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Problem Solving For The Ninth Grader

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Over the years the quality of education in the United States has dropped. The national, state and local mathematical test scores have documented this decline. There are those of us who say; "It is the fault of the student", "It is the fault of the T.V," "It is the fault of the permissive parent". The excuses go on without an end.

The real problem may well be a combination of all of these things coupled with yet another problem, the teacher. Have we written the children off? Is our attitude, "They can not do it so I will skip this part of the text and go on to some less demanding material"? Many of us may be guilty of this. I think its time to evaluate some of our own teaching practices and approach the task at hand with a more positive attitude. They can do problem solving, and I am going to see to it that they do!

In the teaching of mathematics there is a tendency to stick to the book. This will always be a problem, for the texts run on the average of 500 pages. This is a lot to plow through in just 180 days. The need to get to the next page cone hell or high water, is a self imposed one. What do we do? It's time to rework the problem section of our individual texts. Present a good representative sample of the typical problems and then reach out to other sources and deliberately expose the students to more exciting and challenging problems.

There are many state and national mathematics publications which are filled with both information and techniques for problems and their presentation to a class. They can be just the thing to pick up our own spirits and help us combat the dreaded disease Which we have come to call, ''Teacher Burn out".

An Introduction (Delayed)

Let me begin by saying that this is not a six week unit in problem solving. There is no way in which I could get a group of students to sit and work on word problems for such an extended period of time. Such an attempt on my part or on anyone else's part would be a student turn off.

Thisunit will be designed as an "on Going" unit. It will not just be a few painful moments at the end of each chapter. Problems should be plugged into the classroom activities. The instructor will be the key to the success of such a plan. He must know when to feed in the right type of problem, one which will keep the class both moving and involved.

My audience is to range from the slow to average algebra I student. Many of the problems I will suggest will have an appeal to a wide range of students. My main area of concern will remain with those young men and women who traditionally have had matching problems with their reading skills.

Some of these students drift through the four years of the high school without ever having solved a word problem. This bothers me' In a one to one situation these same students are very street wise. They do not lack common sense. It is my feeling that we can tap this energy and focus it into the area or mathematical problems.

To capture and redirect this energy we will have to look for good word problems which will have them in mind. There are lots of good word problems which are appropriate for the slow to average student. Just this week I picked up a copy of the magazine "Psychology Today." In it was an article by Eugene Raudesepp entitled "More Creative Gamesmanship," It is a good source for some starter problems for the students. I have also seen a new magazine called "Games" which Playboy Enterprises puts out. It is totally devoted to puzzles, magic squares and problems. mere are a lot of ideas out there if we take the time to look for them. By making full use of these and other sources we can create a more exciting and more positive class atmosphere, an atmosphere which wi11 encourage all of the students to grow intellectually.

To prepare students for the problem solving process it would be wise to have them buy a loose leaf notebook which will be kept exclusively for this work. In this book they can record new words, definitions, problem solutions, problems in various stages of completion and sketches of their problems. I suggest a loose leaf type of book be used so that you will be able to give the students problem sheets: which can be put directly into the book and not "lost'"

In this unit the student's first encounterswith word problems will be delayed a bit until he has had some lessons in evaluating expressions and in plug in information into formulas. This type of a problem is not a problem in the true sense for the mathematical operations are clearly stated. These exercises do serve a definite purpose in that they are good practice in the observation of patterns, they give the student confidence in his arithmetic ekills and finally they do train the student to speed up his calculations.

I suggest that time limits be placed on these drills. In this way the mechanical process will not overshadow the thought process.

At this point the manipulative skills: which we have sharpened will aid us as we go into the process of translating ideas from the written or spoken word into algebraic symbols and algebraic sentences.

Most of us recall those days when we had to translate Latin into English. We had day after day of word searching and tense finding and then one day it all fell into place. The task had become second nature to us.

In looking for some good material to aid us in this task I have found that problems about percent lend themselves to quick and interesting translation. On the board we usually make a student key and the process looks like t'is. Given the question;

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It is possible to throw out a lot of this: type of problem and get the entire class working and achieving good and quick results.

Here I have to say, “Use that calculator whenever possible!!” Encourage the students to bring their calculators to class for their own use. It’s sad to see a student with a fancy calculator in his hand and he does not have the slightest: idea about how to use it. See him after school or take the time in the class to show how this: tool is to be used to make all. of our jobs easier. There a lot of calculator oriented problems to be found in the National Council of Teachers of Mathematics publications ,Arithmetic Teacher” and “The Mathematics Teacher”. Learn to use them as a teaching tool.

In trying to think of ways to present this paper I toyed with a problem time table of sorts, you know the kind of plan in which you state that on the third day we will do this or that. I think the listing looks good but it is not very useful. The idea was abandoned. In its place I will try to show you some of the things I have tried to do with my classes. I hope that you find some of the ideas useful and interesting. Most of these ideas will be presented in story or play form for that is the way I try to present them in my classroom.

In working with word problems I have found that one must sketch, sketch, sketch. Wherever possible I include my own likeness. The students love it and seem to pay a little more attention to the proceedings. Encourage the students to display their sketches for area, perimeter and distance problems. A poor sketch can lead to a dead end in problem . If you do check the sketches you will see many which will tell you at once if the student understands the problem.

I have thought about some or the difficulty which some of the students have in handling mixture problems. You know the ones: about coffee, nuts or candy and how they are selling for $1.20 a pound and they have to be mixed with another more expensive kind and eventually the entire mixture will be sold for lets say $1.80 a pound? It occurred to me that we are: using problems which made a lot of sense to us but not much sense to today’s student.

A case in point. I grew up in the city of New Haven in the 1940’s. The city was then about 70% Italian. When you walked into an Italian market the food was out in the open. It was there to be seen, touched and experienced: I used to put on ten pounds by just inhaling the aromas.

We did not get a slick package with its coating of plastic. There were, for example tour to five kinds of coffee beans in the bins for us to buy. It could be scooped out, blended to your wishes, poured into a grinding machine, and you had your very own coffee. It was two scoopsof this and one of that. God forbid if you came home with the reverse mix. Those merchants mixed candy, nuts and coffee to your taste or to the taste of your wallet.

When today’s student enters a store he finds none of what I have just described. Everything is in a box, wrapped in plastic or frozen stiff. You take it or you leave it!

So the next time you are at the board (and you are over 40) and the students do not “relate” to the mixture problems, you are just going to have to take some time out and tell them about " the way it was.”

The students will still cry that the problems are not real to them. It is my contention that it is the student who just not observant. A case in point is the ticket problem. I do not know a student who has not gone to the movies or to a play and who has not been intimately involved in the ticket buying process. They are there but they do not pay attention. Here is basically how I try to handle such a problem.

The Cross Players are presenting a Shakespearean drama in the little theater which has a seating capacity of 110. The play is a sell out. The sign on the door lists the prices Teachers $1.50 and Students $.8O. A count of the money at the end of the show reveals that the play took in $158.00. How many teachers went to the play? Silence... A lot of prodding from the teacher and still no luck. Wild guesses are always present but no answers.

To break the ice we agreed to pretend that we were actually there buying tickets, entering the theater and finally counting the money. The first thing we did was to make tickets from scraps of colored paper. One color for the teachers and another distinct color for the student tickets.

Some of the students tried to "walk" into the theater after buying the tickets. They were stopped by that ever present little man who reaches out, grabs and tears up your only souvenir of that never to be forgotten night.

Stop " Who is this man? What is his real duty in the theater? Is he just there to annoy the customer? What does he do with all of the half tickets? Here I try to put in a little reality, I need the students to have a little more input.

Suppose the gal who sold you the ticket was putting some of the money into her pocketbook. How could the manager catch her when he has to be at the door collecting and tearing your tickets? The good class has awakened! They are quick to tell me about the tickets (halves). All you have to do is to split tickets into two piles according to their color. Then each pile is counted and multiplied by its ticket cost and you add it all up and compare this to the night's take and if she is stealing you have her.

The class enjoys this but does not see the connection between the original problem and this last discussion. Here is where I can go two ways to get the idea across. My first try is as follows.

I pick up all of the ticket stubs and display them to the class. How many tickets do I have here? With out question the answer will be 110. While they are still watching I break the pile by removing one type (color) of ticket, say the student's tickets. Now holding the student's tickets over my head for all to see, I ask "How many tickets do I have here?". Silence ... Many times I get an automatic answer of "half" in spite of the fact that the original pile was designed to be heavily one type or color of ticket.

Next a leading question from the teacher. "If we have not counted these tickets then what is the only thing I can name them?" Usually the class will give back an "X!" "Now then what do I have in the other pile?" "Is it still 110?" The class will come back with the fact that it is 110 but without the X. If this is done, than we quickly mark this information on the chalk board, before the thought is lost .

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We now try to make a combination of sketch and equation which will help us visualize the problem. The tickets are put in piles to be counted and if all goes well we should be able to solve the problem.

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The reason for so few students attending the play was of course obvious; it was by William Shakespeare. Before I forget to mention it, I find its always a good idea to have the class actually take and add the 110 X and X to make sure they see it as the original 110. Do not forget to plug in the answers to see if the problem works out.

The ticket problem is one of a type and it does lend itself to the colored scraps of paper. In the event I need another tool for the same or similar problem I grab a long sheet of foolscap paper and quickly scribble a big 110 on it. Then just as quickly I tell the class to think of it (sheet of paper) as my pile of tickets. "How many do

I have?", the answer of 110 comes back quickly. I then tear off a good chunk of the same paper and tell the class, "These are the student tickets!". "Did I count them?" "No I did not". "So what do we call them?" Answer from the class "X!" Mark it this way.

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In my other hand I still hold the original sheet which is still marked 110. "Are there still 110 tickets here?" If no then why not? The class should see that it is really 110 without the X. ' It would be recorded this way.

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Here the sight of you rejoining the torn sheet should drive home the idea that the two parts do indeed account for all of the 110 tickets. The class will then complete the problem in much the same manner that was described in the previous paragraphs.

It is not the time to stop work on this problem. I do suggest that you have the students list the X as the teachers tickets and see for themselves if there is a change in the answers. This is done to dispel the thought which some students have about the teacher having prior knowledge about the problem.

When word problems are presented in story or play form more students seem to get involved. Interject a little intrigue (the cashier stealing money from the theater) and you will catch the ears of more students. Students who might be sitting there letting the "smarter" or more vocal classmates handle the work. Try to get each student involved, so that by the time the problem is over they can truly say that they too had a hand in its solution.

There are times when you would rather not be involved in the act of teaching word problems;. This: summer was one of them. The temperature hovered about the 100 degree mark and all of us would rather have been at the beach. I needed a problem which would break the monotony, stimulate some interest, and lend itself to a good clean cut and quick solution. I decided to up date an oldie. The problem concerns a dude named Clyde.

Clyde went to his corner package store and bought a bottle of their best wine for $2.10. As dudes will do he complained about the high price. The store owner agreed and to point up the fact he told Clyde that this bottle which he had just purchased cost two dollars more than its cork. Later in the evening when Clyde and his girlfriend had a lull in their conversation he mentioned his conversation with the package store owner. His gal then asked him just what the cork cost. Clyde was not a heavy weight in the math department but he gave it his best try. "Why it cost 10¢ I" Was Clyde's answer correct?

The class agreed with Clyde, the cork had to cost 10¢. When they were told this was not the answer they agreed to kick the solution around on the black board. They began by putting a very simple sketch down, and then the parts of the problem began to fall into place.

Bottle plus Cork equals $ (total cost)

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Next they added some simple facts like $ could be replaced with $ 2.10. In place of the sketch of the bottle (to which they insisted on adding a label) they were able to put in (substitute) "$ 2.00 more than the cork ".

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Its an off beat solution. The students had fun putting in Ripple for the wine label. They had a far more important experience for they got away from doing a problem with the typical x variable. I hope that by doing this they were able to see that the sketch of the bottle and its cork conveyed as much if not more understanding than the usual variables. Encourage the students to be creative in this work. They have inventive minds which should not be locked into forms. Try to get them to experience the problem solving process.

Not all word problems can be quickly explained. Some times when the class is very slow you have to resort to basic techniques. One day we took a simple problem about a man who had purchased two Sunday papers. We were given the total cost and we were to determine the cost of the individual papers. I converted the problem to one about Mr. Cochrane who went out on a Sunday morning to buy two newspapers. The cost of the papers was $2.30. This made Mr. Cochrane curious, so he inquired about the cost of the individual papers. "It's because the Times cost 70¢ more than the Register", was the reply the druggist gave him. What then did each paper cost?

Because the class was not able to translate the paper cost relationship, we had to try a different approach. The following line of thought was tried. There is some connection between the cost of the two papers. Let us put down that very simple thought. The Times is related to the Register.

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Here we stress that we are only saying that there is some kind of a connection (mathematical) between the cost of the two papers. Our next thought is can we tell which paper cost the most? Use the appropriate inequality symbol to show this.

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The next thought is also very simple. "If the two newspapers were on some device like a child's seesaw how would they look?"

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Its quite clear that the Times outweighs the Register. While this helps us, it does not establish an equality relationship between the two newspapers. The thought is not a difficult one to complete for the class realizes that the Times outweighs the Register by $.70. From their playground experience they know that to effect a balance they need to add this $.70 to the Register's side of the seesaw.

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Now to wrap up the problem. The original problem shows the two newspapers have a total cost of $2.30. We can not put this into an equation form and solve it for we will have an equation with two unknowns in it. If we take the fact that the Times equals the Register plus 70¢ and plug this in place of the T in the equation we can solve our problem. "Let us try it!"

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Again we try to get the students to relate to the work. Here we were able to draw on a childhood play experience, the seesaw, and solution flows from it. Some students have said that its like doing math with out math.

When your doing word problems there are bound to be a few favorites. Here is such a problem. It has many solutions some complex and some simple. The general information about the problem is this ; we have a square and within that square is an area formed by the overlapping of four semicircles. Our job is to determine the exact area of the shaded part.

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When this type of a problem is presented to a class most of the students will tell you that it just cannot be done. They hope that you will agree with them and go on to some other problem which will require less thinking. One is then forced to offer some sort of a suggestion or hint to the class.

"Would anyone object to studying just one "petal" of the figure?" There is usually no objection. The class is quick to realize that the partial solution would have to be multiplied by 4 to get a final solution. The class is waking up! At times a petal looks a lot like a leaf. This may aide us in our search for a solution. If we had a leaf line or rather an axis of symmetry line through the leaf (petal) would we see anything? "Lets take a good look."

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Do not forget to bring those dimensions along to keep this inset in its mathematical framework. "You know I am getting an idea but there is still too much in my sketch". "Lets just look at half a "petal." Any solution gotten from this would have to have a factor of 8 to get back to the original area. There are some old shapes here which we can all identify. "What are they?" Play with the sketch and see if YOU can come up with any ideas.

Here is a very interesting time for me and hopefully for the students. With some sketches a complete turn up side gets us to see a solution. Here is What worked this time.

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From what I can see we have two basic shapes here. It is hoped that this class (not a dull class) would pick up on the circle or rather the 1/4 circle shape and the right triangle shape. In simple terms we are looking at the area expressed as the difference between the two figures.

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As I just finished these sketches it occurred to me that the basic dimensions should be changed to another letter other than R to keep from being confused with the radius r. The R does lead the way to the problem's solution by suggesting r which is associated with the circle. Lets keep it in.

Some of the class thought the problem could be done in a better way so we went back to our Sketching. On the next attempt the figure was torn apart and reconstituted. The sketch was only one half of the original area but by now the students could add the factor of 2 with ease. Lets see what they did.

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Wow! This solution leaps off the board at you! It is clear! A circle minus a square. Is it as easy as it looks? The length of the sides of the square can be determined if we sketch in some diagonals, which will lead to the idea of right triangles, which leads to the dreaded beast "Hypotenuse." Yes the easy way looks a little less easy now that the thought of the pythagorean theorm has come up. The class is brave and it went on with a big loss of its enthusiasm. The sketch and first thoughts follow.

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The area of the circle is not too hard to determine, the usual rrr2. The determination of the length of the side of the square did require a good background in the mathematics. Do not expect that all members of the class can follow the proceedings.

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Now wait a minute! Was all this work really necessary? What are all those little shapes within the square, you see them, the ones created by the diagonals. They are right triangles, whose area we can easily compute by the 1/2bh rule. “We did not have to go and get poor old Pythagoras to help us!” Let the poor guy sleep! The parts will fit in and we have another good solution to the same problem.

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Just when I thought it was all done, a voice from the back of the room said “I could do that problem in a faster and better way.” Seeing is believing so we took a look. Her solution involved looking at just two of the semi circles and their shaded area. Her plan was to compute the shaded area subtract this from the original square, doubling the difference and then subtracting this difference from the original square for the second time. It was hard on the ears. The problem looked a lot better in sketch form as you will see. What you are about to see requires that I can not only recall it, but also that I sketch it. I am not an artist so bear with me.

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I did have a choice in sketching this and I as you see tried for the very rough sketch as a change of pace. The work on the board looks like this an in a way it makes the work and ideas flow .

This is, as I have mentioned in the sketch itself, not unlike a negative image. `The solution was backed into but it was fun and showed a lot of imagination. The mathematics was very simple .

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As you can see this is a “better yet” solution. I can see one or two of you trying for your own solution. The process is very infectious.

There is no easy way to teach problem solving. What works for the first class of the day will not work in the next class. The hardest thing to do is to get the student to do most of the work. There is an old saying by Kam Fong which says in effect “I hear, and I forget I do, and I understand”

Lets get the student to doing!!!!

Student Readings

The National Council of Teachers of Mathematics (NCTM) publishes The Mathematics Teacher, Arithmetic Teacher, The Mathematics Student. These publications contain many ideas for the student, teacher and the class as a whole. Get your hands on them and learn to read them.

Krulik, Stephen, Kaufman Irwin; Shostak Jerome The Handbook of College Entrance Examinations Pocket Books A must for the student who wants to practice for the college boards

Polya, G. How To Solve It Princeton University Press Very enjoyable book on how to solve problems, contains many good ideas and techniques.

Teacher Readings

The student readings above contain most of what I use in my class. Add to this your own classroom text.

Corcoran, Clyde; Delvin, John; Gaughan,Edward; Johnson, David; Wisner Robert. Scott Foresman and Co. 1977. Algebra I Contains a great set of problems sprinkled throughout the text under the labels of “Take A Break”.

Games A publication of Playboy Enterprises is an interesting new magazine on the market. It can be read by all levels of students. I would use it to get the students attention and then use other more serious sources to more fully develop their problem solving powers.

Brownstein, Samuel; Weiner, Mitchel Barron’s How To Prepare For College Entrance Examinations 4th Ed 1969 An old but very useful series of problems I would suggest a more recent edition of the work.

Raudesepp, Eugene “More Creative Gamesmanship” an article from Psychology Today July 1980