born. This class will also address the square root of 2 and how it presented a problem in the Vedas, which gave precise directions on how to construct a Hindu altar. While preparing for the lesson it is recommended that the instructor review and reflect on the following quote:

- “It is no surprise, then, that math explorers can be found in every society throughout history. This is most readily apparent in the games that people play, especially games of strategy, which generate interesting mathematical questions. Achi is a game played by the Ashanti people of Ghana in West Africa.”

Activity 4 Details

Essential question: When you multiply a number by itself it is often called “squaring it”, why is that so?

Sound track for the day: “Lost ones” By Lauryn Hill

Materials: Math notebook, The Number Devil, calculators, rulers, Wipe off boards, dry erase markers

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I just created this game, and at the time of writing I’m unaware of whether it has been invented or studied before. So there are lots of open questions, and I’m playfully exploring the game.”
Activity 5 Summary

In the fifth activity students will explore square numbers and how they can relate to Pythagorean triples. Additionally, students will hear the story of the mathematician Ramanujan and his conflicts around the concept of proof while he studied at Oxford. While preparing for the lesson it is recommended that the instructor review and reflect on the following quotes:

- “I point out to my students that the Egyptians, Chinese, and Indians used different styles of mathematical generalizations in algebraic problem solving.”
- “Math is way of life, kind of sort of like how racism takes place in life...you’re going to always have to deal with math in life...you’re going to always have to deal with racism in life.” - Julian Davis

Activity 5 Details

Essential question: Is racism a part of mathematics?

Sound track for the day: “Ain’t gonna let nobody turn you round” by The Freedom Singers

Materials: Math notebook, The Number Devil, calculators, rulers,

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**Introduction**

- The story of Ramanujan
  Srinivasa Ramanujan: He wrote his findings, things he knew to be true, into his notebooks. He believed that the Goddess Namagiri brought his discoveries into his mind and therefore he didn’t require proof. The world of Western Mathematics is built upon axiom proof.

**Classwork**

- Quote for the day
  “Math is a way of life, kind of sort of like how racism takes place in life...you’re going to always have to deal with math in life...you’re going to always have to deal with racism in life.” -Julius Davis

**Writing & Thinking**

- Chapter 5 of *The Number Devil*[^46]
- Square numbers

**Conclusion**

- The interest of the Pythagoreans in numbers goes way beyond the purely arithmetical. For them, number was the universal principle that underlies the cosmos and allows it to be understood. As part of a unique blend of a rational approach to understanding nature with numerology and other mystical practices, the Pythagoreans saw the natural numbers as a clearly discernible, stable element that hides behind the apparent chaos of day-to-day experience and helps to make sense of it. Relations among numbers explain, in their view, such disparate phenomena as the properties of geometric solids, the relative motions of celestial bodies and their possible configurations on heaven, and the production of musical harmonies. -Leo Corry
- “To pythagoras, playing music was a mathematical act.” Like squares and triangles, lines were number-shapes, so dividing a string into two parts was the same as taking a ratio of two numbers. The harmony made by two notes was the harmony of mathematics -and the harmony of the universe.” -Charles Seife

**Activity 6 Summary**

In the sixth activity students will pattern sniff using the Fibonacci numbers. Additionally, students will explore 3 by 3 magic squares and try to find as many of them as possible (there are 8). While preparing for the lesson it is recommended that the instructor review and reflect on the following quote:

- “A concise and meaningful definition of mathematics is virtually impossible. Mathematics has developed into a worldwide language with a particular kind of logical structure. It contains a body of knowledge relating to number and space, and prescribed a set of methods for reaching conclusions about the physical world” -George Joseph
Activity 6 Details

Essential question: What kinds of addition patterns exist?

Sound track for the day: “Why” by Tracy Chapman

Materials: Math notebook, The Number Devil, calculators, rulers,

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| Introduction | Magic Squares  
The expression Magic square is commonly used for any arrangement of squares in which the cells contain numbers such that any column, row, or diagonal produces the same sum.  
“There is evidence that magic squares were brought to China during a period of Islamic invasion. To this day, in northern India, healers make use of magic squares of order 3 in the treatment of malaria.” -Jean-Luc Chabert | · Students work in pairs to construct a 3 by 3 magic square (there are 8 possibilities)                               |
· Poem/Quotes for the day  
· “A concise and meaningful definition of mathematics is virtually impossible. Mathematics has developed into a worldwide language with a particular kind of logical structure. It contains a body of knowledge relating to number and space, and prescribed a set of methods for reaching conclusions about the physical world -George Joseph  
· Fibonacci was educated by Muslims in North Africa and then took his studies back to Italy where he wrote Liber Abaci in 1202. This introduced 0 to the European world. | · Students write reflectively in response to the podcast and to the quotes                                                   |
| Writing & Thinking | Chapter 6 of The Number Devil Fibonacci numbers                  | · Students use think/write, pair, share to respond to the chapter                                                   |
| Conclusion | Discuss how their writing is going. Is there anything we should agree to change on the rubric? | · Personal mathematics narrative check-in                                                                 |

Activity 7 Summary

In the seventh activity students will create Pascal’s triangle. Additionally, students will briefly be introduced to the life and work of Bob Moses from his early days in the voting rights movement to his later work with the Algebra Project. While preparing for the lesson it is recommended that the instructor review and reflect on the following quote:
• “One of the best ways to claim your heritage in mathematics is to find a game of strategy from your own cultural history and embrace the kind of thinking the game requires. Probe it with exploratory questions.”

• “If we can do it, then we should do it.” -Bob Moses

**Activity 7 Details**

Essential question: Is mathematics living or dead?

Sound track for the day: “Lord, don’t move the mountain” by Inez Andrews

Materials: Math notebook, The Number Devil, calculators, rulers,

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</table>
| Introduction | Pascal’s triangle     | · Present the first four rows of Pascal’s triangle  
· Have students brainstorm what the pattern might be  
· Have students construct more rows of their triangle  |
| Introduction | Each cell is the sum of the two cells above it |                                                                                                                                                                                                       |
| Classwork | · Storycorps: https://storycorps.org/stories/ralph-catania-and-colbert-williams/  
· Poem/Quotes for the day  
“If we can do it, then we should do it” -Bob Moses | · Students write reflectively in response to the podcast and to the quotes  |
| Writing & Thinking | Chapter 7 of *The Number Devil*[^49]  
Pascals triangle | · Students use think/write, pair, share to respond to the chapter  |
| Conclusion | Discuss how their writing is going. | · Personal mathematics narrative check-in  |

**Activity 8 Summary**

In the eighth activity students will explore factorials and calculate them. While preparing for the lesson it is recommended that the instructor review and reflect on the following quotes:

• “At other times and places, people used other ways to help their calculations: pebbles, marks in the dust, knotted strings, tokens on a counting board, beads on an abacus.”

• “So if you ask me, “why do mathematics?” I will say this: “Mathematics helps people flourish” Mathematics is for human flourishing.” -Frances Su
### Activity 8 Details

**Essential question:** What does it mean to be “good” at math?

**Sound track for the day:** “Calypso Freedom” by sweet honey and the rock

**Materials:** Math notebook, The Number Devil, calculators, rulers,

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<td>- Students use think, pair, share strategy to decide which one of the four images doesn’t belong.</td>
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</tbody>
</table>
| Introduction | Factorials         | - Present the first four factorial numbers  
|             |                       | - Have students brainstorm what the pattern might be  
|             |                       | - Have students calculator 5!, 6!, 7!, etc |
|             | - Poem/Quotes for the day  
|             | - “In order for us as poor and oppressed people to become a part of a society that is meaningful, the system under which we now exist has to be radically changed. This means that we are going to have to learn to think in radical terms.” -Ella Baker  
|             | - “So if you ask me, “Why do mathematics?” I will say this: “Mathematics helps people flourish.” Mathematics is for human flourishing”. -Frances Su | - Students write reflectively in response to the podcast and to the quotes |
| Writing & Thinking | Chapter 8 of The Number Devil  
|                     | Factorial          | - Students use think/write, pair, share to respond to the chapter |
| Conclusion         | Discuss how their writing is going. | - Personal mathematics narrative check-in |

### Activity 9 Summary

In the ninth activity students will contemplate the infinite and infinitesimal as well as look at the irrational constant Pi. While preparing for the lesson it is recommended that the instructor review and reflect on the following quotes:

- “I am in a sense something intermediate between God and Naught.” -Rene Descartes
- “Where there is infinite there is joy. There is no joy in the finite.” -The Chandogya Upanishad
**Activity 9 Details**

Essential question: What is more impressive: Everything that exists inside a cell or everything that exists inside a galaxy?

Soundtrack for the day: “All along the watchtower” by Jimi Hendrix

Materials: Math notebook, The Number Devil, calculators, rulers, cardboard circles, string

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</tr>
<tr>
<td>Introduction</td>
<td>Pi &amp; irrationality</td>
<td>· Have students measure the circumference and diameter of various cardboard circles and create a ratio between the two · Discuss this relationship as Pi</td>
</tr>
<tr>
<td>Classwork</td>
<td>· Storycorps: <a href="https://storycorps.org/animation/the-saint-of-dry-creek/">https://storycorps.org/animation/the-saint-of-dry-creek/</a> · Poem/Quotes for the day “Where there is infinite there is joy. There is no joy in the finite.” -The Chandogya Upanishad “Concealed in the heart of all beings is the Atman, the Spirit, the Self, Smaller than the smallest atom, greater than the vast spaces.” -Charles Seife ‘Nothingness is benign and being nothingness our limited mind can not grasp or fathom this, for it joins infinity.” -Azrael of Gerona ‘I am in a sense something intermediate between God and naught. -Rene Descartes</td>
<td>· Students write reflectively in response to the podcast and to the quotes</td>
</tr>
<tr>
<td>Writing &amp; Thinking</td>
<td>Chapter 9 of The Number Devil[52] Infinitesimal vs Infinite</td>
<td>· Students use think/write, pair, share to respond to the chapter</td>
</tr>
<tr>
<td>Conclusion</td>
<td>Discuss how their writing is going.</td>
<td>· Personal mathematics narrative check-in</td>
</tr>
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</table>

**Activity 10 Summary**

In the tenth activity students will return to their work with the Fibonacci numbers and explore Phi (The Golden Ratio). In this lesson students will also construct a pentagram and read about its historical significance. While preparing for the lesson it is recommended that the instructor review and reflect on the following quote:

- “Every being cries out silently to be read differently.” -Simone Weil

**Activity 10 Details**
Essential question: Is mathematics divine?

Sound track for the day: “I wish I knew how it would feel to be free” by Nina Simone

Materials: Math notebook, The Number Devil, calculators, rulers, strips of paper

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<td>· Students use think, pair, share strategy to decide which one of the four images doesn’t belong.</td>
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<tr>
<td>Introduction</td>
<td>The Golden ratio</td>
<td>· Have students construct a golden ratio on a strip of paper</td>
</tr>
<tr>
<td></td>
<td></td>
<td>· Have students construct a golden spiral on a piece of paper</td>
</tr>
<tr>
<td>Classwork</td>
<td></td>
<td>· Students write reflectively in response to the podcast and to the quotes</td>
</tr>
<tr>
<td></td>
<td>Storycorps: <a href="https://storycorps.org/animation/lessons-learned/">https://storycorps.org/animation/lessons-learned/</a></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Poem/Quotes for the day</td>
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</tr>
<tr>
<td></td>
<td>Every being cries out silently to be read differently - Simone Weil</td>
<td></td>
</tr>
<tr>
<td>Writing &amp;</td>
<td>Chapter 10 of The Number Devil[53] Golden Ratio and pentagram</td>
<td>· Students use think/write, pair, share to respond to the chapter</td>
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<tr>
<td>Thinking</td>
<td></td>
<td></td>
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<tr>
<td>Conclusion</td>
<td></td>
<td>· Students will construct a pentagram while listening to some of the history around this symbol</td>
</tr>
<tr>
<td></td>
<td>Pentagram</td>
<td>· Personal mathematics narrative check-in</td>
</tr>
<tr>
<td></td>
<td>Discuss how their writing is going.</td>
<td></td>
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</table>

**Conclusion**

As Su states, “Those of us who have experienced the freedoms of mathematics have a significant responsibility to welcome others to those freedoms as well.”[54] This unit seeks to give students a chance to think critically and creatively about the concepts and histories of Mathematics. As the instructor and developer of this unit it is my goal to reveal algebraic and historical truths to the students in ways that are socially just and that cultivate their mathematical potential. As Anderson states, “The task of progressive educators is not to try to fix or prop up the capitalist system of education. Those of us who want to see true mathematical and scientific knowledge flourish among our youth and workers must break from the Eurocentric perspective and begin the reconstruction of the rich and complex fabric of world mathematical and scientific knowledge. This requires breaking with most of the pedagogical and curricular traditions that exist today.”[55] For other educators who are considering using this unit or modifying it to suit the needs of your own institutions I’d like to leave you with a final thought from Frances Su: “The importance of having an advocate, someone who says, “I see you, and I think you can flourish in mathematics.” Everyone can use this encouragement, but this can be especially important for marginalized groups who already have so many voices telling them they don’t belong. Can you be that advocate?”[56]
Appendix on Implementing District Standards

This unit is designed to provide an overview of strategies to unveil truths in mathematics and the way it is taught. As such, the unit does not focus on specific math standards like a typical Algebra I unit would. The Common Core State Standards listed below are addressed across all ten sample lesson plans in this unit.

CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.

CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.

CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.

CCSS.Math.Practice.MP4 Model with mathematics.

CCSS.Math.Practice.MP5 Use appropriate tools strategically.

CCSS.Math.Practice.MP6 Attend to precision.

CCSS.Math.Practice.MP7 Look for and make use of structure.

CCSS.Math.Practice.MP8 Look for and express regularity in repeated reasoning.

Annotated Bibliography


Notes


4 Richards, “Math scores stink in America. Other countries teach it differently -and see higher achievement.”


8 Su, Mathematics for Human Flourishing, 190


14 Davis, 202.

15 Su, 12.


17 Ascher, Mathematics Elsewhere: An Exploration of Ideas Across Cultures, 2.


22 “which one doesn’t belong,” accessed May 1, 2022, https://www.wodb.ca

23 Su, 50.


28 Su, 120.

29 Leo Corry, A Brief History of Numbers (Oxford, UK: Oxford University Press, 2015), XiX.


35 Barbin et al., A History of Algorithms from the Pebble to the Microchip, 22.

36 Ibid, 29-46.


40 Hans Magnus Enzensberger, The Number Devil: A Mathematical Adventure, 49-64.

41 Su, Mathematics for Human Flourishing, 51.

42 Su, 23.


45 Barbin et al., A History of Algorithms from the Pebble to the Microchip, 24.

46 Hans Magnus Enzensberger, The Number Devil: A Mathematical Adventure, 90-104.

47 Ibid., 107-122.

48 IBID, 25.

49 Ibid, 125-146.


51 Ibid, 149-168.

52 Ibid, 171-188.

54 Ibid, 183.


56 Su, 160