Yale-New Haven Teachers Institute®

Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
1980 Volume IV: Language and Writing

Practicing Precision: Lessons from Mathematical Language and Writing

Curriculum Unit 80.04.05
by Helen Sayward

INTRODUCTION

We teachers of mathematics have our work cut out for us today. In this present world of casual attitudes and selfgratification, precision and pride in one's work have notably vanished. The media, prices of consumer goods, and the unemployment rate all urge our young people to live for today and stop working toward or planning for the future. Presenting mathematics as a solid foundation for is, therefore, an unimpressive argument. To permit our students to fail continuously under these circumstances is to neglect terribly the development of their minds We must begin employing new tactics in teaching mathematics to begin to make the subject matter meaningful and intrinsically valuable in our students' eyes. Part of this fresh approach may involve devaluation of strictly practical applications such as those found in "consumer" math courses. We must begin to stress competency, precision and pride in one's own capabilities. Students from innercity schools are often youngsters already burdened with a complex array of medical, social, and emotional problems, as well as learning disabilities. Therefore, they are often not motivated to try a subject which will need considerable effort to master. Additionally, many of these students are used to failing and will withdraw from a situation where they will risk repeating the failure experience. Since they can no longer see any future for themselves, they will usually choose experiences which offer immediate gratification and will cooperate only in those plans which offer short term goals. I feel that teaching mathematics under these circumstances is going to require incredibly secure planning and presentation on the part of the teacher. In essence, we have to teach an affection for a rather pretty but demanding branch of knowledge to students who often feel burned out, have experienced very little affection themselves, and who are not easily turned on by the beauty of the mind.

OBJECTIVES

What I hope to achieve in this curriculum unit is the presentation of a unified package of devices for achieving the goal I outlined above, specific exercises designed to turn students on to the beauty of mathematics and the life of their own minds. Before outlining specific procedures for doing this, I need to discuss some special problems and circumstances.

I feel that it really helps a teacher of mathematics to view mathematics writing (such as word problems) as a student sees it: as a mysterious foreign language. Mathematics writing is unique, with its combination of words and symbols and special, compact style, and inner city children are often terribly disadvantaged when it comes to communications skills in almost all areas. This is particularly devastating in learning mathematics where every word must be precisely read (or written) and understood. Students need experience in dealing with math topics in more than one form; for example, some math writing is all writing, but other math, like equations, is all symbols. Students must become adept at "translating" from words to symbols as they search for meaning and solution. Additionally, reading mathematics equations has a unique directionality, quite different and frustrating for a newcomer to mathematics. For example, correctly reading this illustrated equation must follow the direction of the arrows:

(figure available in print form)

One main objective of this unit is to help students feel confident around mathematical language and writing style. A measure of achieving this objective can be sought in your students' increasingly improved attitude and involvement in accepting the challenge of understanding word problems. This has to be a first step in developing the confidence to attempt to find the proper function necessary to solve the problem.

Another unique feature of mathematics as a foreign language is its vocabulary. Some words are special to mathematics, some are borrowed from ordinary usage, and some are familiar words with new and different meanings. Another objective of this project will be to help students increase their mathematics vocabulary and develop a more discriminating attitude toward the contextual use of words. Mathematical vocabulary's departure from ordinary usage can be illustrated in the following example

word	ordinary meaning	mathematical meaning	
prime	to prime a pump	a prime number has itself and one as factors	
power	God has great power	two raised to the	third power

It is also true that many mathematical terms are commonly used in everyday language. For example, the following concept words:

word	concept
negative	a loss, a penalty, losing ground
positive	a gain, a reward, a raise, an increase
volume	the contents, the fullness
set	a collection of items, a pair

Mathematics language and writing tolerates no ambiguity of expression or longwindedness of style. Every word in a mathematics word problem has an important function in the overall meaning and provides essential clues for how to solve the problem. Students need to have pointed out to them that much of the correct approach to solving word problems involves getting in tune with this clean, precise style. First exercises in this type of appreciation and recognition can be literally translating mathematical word sentences to symbols and conversely translating equations to words. An example of a translation from words to symbols is

What is ten percent of three hundred?

? = 10 % x 300

Putting the translated symbol or numeral directly under the corresponding written item is essential for the success of this type of exercise, as precision is one of the main goals of this type of learning. This involves identifying that “is” means equals and “of” means times. Also demanded is the correct recognition of an unknown (?) which could be replaced with a variable symbol (x, y, etc.). Proper transcription of words for numbers to numerals is essential for answering of course; not surprisingly, many of our students cannot reliably write the numerical equivalent of numbers in words, especially decimals, mixed numbers, and fractions.

More reading must be an objective of learning to love and appreciate mathematics language also. Recreational reading which has high mathematics content is not hard to find. Understanding ERA s in baseball and gleaning the statistical information from surveys are two ordinary avenues for practice.

Increasing interest and motivation is an important aim also. Students must be led to feel excited and successful in understanding something new. Almost all of my students have strong competitive urges, once drawn out of themselves long enough to become involved. Activities involving mini computation races, logic word puzzles and other such exercises are really helpful in this area and will be discussed in more detail below.

Our students in the lowest math learning groups have a negative mindset against any kind of printed material given to them in class. We must break that mindset by requiring our students to read and begin practice in writing by requiring them to take at least some notes in class.

STRATEGIES

There are many vehicles for teaching mathematics; the ones which I will outline here are not brand new, but I feel that their combined presentation can make a useful contribution to furthering a student s competency in mathematics. The following objectives list things I feel should be achieved by the lessons and classroom exercises presented in the concluding one third of this project.

Objective One

Learning to feel confident around mathematics language and writing style. Facing a piece of mathematics writing, especially a word problem, is an instant turnoff for many students; they automatically assume that they can’t possibly understand it, not to mention find a solution. Reading aloud is a very helpful technique for overcoming this difficulty. I think that it has been a mistake to get away from this practice in our classrooms. The process of reading the entire passage is therefore stressed as an automatic response to encountering a piece of writing. Even though this sounds extremely obvious, it is an important instruction to the student, and one which parallels another big reality: just as your attendance at school is essential for learning, likewise getting into the middle of the problem by reading it must happen before you can search for a solution. Reading together or individually helps the teacher to pick up key words and offer explanations as well as introduce new vocabulary words. Writing notes in class should be a strict requirement also; several special skills which badly need practice are involved in transferring knowledge from concept to writing. A challenging

exercise and one which can help a teacher evaluate her students' facility with mathematical language is the a CLOZE procedure which works in the following way. Select a passage from a mathematics textbook, or other writing. Remove every tenth word from the passage and have the student fill in the missing words. Increased skill can be exhibited by the ability to complete the exercise with every eighth word missing, etc., in decreasing order.

Objective Two

Help students increase their mathematics vocabulary and develop facility with the contextual use of words. Much of improving a student's mathematics vocabulary and usage lies in the student's memorizing word meanings, and the teacher's insisting that he do so. At present so many areas of formal learning have been labeled "not important," that we are reaching really low content in some subjects. Vocabulary should be written, memorized, and tested regularly. Word search puzzles, logic word games, and the a CLOZE exercises are very helpful here.

Objective Three

Learning to read more for enjoyment and increasing comprehension. Reading for enjoyment is very a student reach his fullest potential in seeking knowledge. A young person will develop intellectual curiosity if he finds the learning pleasurable and interesting, and conversely, he will grow intellectually when he becomes proud of what he can understand. Finding reading materials with high mathematical content is easy. Beside newspapers and magazines, a favorite resource of mine is the type of reading found in The Book of Numbers by the editors of Heron House Publishing Co., New York (1978).

Objective Four

Increasing interest and motivation to learn. Teaching inner city students is a real challenge. They have a host of problems, as I've mentioned before, and a fairly bad attitude toward learning. The presentation and attitude of the teacher are, therefore, tremendously important in creating a learning environment. One issue of critical importance must be discussed here: Class sizes must be reduced. We must accept the fact that students with a lot of obstacles to learning have to be taught in small numbers (1015 at most). Many students have poor attention spans and severe behavior problems; poor attention can be partially addressed by changing direction frequently in each lesson, as much as three times a period, if necessary. Part of each lesson in mathematics can be spent reading aloud, part drilling basic skills (like the multiplication tables), and part can be spent on a small recreational project like word search or function games, or a long range plan like building a mobile, etc. Severe behavior problems have to be dealt with through a lot of individual attention. Here I plead with the administration of our schools to give us smaller classes so that more time may be spent teaching and less time spent on discipline.

Objective Five

Learning mathematics should help develop your students' minds in Precise and logical thinking. Problem solving skills are very difficult to master and are really a culmination of the capabilities earned in the previous objectives. Reading a word problem correctly, understanding its vocabulary, realizing the tremendous importance of every word, and comprehending what is being asked for are four enormous tasks. All four involve definite skills which rely heavily on a student's self discipline and the ability of a teacher to turn her students on to being proud of errorfree work and zero defect thinking.

CURRICULUM CONTENT

This curriculum unit is designed for approximately a five week duration. It is arranged in five oneweek cycles; a structured plan was chosen as a reinforcement of the structure and orderliness which is the foundation of mathematics. A unit like this could be best used at the beginning of a semester as a mental “shifting of grain” to mathematical thinking as well as a review of basic skills, concepts, and functions. Presented as an intensive review and preparation, it could be very helpful in insuring success further on in the semester’s work, much like the facilitation principle of warming up one’s muscles before an athletic event. The interior structure of each cycle is designed to permit occasional elimination of single parts of each day’s lesson. Basically, the plan for each day of the one week cycle will have three parts.

Part I

Each day will begin with some exercise in drilling basic facts which should be memorized, such as the multiplication tables, percent to fraction conversions, decimal and fraction equivalents, English and metric equivalents, and measurement, volume, area etc. conversions. It is extremely difficult to get students to participate in repetitive review such as drills and memorization exercises over a long period of time. This is the advantage in setting aside one or two five week periods per school year where drilling is done intensively; telling the student that this special type of learning will occur for a finite period of time helps a great deal. These skills, well learned, will give the student an enormous edge in approaching the specific mathematical material to be covered in the rest of the semester. Drilling, especially multiplication and measurement tables, introduces a certain speed, dexterity, and rhythm to thinking which is useful in a pattern oriented science like mathematics. Confidence ~ gained by really knowing the basics, and a lot more attention can be focused on new concepts without having to stop to correct lowlevel errors. The main obstacle to overcome is our students’ views of drill as boring and infantile. Stressing its limited duration, varying the way the drills are done, and feeling excited and proud of their accomplishing the task are the three ways a teacher can make this practice successful.

Part II

Part of each day should be spent reading aloud, with the exception of Wednesday. This special day should be saved for ongoing work on a special project and drill only. More about this arrangement later.

Reading aloud, individually or in a group, is really an invaluable way for you to get to know your students’ facility with printed words and their potential difficulties with comprehension. Reading teaches them the aggressive approach to solving a problem, enabling a teacher to take time out to introduce new concepts and give practice in seeing mathematical language in context. Reading should be accompanied by notebooks; writing notes about what is encountered in the reading is important for putting the material learned into your own words as well as learning neatness, good writing style and good work habits. Notebooks should have a safe repository in the classroom and should be checked periodically. A lot of the student’s grade should be dependent on his keeping neat, accurate notes (and not losing his notebook). While there is no intellectual value in keeping a neat notebook there is a great need for following format instructions and achieving a measure of neatness. Illegible notes cannot be read or studied later.

Part III

This part of the daily plan includes small recreational projects like puzzles or number games. Part I should

occur every day for five weeks. Part II should happen every day except Wednesdays, which are special subject days. Part II items can be used at the end of the lesson or not, depending on the progress of the reading and note taking and confusion around the beginning of the school year schedule. These final little exercises are fine for being dropped out or sent home, or as assignments during this time of heavy review building skills. The best things for this category are 1015 minutes in duration, varied, and sufficiently challenging. Materials like these are also great substitute work for the sudden appearance of new students. Plans which stress order and present a fairly steady stream of work are two facets of mathematics class which students should learn to expect.

CLASSROOM ACTIVITIES

Classroom activity suggestions are divided into those suitable for parts I, II, or III of the lesson day and are presented within each part in order of increasing difficulty. My suggestion is to use the most simple exercises in the first week of the five week cycle and increase the difficulty in a straightforward way.

Part I: Drill and Basic skill exercises

1) Use flash cards to get the class to respond individually or as a group to the following types of exercises. (A) Multiplication facts (week one). (B) Percent to decimal equivalents (week two). (C) Percent to fraction equivalents (week three). (D) Area, measure, volume conversion in either the English or metric system (week four). (E) Basic concepts, vocabulary words (week five).
2) Write the informational content with random missing item on the board; have students supply missing items verbally or in writing. To illustrate, an example for category (B) above would be
1% = .01
2% = .02
3% =
4.5% = .045
5.5% =
6% = .06
% = .07
3) Instruct the students to draw, for example, their own multiplication master table (week one) in a special reference section of their notebook.

Part Two: Reading aloud and taking notes.

Reading aloud material can be from several sources. One good and universally available source is all the special career capsule or special topics sections of your student's text book material for the year's course. Difficulty of material and content is not uniform in these exercises. One way they can be made useful as comprehensive exercises is by using the CLOZE format: copy the passage eliminating every tenth word. A criterion for readability of a text, incidentally, is its ability to hold together contextually in this exercise. Present the passage to students and have them supply the missing words (week one). Weeks two and three can be spent reading similar passages from the textbook or recreational mathematics material, textbooks, etc. and copying notes and vocabulary word meanings to be memorized. An excellent resource for one to two page treatises on special topics is the Book of Numbers. The entries in this book have brief (two pages) statements on interesting topics like makeup habits around the world, alcohol consumption, the certainty of a third world war and world wide opinion on these topics arranged in a data table.

Week four techniques for tackling word problems can be begun here. Volumes on the methodology of problem solving have been written. For low level or average students the direct approach is the best. Issue the student a page of word problems. (1) Read a problem together as a class. (2) Underline or circle the part of the problem that asks the question (identification). (3) Write down what has been asked. (4) Write down given facts (data).

In week five issue a page of word problems and have the students do the following: (1) Identify the question asked (often just involving finding the question mark and adjacent words) (2) Write the data given. (3) Choose the correct function (look for clue words, deduct means, subtract, increase by mean, add, etc.). (4) Write an equation. (5) Solve the equation. (6) Check your answer for reasonableness.

Part III: Small Recreational Puzzles and Games

These are great for teaching students alternative ways of thinking, creativity, and competitiveness. Dangers are the temptation to overuse them or use them at random. The math learning atmosphere should be one of order, structure, and logic; games should be presented in a straightforward fashion, not when the class is about to break down in maximum disorder.

Some good examples of games to play are: (1) Blackboard races (best for small groups of five to 10). Students stand at the blackboard and compete in calculating simple word problems presented verbally by the teacher. Knowing your students is important here. This is good for increasing listening skills and comprehension and the competition increases the individual's confidence. Usually each student has his own special talent (time problems, or making change, etc.), and some sensitivity in presenting the everyone feel like a winner at something. (2) The function game is well known. Pairs of students can play or it can be teacher versus class. The principle of the game is identifying the functional relationship between ordered pairs of numbers:

(figure available in print form)

(3) Word search puzzles can be made up for the vocabulary words currently under study. Alphabet letters are arranged two dimensionally on a page with the words hidden. Words of correct spelling must be circled. (4) Cross number puzzles are puzzles using mutually satisfying numbers instead of words for across and down junctions. (5) The geometric and spatial relationship puzzles. For example, connect the nine dots using only four consecutive lines without raising pencil from paper.

SPECIAL LONG TERM PROJECTS

Wednesday should be set aside for a special ongoing project. I favor plans centering around the class size of ten students which makes some small field trips possible.

1 One type of project I would like to see involves introduction to banking through field trips. I would like to be able to take students to a local bank, have them all open savings accounts, and get to speak to some member of the staff like a teller or loan officer on banking procedures as well as how the bank operates.
2 Inschool field trips are another A mathematics teacher could plan a series of five lessons with teachers in each of the following areas: home economics, shop, automotive mechanics, construction science, and chemistry. This would be particularly good for freshmen new to the school who are curious about these areas. Each presentation could be jointly done to demonstrate the role of mathematics in these specialized areas.
3. A third type of project which can be done in class over a period of five meetings requires mainly a safe place for half completed projects. Building mobiles is a construction project requiring precise measurement, good coordination, and a fair knowledge of fractions. To build a balanced mobile requires accurately measuring and cutting cross members and joining them according to a mathematical formula. This is necessary to build a wellhung mobile.

Annotated Bibliography for Teachers

Conference Board of the Mathematical Sciences. The Role of Axiomatics and Problem Solving in Mathematics. 1966.

A fine collection of articles presented at the International Congress of Mathematics, Moscow, 1966. Particularly interesting are papers dealing with the role of problems in development of mathematical activity in students. See especially Paul Rosenbloom's "Problem Making and Problem Solving", p. 130.

Harnedek, Anita. Mathematical Reasoning. Midwest Publications, Birmingham, Michigan, 1969.

An interesting group of written exercises meant for reading and logical interpretation. Thought provoking and challenging.

Hater, Mary and Kane, Robert. "The CLOZE procedure as a Measure of Mathematical English", Journal for Research in Mathematical Education 6,2 (1975), pp. 121127.

Land mark article introducing the renowned CLOZE procedure.

Hoffman, Ruth. "The Slow Learner—Changing his view of Nath", Bulletin of the National Association of Secondarv School Principals, 1968.

Good examples of locally made curriculum planning and pointers on how to make it work in the classroom.

Johnson, Stanley. Arithmetic and Learning Disabilities. Allen and Bacon, Boston, 1979.

A new book which could be very helpful in aiding understanding of some of the problems our students have with numbers and conceptualization. Includes some excellent diagnostic techniques and remedial exercises for the classroom.

Kohl, Hubert. Mathematical Writing and Games in the Open Classroom. Random House, New York, 1974.

Good explanation of the role of games in the learning process. An interestingly written easytoread book.

Polya, George. Mathematical Discovery. Volumes I and II. John Wiley and Sons, 1967.

Highly regarded as the definitive textbook on heuristics and problem solving methods. Really excellent and rather advanced.

Shoemaker, Terry. Involving LowAchievers in Mathematics. Jefferson County School System, Lakewood, Colorado, 1976.

An excellent collection of curriculum materials designed especially for low level students. Gives detailed instructions for a number of classroom projects and problem solving techniques.

Annotated Bibliographv for Students

Amerongen, G The Way Things Work. New York: Simon and Schuster, 1971.

A book explaining how various everyday machines and appliances operate.

Court, Nathan. Mathematics in Fun and in Earnest. New York: Macmillan Company, 1954.

A collection of interesting problems which get more and more difficult to solve as the book progresses.

Heron House Editors.The Book of Numbers. Heron House, New York,1978

A beautifully written book which gives written and graphical information on a great many topics of mathematics. Well done and enjoyable reading.

Hraitchik, Maurice. Mathematical Recreations. (second edition). New York: Dover Publications. Inc.. 1953.

Interesting collection of games type problems with cute aspects to catch the students' attention.

Wylie. C 101 Puzzles in Thought and Logic. New York: Dover Publications, 1957.

The content of the book is completely described in the title.