A Mathematical Voyage of 20th Century America

Curriculum Unit 79.02.07
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Part IA. Introduction

This “voyage” is planned to provide an integrated body of resource material for a high school general mathematics course. The integrating theme, 20th Century America, provides the general area to be considered and five “focuses” supply the specific subtopics from which mathematical data to be used in class will be drawn. The five focuses are (1) Orientation, (2) People, (3) Science and Technology, (4) Ecology and (5) Communications.

This unit is interdisciplinary, primarily combining mathematics and history, and is planned to supplement the basic skill lessons by providing relatively integrated data that can be used in various ways throughout the year. The general plan is that the required basic mathematical skills would be taught in from two to four days a week and the math concepts would be used in the area of 20th Century American history for from one to three days a week depending on the situation.

Dividing the school year into four quarters, the use of materials might be developed as indicated on the schedule below. Once an area has been introduced, it can remain alive as an area for assignments and projects. This would allow for greater use of material and deeper development of certain topics. For example, the line graphs on United States and World population constructed in Focus Two on people could be used in a population crisis project in Focus Four on ecology.

Part II contains the five focuses, each of which contains (a) an introduction, (b) objectives, (c) general strategy and (d) a lesson plan. The historical objectives are stressed in the general strategy and lesson plan sections; the mathematical objectives are incompletely listed and are primarily for illustrating linkage between math and history. Part III contains a time line indicating the correlation of some events from the various focuses. It is to be put up in the classroom at the beginning of the year, used as a reference, and developed further as the year progresses. Part IV contains an annotated bibliography.

Possible Schedule for Using Materials

Quarter Focus and Areas of Emphasis

1st Focus One, Orientation
20th Century America
longitude and latitude
Focus Two: People
graphs illustrating population and employment trends

2nd Focus Three: Science and Technology 20th century advances
atoms
radio
straight line curves

3rd Focus Four, Ecology history and meaning of population crisis

4th Focus Five: Communications
20th century advances
classroom communication
communication satellites

Basic Mathematical Skills to be Covered Throughout the Year

a. Counting concepts and place value
b. reading and writing large numbers
c. addition, subtraction, multiplication and division of whole numbers and fractions
d. ordering of whole numbers and fractions
e. ratios and proportions
f. maps, charts, tables and graphs
g. mean, median and mode
h. percent
i. addition, subtraction, multiplication and division of decimals
j. exponents
k. geometric concepts
l. word problems
m. measurement
Part IB. Theory Behind Unit

Many of the students in a high school general mathematics class have been given years of work with addition, subtraction, multiplication and division of whole numbers, decimals and fractions. They have repeatedly encountered these and a few other basic concepts and have retained relatively little. My last year’s general mathematics (ninth grade) students began the year testing on the average at a fifth grade level. This unit is an attempt to help such students take a fresh look at some of the concepts with which they have had so little success and so much frustration.

In this curriculum unit the mathematic skills studied would be related to a particular area, history, consistently throughout the year and the idea of conceptual structures, or schemas, would be used as a teaching-learning tool in as many ways as time allows. Both ideas are attempts to move away from the discrete presentations of separate mathematical skills and applications and towards greater integration of mathematical concepts and of mathematics with other disciplines.

The use of conceptual structures, models of the abstract, seems to be one of the most promising teaching-learning tools being developed. Conceptual structures are sometimes called schemas, cognitive maps, large-scale representations, frames and other similar names. The following four references may be helpful in understanding the meaning and significance of this trend.

In *The Psychology of Learning Mathematics*, Richard R. Skemp explains the development and implications of conceptual structures or schemas. Schemas are complex structures built up from relatively simple structures based on primary sensory-motor activity. What holds these structures together is an understanding of the relationships among the various parts—it’s all related, it fits together, it makes sense.

The schema has two main functions—to integrate existing knowledge and to provide a reference frame for new concepts. If a new concept is valid for a particular schema, it will fit; if there is no schema for which it is valid, then the concept must be memorized as a separate concept or forgotten.

Skemp suggest that one of the causes of anxiety and failure in the mathematical-learning situation is the authoritarian teacher. Students often will accept and memorize what they are told without really understanding it. In order for a student to understand a concept s/he must have the schemas necessary for comprehension present and available—otherwise it will be just rote-learning, not schematic-learning. It is often difficult to tell rote-learning from schematic-learning. A bright and willing student can quickly memorize a lot of elementary mathematics, but it will be only a matter of time before the student’s confidence and progress will slow down or stop. The reasons for this are (1) as the mathematics becomes more advanced and complex it imposes an impossible burden on the memory and (2) the memorized concepts will not allow the adaptation needed. Schematic-learning is both more adaptable and easier on the memory.

In “Large-Scale Representations: Some Characterizations and Educational Implications,” John Eliot lists three educational implications of conceptual structures, which he calls large-scale representations. (1) The study of such structures is becoming an important new dimension in our understanding of thinking. (2) The acquisition and refinement of these structures may be more related to a student’s comprehension and attitude than we suspect. And (3) careful study of a student’s errors in solving large-scale problems could help us better understand the student’s style in problem solving.

In “Conceptualizing the Structures Underlying Cognitive Behavior—The Usefulness of ‘Frames,’” Robert B. Davis explains that the study of the learning of mathematics today is very different from the study of
mathematics learning twenty years ago: one of the main reasons for this is the development of a cognitive science. The rise in the use of computers has been influential in this development. Davis states that recent studies in mathematics learning have been concerned with (1) identifying “bugs” in student procedures and (2) using metaphoric models of human information processing. It is the second area which is most relevant here.

He holds that the development of such metaphoric models in mathematics is analogous to the physics/chemistry metaphor models of atoms, molecules and other particles. Such models have been valuable in physics and chemistry; and he suggests they may prove quite useful in mathematics too. He identifies and discusses eleven main hypothetical mechanisms of human information processing, one of which is the schemata or frames.

In Freedom and Beyond, John Holt writes about freedom, choice, authority, education and society. He makes the point that the better job we do in education, the more we will have to face the students’ awareness of the connection, or lack of connection, between what is happening in school and the larger reality outside the school.

Though Holt does not directly address the problem of conceptual structures as Davis, Eliot and Skemp do, he does address it indirectly by trying to spell out the relatedness of freedom, choice, school and society and the consequences when this relatedness is not recognized or doesn’t fit together.

Though each of the above references is different from the others, they share a common commitment to concepts and exploring the relatedness of them. Eliot and Davis are primarily addressing the cognitive dimensions. Holt suggests that we take a serious look at our own and our culture’s reference frame pertaining to freedom and choice, school and society and to ask if it all fits together, if it makes sense. Skemp very clearly states that schematic-learning in any area helps a person to better understand, predict and control his physical environment.

The implications for planning a curriculum unit are as follows. (1) If we begin to think in terms of schematic-learning then we will have to become more aware of the reference frames needed for whatever we want to teach. It’s a different way of saying prerequisites—a way that can begin to impose a structural order that could be useful in visualizing relationships and helpful in reorganizing and evolving them. And (2) it can help in analyzing classroom problems like what is happening when you see a student seriously struggling to multiply two decimal numbers by keeping the decimal points lined up. Suppose you say, “Keep the decimal points lined up’ is the rule for adding and subtracting decimals, not multiplying.” And then the student joyfully responds, “Oh, now I know. I have to count these:” Rut does s/he “know” now or have you helped the student select one of two not-so-well anchored concepts? This could prove to be an infinite loop, ineffective and demoralizing for both student and teacher.

As we develop and use conceptual models we will at a minimum be able to identify with greater precision where a problem begins and either be able to get effective remedial help for the student, if it’s available, or at least have it in our reference frames to include the earlier steps needed to understand adding and multiplying decimals the next time around. Good teachers have always done this. Skill in using schemas could make it easier and more efficient to do so.

There is one last aspect of conceptual structures that I think is relevant to mention here. This is that there can be structures of conceptual structures—super-structures and hierarchies of these. This is one way we can consider looking at Holt’s concerns about trying to get some consistency in what we say and how we act at
various levels concerning freedom and choice, schools and society. He is asking us to look at a large superstructure of society and to focus in on a few chains of substructures—looking for consistencies and contradictions and consequences. We can use these structures to impose a working order on whatever we are concerned with, from religion to adding numbers. Once we have a working order, then we can look at consistencies and inconsistencies and consider change.

A lot of work has been done in this area and, as a mathematics teacher, I can see part of my future work being (1) learning to use the structural analysis of various mathematical concepts that have already been worked out—those relevant to the classes I teach and (2) developing others as seem appropriate. The more visual we can make the abstract, the more clearly can we understand what we are dealing with and the more exactly can we check our communication with students. John Seely Brown in “Diagnostic Models For Procedural Bugs in Basic Mathematical Skills” gives examples of such models only in a more elaborate development that would be practical for classroom work.

Mathematics is the discipline that helps to order all other disciplines. As fascinating as the human skeleton may be, it is even more wondrous when part of the whole human body. Making a consistent effort throughout the year to relate mathematics and history is an effort to move in this direction. Using conceptual structures as a teaching-learning aid in as many ways as possible seems to hold promise. Perhaps the combination will be able to reach a few more of our “turned off” high school general mathematics students.

**Part II. Focuses**

*Focus One: Orientation*

I. Introduction

We are all located in our own time and space and this “voyage” has its own time, the 20th century, and its own space, America. The purpose of this focus is to orient students by working with (a) some of the relationships found in the Milky Way Galaxy, our Solar System and Earth—with lots of large numbers and mathematical possibilities, (b) the location of any point on Earth using longitude and latitude and (c) time measurements pertaining to the age of Earth, the length of recorded history and the meaning of a millennium and a century.

II. Objectives

A. Mathematical: student should be able to
   1. accurately read and write large numbers
   2. round whole numbers to specified place
   3. use ratios and proportions
   4. make scale drawings
   5. use longitude and latitude skills
   6. draw and label a circle, ellipse and sphere
   7. use basic measuring techniques for time and linear measurement
   8. estimate calculations
   9. optional: use scientific notation
   10. optional: use astronomical units

B. Historical: students should be able to
   1. fill in a blank time line by 100-year periods going from 2000 B.C. to 2000 A.D. and label each century
2. be able to state the meridians of longitude and parallels of latitude for the United States, give longitude and latitude for specified places and locate places in the U.S. given latitude and longitude

III. General Strategy
   A. Time: 2 days a week / 4 weeks
   B. Specific plans and activities
      1. Area: galaxies, solar system, Earth
         Introduction: use (a) three slides—one of a galaxy, one of our solar system and one of Earth (b) student information cards with data about planets’ sizes, distances from sun and length of revolutions and (c) a mobile of our solar system
            Activities:
            1. answer questions about planets from information on student cards
            2. draw an illustration showing the order of the planets going away from the sun
            3. make a scale drawing showing the relative sizes of the planets
      2. Area: longitude and latitude
         Introduction: is today work with circles (see lesson plan below) and 2nd day work with maps
            Activities:
            1. cut out, fold and solve problems pertaining to circle
            2. draw a map of the world and indicate on it lines of latitude and longitude and specific locations asked for
      3. Area: time
         Introduction: review number chart to recall place value idea going to trillions, then get oriented by making a time line indicating the oldest things the students know about
            Activities: Ask students how could we go about constructing a number line that would represent the beginning of the Earth and the beginning of recorded history. As a class work together with these ideas. Then have class construct time lines going from 2000 B.C. to 2000 A.D. and to divide it into centuries and millennia

IV. Lesson Plan: Locating Any Place on Earth (longitude & latitude)
   A. Previous work needed: the Earth is spherical, kind of like a ball but slightly flattened at two ends, the North Pole and South Pole, and it has one area where the sun’s rays hit directly all year, the Equator.
   B. 1st Day: work entirely with the circle. Work with paper and cut out a circle. Fold near end and get a chord—a line touching two points on the edge of the circle. Now fold to get a chord going through the center of the circle. How many even parts do you get from just the last fold? Problem: how many chords, going through the center of the circle, do you need to divide the circle into 360 even parts? If things go quickly and someone can convince me and the rest of the class what the answer is, then we can move on to possibly talking about Claudius Ptolemy who in 150 A.D. had the idea of dividing the circle into 360 equal parts, or we might work with this activity looking for a rule to state what we did by setting up a rule (function) table. If it doesn’t go well, we will keep struggling and I’ll show them by the end of class.
      2nd Day:
      1. Briefly discuss dividing the circle into 360 degrees and what we would have half-way around and a quarter of the way around.
      2. Review what is special about the Equator and develop lines of latitude—probably using a styrofoam ball cut appropriately.
      3. Look for some kind of natural way to mark lines going north and south—something
comparable to the Equator. If we can’t find one, what’s the next best thing to do? Develop the
ides of the prime meridian and meridians of longitude.

4. Project: students are to make a sketch of the world, put in lines of latitude and
longitude and locate several places.

C. Words to emphasize:
meridians of longitude cartographer
parallels of latitude geographer
Focus Two: People

I. Introduction

It seems appropriate to allow time to focus on people. In this area I chose to consider people in view of numbers, or population, and employment. Population was chosen because the population crisis is sufficiently serious to be ranked by the United Nations to be equal in importance with the problems of world peace, peaceful control of atomic energy and human rights. Employment was chosen because high school students are continually making decisions that will affect their current and/or future employment.

II. Objectives

A. Mathematical: student should be able to
   1. write ratios
   2. make bar, line and circle graphs
   3. optional: use set notation

B. Historical: student should be able to
   1. make a line graph showing the United States population for each 10-year period from 1900 to 1970
   2. make a line graph showing the world population for each 10-year period from 1900 to 1970. Indicate on this the predicted population data for 1980, 1990 and 2000.
   3. optional: project on 1980 census

III. General Strategy

A. Time: 2-3 days a week / 4 weeks
   2 weeks each for population and employment

B. Specific plans and activities

In both areas, population and employment, give a library assignment the week before. If two days for two weeks are used, the first day can be used to work with and organize data (see lesson plan below). The second and third days of the allowed four days can be used using the same data to make different kinds of graphs. The fourth day would be spent primarily interpreting data and evaluating the different kinds of graphs used. The basic format would be the same for both population and employment.

IV. Lesson Plan: Population

A. Library assignment given previous week: (1) go to the library and list the number of people in the United States in 1900, 1910, 1920, 1930, 1940, 1950, 1960, and 1970, List source. (2) Do exactly the same for the world population for each of those years. (Give students a handout which has on it the assignment, the librarian’s name, various possible sources and blanks where they are to list data including sources.)

B. 1st Day: Main objective is to make data more visual using graphs but on the first day we’ll work only on the data. Compare data and sources. Agree upon a list of numbers and round to nearest million or billion as agreed upon. If do not complete in class, give for homework the task of completing.

2nd Day: Use U.S. data. What two factors do we know about population? Time and numbers. Work out appropriate scales for time and numbers. Make horizontal and vertical bar graphs together. Assignment for homework is to use same data and scales to make a line graph.

3rd Day: Go over line graphs from homework. Determine appropriate scales to use for world population numbers and time. Repeat as on 2nd day only using world population data.
4th Day: Go over line graphs from homework. Try to get some ideas on how we can use the information we have to predict the world populations for 1980, 1990, and 2000. Get a consensus for the three predictions and then compare with official predictions.

Focus Three: Science and Technology

I. Introduction

The 20th century has been overwhelmingly crowded with scientific and technological advances. Einstein’s theory of relativity alone effected our understanding of the motion of the electrons in the atom, the planets in our solar system and the galaxies in distant space. This century also saw the first airplane, radio, A-bomb, computer, automobile and moon landing—to mention only a few others. I would like to spend time making a class time line showing 20th-century science and technology events that students can come up with, doing classwork and written reports on the radio and atoms and end this section illustrating a relationship between generating curves from straight lines and the patterns formed by orbiting planets, a beautiful lesson I recently saw demonstrated by Professor Kathy Kharas from Mesa Community College in San Diego.

II. Objectives

A. Mathematical: student should be able to
   1. construct line curves of various degrees of complexity
B. Historical: students should be able to
   1. List five significant advances that happened in science in 20th-century America
   2. prepare a report on the radio or one of the other significant advances
   3. List three ways our understanding of the atom has advanced
   4. answer the question “What did the film Infinite Design illustrate?” (The film is an excellent example of 20th-century technology in film making and computer use along with having a message.

III. General strategy
   A. Time: 2 days a week / 8 weeks
   B. Specific plans and activities
      1. Discuss what we mean by science and technology and as a class construct a number line illustrating when many 20th century events took place. Give relevant math problems. (about 2 periods)
      2. Have students do some research on the radio—enough to have a minimum of two facts before we discuss it in class. In class pool knowledge on radio and develop some questions and problems. Discuss reports each student is expected to do and how to go about it. Some students may give their reports orally and therefore not have to hand them in. (about 7 periods)
      3. Work in class on what an atom is, what are its component parts, when did the theory of atoms develop and what are some of the contributions to our knowledge of atoms from this century. ( about 4 periods)
      4. Work with straight line curves. See lesson plan below. (about 3 periods)

IV. Lesson Plan: Straight Line Curves
   1st Day: have students construct curves with straight lines establishing the rule to connect P with P’ and continue on one place at a time for a circle, angle and a kind of pulled-over angle. See a, b and c in illustration below.
   2nd Day: do as 1st day, only using a variation and doing so with perpendicular lines and sections of a circle. See d and e in illustration below. Homework assignment: using a circle divided into 48 equal parts (given to them—see f in illustration below), connect P with P’ using rule “move P one unit and P’ two units each time.”
   3rd Day: view results of homework. Show film Infinite Design. Discuss film and how it relates to making straight-line curves. Discussion could range from very simple to complex.
Focus Four: Ecology

I. Introduction

Although a considerable amount of ecological knowledge was known prior to 1900, it was not until the 20th century that ecology emerged as a distinct science. Ecology, which deals with relations between organisms and their environment, has developed into many subdivisions such as population ecology, eco-system ecology, marine ecology, terrestrial ecology, human ecology and social ecology. This discipline probably will increase in importance as we try to develop patterns of living that are harmonious with, and not destructive to, our environment. Energy sources, population crises, holistic approach to nutrition, pollution control, environmental protection and endangered species are some of the major ecological issues with which Americans are currently concerned.

Straight line Curves

I have chosen to work with the population crises here and plan to develop one other ecology unit in cooperation with the class during the third quarter.

II. Objectives

A. Mathematical: student will be able to

1. calculate crude birth, death and growth rate when given appropriate data.
2. find the mean growth of United States population for a given ten-year period: same for world growth.
3. do s nutrition-cost ecology project that relates to students own eating habits

B. Historical: student will be able to

1. give an acceptable definition of ecology and state the century in which this science came into existence.
2. state at least two reasons why the world population crisis is considered part of ecology
3. state the approximate years when the world population reached 1, 2, 3 and 4 billion
4. define population density and state why such data can be misleading
5. use a table to find the life expectancy for men and women in different countries.

III. General Strategy

A. Time: 2 days a week / 4 weeks
B. Specific plans and activities

1. 1st week—birth, death and growth rates
   2nd week—life expectancy and population density
   3rd week—population crisis
   4th week—time spent on what is being done and what can be done
2. For weeks 1, 3 and 4 the general plan would be the first day to introduce the specific area and work with related problems. The second day would be for further work with related problems and time to discuss the meaning and implications of data being used. For the 2nd week
allow each of the topics a day.

IV. Lesson Plan: *Population Crisis*

A. Previous work: reading and writing large numbers, rounding off numbers and using them to make U.S. and world population graphs.

B. 1st Day: It is estimated that civilization began about 20,000 15,000 B.C. From the beginning of civilization to the time of the Roman Empire, it is estimated the average world growth rate was 1000 per year. (On board roughly illustrate with a time line.) The first question is: given the data we have on the world population in 1970 and 1977, can we determine an average world population growth rate for the 1970s? If not, why not? If so, how so we so it? (Let students work together or alone to figure it out.) Look at answers obtained and agree upon an average growth rate for the 70s. Compare this with the pre Roman Empire rate. Next question: using the rate we just determined, about how long does it take us today to increase the world population by 1000? Let them work on it and take it for homework.

2nd Day: Get homework answers and work out problem in at least one way, maybe more if there is interest. (The answer is about 8 minutes.) Discuss results. Try a simple simulation game if any time remains.
Focus Five: Communications

I. Introduction

In “The Human Side of Tomorrow’s Communication” Lane Jennings discusses many current and predicted developments in communication. From the 1972 launching of Pioneer 10 which carried a message-plaque and is expected to last for hundreds of millions of years in interstellar space, to conversing with porpoises, chimpanzees and gorillas; from the radio, television, domestic satellites and optical fibers (light pipes for laser beams) to ten-second or less “super-short” TV commercials; body language as a regular part of school curriculum and thin-screened televisions flat enough to hang on the wall, he presents many possible topics from which to develop math problems and projects. We could add the findings of psychology and sociology, advances in newspapers and other periodicals, travel, computer use and many more. I chose to work with communication satellites and classroom communication.

II. Objectives

A. Mathematical: students will be able to
1. keep accurate record of his/her own grades
2. using mean and percent determine own grade
3. use newspapers in library or at home to do various projects and answer questions pertaining to (a) shopping for food, clothing, household items and cars, (b) events from the sports sections, (c) changes in stock market and (d) current events in which mathematics is used
4. keep a record of how many hours of television s/he watches and what programs for a two week period
5. use data gathered in 4 to determine total average hours of TV watched per day by students in that class and to classify programs into entertainment, violence, sports or news to determine what percent of each we watch

B. Historical: student should be able to
1. give an acceptable definition of communication
2. to list five 20th century advances in communication
3. state what a satellite is and why it is an important communication issue today
4. List three ways mathematics helps us better understand history

III. General Strategy

A. Time: 2 days a week / 8 weeks

B. Specific plans and activities
   1st week—establish record-keeping and grade system to be used last quarter and practice it
   2nd and 3rd weeks—assignments in newspapers and other periodicals
   4th, 5th and 6th weeks—TV assignments, general history of communication this century, and two days on satellites alone
   7th and 8th week— determine grades and evaluate communication between teacher and student and among students.
IV. Lesson Plan: History of Satellites (1 Day)

In class begin by asking what a rocket is. Either by teacher or interested student a simple sketch of a rocket’s general shape, simple combustion chamber and fuel part should be put on the board. Next establish what a satellite is and give examples of both natural and artificial satellites.

Go through the history of the Chinese using rockets to defend themselves as far back as 1232. They were used as military weapons from then on until the War of 1812 (where under rocket attack Francis Scott Key wrote the words to the “Star-Spangled Banner”) and the Mexican War of 1814, when artillery cannons were used more. Rocket were still used for flares to light battlefields, as distress signals and fireworks. Give dates and data of Dr. Robert Goddard, American physicist, experiments. Put them on board. Bring in Count Wernher von Braun and Sputnik.

Classwork Using dates and numbers listed on board, make up a math problem. For example: “How many years was it between the Chinese first use of rockets to defend themselves and the sending up of Sputnik?” or “Dr. Goddard’s 1935 rocket went about how many times as far as his 1926 rocket?”

The 2nd day would use same format but cover the history of communication satellites.

Part III. A 20th Century Time Line

(figure available in print form)

Part IV. Bibliography


Davis, R. and McNight, C. “Modeling the Processes of Mathematical Thinking.” *Journal of Children’s Mathematical Behavior*, Spring 1979. This is a revision of “Conceptualizing the Structures Underlying Cognitive Behavior—The Usefulness of Frames” that Dr. Davis presented to the AERA annual meeting in San Francisco, April 1979.


