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Lunar Eclipse: Fact and Myth

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Mathematics studies in middle schools are generally focused on sharpening students' skills working with decimals, percents, and fractions; introduction to geometry; and basic reading of charts and graphs. The average student considers their math requirement a time filler, 45 minutes of drill, with no relationship to other subjects. This unit proposes to change that misconception of mathematics and develop lessons to be utilized as a culmination of a year's work. This unit will introduce the eighth grade student to the versatility of mathematics and its applications in other fields of study, specifically astronomy. It is imperative for students to realize that mathematics effects many fields of study, science, history, business, and the social sciences. A unit involving math and science will enable students to recognize that mathematics is not an isolated discipline.

I propose to draw upon the mathematical skills learned during the year and apply them to a unit on computing lunar eclipses. Granted, calculations used in astronomy are too sophisticated for average middle school students. However many of the skills learned, i.e. graphing, geometric constructions, solving proportions, and chart reading can be utilized so that students can achieve the desired outcome, charting and calculating the elements of a lunar eclipse.

In addition to allowing students to study an application of mathematics to science, the unit will also allow students to observe a scientific phenomena as an individual would have hundreds of years ago by reading and eventually writing a story or fable explaining the occurrence of an eclipse. The unit will cover a four week period, ideally the last weeks of school.

The material I am presenting will be new to all students. It will be a challenge to many and possibly too difficult for some. However, there are enough different steps requiring different skills that will allow each student to benefit from the unit. The intention is for the unit to be used as an end of the year activity. Students can work individually, however, students may work in groups of two or three and still benefit greatly. No one student will feel overwhelmed by the information and each student can share the information on eclipses and the folklore interpretations that they have found. The novelty of this information and the realization that much of the work can be accomplished by using skills of which students are familiar, will motivate students to learn and enjoy the unit. Ultimately, I hope students will begin to understand mathematics and its relationship to science and acknowledge the use of mathematics in solving problems outside of the math classroom.

The unit is comprised of three major portions: 1) the scientific or physical explanation of a lunar eclipse 2) computing and graphing of particular elements of a specific eclipse and 3) the study of fabled eclipses and the writing of an original fable explaining an eclipse.

I. The Physical Explanation

Students will need a working vocabulary of terms, and a basic understanding of what an eclipse is and how it occurs. Initial lessons will explain how an eclipse occurs with subsequent lessons instructing students to create a model of this phenomena. For our purposes, and our calculations, we will consider the earth the center of our system (the Ptolemian system as opposed to the Copernican system). We will use this system only as a means of explanation. Hopefully students understand that the sun is the center of our solar system with the earth orbiting the sun and the moon orbiting the earth. The moon completes its trip around the earth in one month while the earth completes its trip around the sun in one year. Occasionally the sun and moon cross in front of one another. When the moon passes between the sun and the earth so as to block the sun from sight, a solar eclipse occurs, the moon's shadow blocking out the sun's rays. The moon blocks a portion of the sun's rays causing a dark "spot" on the earth's surface. People living in the region can observe, with necessary precautions, the moon passing in front of the sun with darkness following in its wake and the eventual passage of the moon with the return of light. (Figure I) Similarly, when the sun and moon are at opposite sides of the earth, i.e. the sun and moon are in opposition (180° apart), a lunar eclipse occurs. Essentially the sun illuminates one side of the earth causing daylight while the opposite side of the earth remains dark. The opposite or night side of the earth casts a shadow, called the umbra. A lunar eclipse occurs when the moon passes through this shadow of the earth blocking it from the sun's illuminating light and consequently from sight. The moon itself casts no light and is dependent upon the sun's rays for its illumination. When the moon passes through the shadow, sun rays its source of light, are blocked and the moon can not be seen.

(figure available in print form)

Lessons explaining the movement of the sun and moon should insure the students' understanding of what is physically happening when an eclipse occurs. Students can draw diagrams or construct models using light bulbs and Styrofoam balls to show the sun and moon at opposition. Also many school science laboratories have modelsof the earth, sun, and moon that can be used to illustrate an eclipse.

The first week of the unit should concentrate on students understanding the scientific explanation of an eclipse. Students should not begin the computational work until they have a viable working image of the physical occurrences during eclipses. In addition to a working Knowledge of the occurrence, a list of working definitions should be introduced and reinforced throughout the unit. The basic terms, some of which have been introduced and those that will be introduced are listed below. Definitions and explanations will follow.

Vocabulary Terms **:**

1. Longitude 6) Greenwich Hour Angle
2. Latitude 7) Greenwich Mean Time
3. Eclipse 8) Prime MeridianCelestial Meridian
4. Opposition 9) EquatorCelestial Equator
5. Declination

II. Computing an Eclipse

I will preface this portion of the unit with a list of skills that should be reinforced before attempting the computations and an explanation of how we will determine where the sun and moon are positioned with respect to the earth.

Certain skills should be reviewed before actual computations begin. The basic and most generally used computations are:

1. changing from degrees to minutes and seconds, and the reverse,

2. solving proportions,

3. operations involving decimals,

4. setting proportions for interpolation,

5. constructing coordinate axis,

6. constructing circles of given radii,

7. scaling and marking measurements.

(figure available in print form)

Each step will be preceded by the list of skills needed to complete the step. I suggest that a worksheet that' provides adequate practice in each of the above areas be assigned each day.

In addition to skills review, students will also need a picture of how we determine the positions of the sun and moon in the sky. For our purposes we can imagine the sun and moon traveling across a celestial sphere. (Figure III) The celestial sphere will contain a projection of the earth's equator, called the celestial equator, and the earth's prime meridian, the line of longitude with degree measurement of zero, will be the celestial meridian. We can then locate the positions of the sun and moon with respect to the earth in terms of 1) Declination: The distance in degrees north or south of the celestial equator and 2) Greenwich Hour Angle: Distance in degrees west of the celestial meridian (degrees will vary from 0 positioned directly on the meridian, to 360'westwardly approaching the meridian).

When making our calculations we will assume that all days are exactly twentyfour hours long, this will be referred to as Mean Sun Time. Day lengths vary each day but use of a 24 hour day will make our calculations easier. (If all days were 24 hours long, would we need Leap Year?). Finally we will state that when the sun is directly at the celestial meridian (noon at Greenwich, England) the longitude or the Greenwich Hour Angle of the sun is O' The sun will then travel at a speed of 15— per hour (15—X 24 = 360—). For example, when the sun is 15— west of the celestial meridian, it is 1:00 in Greenwich. The hours of time we use will be based on Greenwich time and is called Greenwich Mean Time.

A. Computing Steps

The following portion of the unit includes steps which each teacher must progress through with his students. Each instructor must pick a year in which a total lunar day and time of said students with the proper eclipse has occurred, find the occurrence and then supply the students with the proper chart necessary. All of this information can be found in the *Nautical Almanac* , published by the U.S. Government. As of 1976, the style in which the *Nautical Almanac* (the source of the tables we will use) has changed, therefore I suggest teachers choose a year before 1976, i.e. the students' year of birth, their first year of school, or the year they learned to ride a bicycle etc. as the year of the almanacto choose. Although students will not be able to predict an eclipse, they will still benefit from the work involved. Also, be careful to choose a year in which a *total* eclipse occurs, the graphs are much more interesting.

(figure available in print form)

There are some aspects of calculations that will be too sophisticated for the eighth grade students to perform. Again, I will note these exceptions and suggest that each teacher supply that information to their students.

Interpreting the Nautical Tables

I have chosen as an example the lunar eclipse that occurred May 25, 1975. The information in the table below was obtained from the *Nautical Almanac 1975* .

	GMT	SUN		MOON		
DAY	HOUR	GHA	DEC	GHA	DEC	HP
25	0:00					
	4:00	240—48.2’	N 20—49.6’	61—48.1’	S80—30.3’	58.2’
	5:00	255—48.1’	N 20—50.0’	76—14.0’	S20—33.9’	58.2’
	6:00	270—48.1’	N 20—50.5’9	0—40.0’	S20—37.4’	58.2’
	7:00	285—48.0’	N 20—51.0’	105—0.9’	S20—40.7’	58.1’
		SD 15.8’		SD 15.8’		

Table Explanations:

GMT : *Greenwich Mean Time* : Time based on a 24 hour day starting at Greenwich, England.

GHA : *Greenwich Hour Angle* : States the westward position of the sun (moon) with respect to the celestial meridian. (O—longitude) (Figure III)

DEC: Declination : The north or south position of the sun (moon) with respect to the celestial equator (O—latitude). (Figure III)

HP: Horizontal Parallax: The measure of the angle formed by two rays starting at the center of the sun. One ray passing through the center of the earth and the other ray tangent to the earth.

SD: Semidiameter: Angular Radius in minutes of the arc. Information obtained from the table.

Once we understand the kind of information given in the nautical tables and we are able to read the tables successfully, work on the computing of the eclipse can begin. The following is a basic outline of the steps involved in computing an eclipse. Each step will eventually be worked through with a specific example.

Outline of Computing, Steps

I. Calculate the time of opposition: The time at which tenth sun and the moon are 180— apart.

II. *Determine the declination of the sun and moon at the moment of opposition* . Using information from step I, find the sun and the moon's declination (degrees from the equator) at the time of opposition.

III. *Radius of the Shadow* : With a given formula, we will determine the size of the shadow cast by the earth during the eclipse.

IV, *Coordinate axis* : Setting up our coordinate axes to plot the moon's motion through the earth's shadow,

V. *Determine the NorthSouth movement of the sun and* moon: The degree changes in declination each hour.

VI. *Determine the EastWest rate of change for the sun and moon* : The degree changes in GHA for the sun and moon.

VII. *Plot the path of the moon* : Using the East West and NorthSouth differences, plot the path the moon will follow through the earth's shadow.

VIII. *Construct models of the moon* : The models will be centered of the moon's path, passing through the earth's shadow.

IX. *Calibrate the graph* : The graph will be calibrated to determine the time the eclipse occurs.

Step I: Calculate time of Opposition

Students will use the table to determine at what time the sun and the moon are in opposition, i.e. 180° apart.

PreStep work suggestions:

1) Changing degrees to minutes and seconds

2) Writing proportions

3) Interpolation

Procedure:

Students should find the difference between two consecutive GHA readings for both the sun and moon. Continue operations until the smallest interval in which 180—. is contained is found.

(figure available in print form)

Then interpolate to find the exact time in which opposition (180—) has occurred.

(figure available in print form)

The sun and moon are in opposition at 5h 45.1min or 5:45 AM.

All information may be stored on a Student Information sheet. An illustration of such a sheet is included in the lesson plans.

Step II: Compute the declination of the sun and moon at the moment of opposition.

Students will use the table to find the declination of the sun and moon at 5h 45m

PreStep work suggestions:

1) setting and solving proportions

2) simple interpolation

3) estimation

4) rounding numbers

Procedure:

Using the table, find the declination of the sun and moon using the values for 6h and 5h and interpolating to determine declination at 5h 45 m

(figure available in print form)

The moon’s declination at opposition is approximately S 20— 36.5’

We have computed that the sun’s declination at the time of opposition is N 20— 50.4’ This is also the southerly declination of the center of the earth’s shadow. Finding the difference in the declination of the sun and the moon will yield the difference between the center of the earth’s shadow and the center of the moon.

(figure available in print form)

Therefore the center of the earth’s shadow is 13.9’ below the center of the moon.

Step III **: Find the Radius of the Shadow,**

Certain information need not be explained to students due to its sophisticated nature. After students feel confident with their skills and have successfully completed steps I and II, each instructor can provide the students with the value for the Radius of the Shadow. The following information can be given to each student via the information sheet at the end of the unit.

The radius of the shadow is determined by the following formula:

Radius of Shadow Solar Parallax + Lunar Parallax

Sun’s Semidiameter

The value of the Solar Parallax is a constant: 8.8 secs. (SP)

The value of the Lunar Parallax is a constant: 59.5 min. (LP)

The value of the sun’s Semidiameter is obtained from the table: 15.8 min. (SD)

Plugging in the above values in the formula yields a radius:

Radius = 8.8” + 59.5’ 15.8’

Radius = 42.55’

Step IV; Setting up a Coordinate Axis that will be used to plot the moon’s motion. (Diagram I)

Prestep work suggestions:

1) converting and scaling measurements

2) setting coordinate axis

3) marking intervals on numberlines

4) constructing circles

Procedure:

Students are now ready to start graphing the information they have acquired in the previous steps,

I. Allowing 1 inch = 1000 seconds of an arc, students should convert all their degree measurements to seconds. This information can be stored on the Student Information Sheet. Converted measurements:

    1. The declination of the earth's shadow with respect to the moon: 13.9' 834" (Step II)
    2. The Radius of the shadow: 42.55'.= 2553" (Step III)

II. Students should set up a coordinate axis (NS, E W). The origin represents the center of the *moon* at opposition.

III. Instruct students to find the point of the NS axis that will represent the center of the earth's shadow. The value from Step II is 834".

IV. Construct a circle representative of the earth's shadow centered at 834" with a radius of 2553". (Step III)

***Step* V: Find the NorthSouth movement of the moon and sun.**

Prestep work suggestions:

(figure available in print form)

1. subtraction of decimals

2. change from degrees to minutes to seconds.

Procedure:

The NorthSouth motion of the sun and noon can be obtained directly from the nautical table. Find the difference between the hourly declination of the sun.

(figure available in print form)

The sun moves north at a rate of 0.5' per hour, therefore the earth's shadow moves south at 0.5' per hour.

The NorthSouth motion of the Moon is similarly found.

(figure available in print form)

The moon moves south at a rate of 3.5’ per hour.

Given this information we can determine that the moon with respect to a stationary shadow rises at:

3.5’ 0.5’ = 3’ or 3 minutes each hour.

Step VI: Compute the EastWest rate of change for the sun and moon.

PreStep work suggestions:

1. change degrees to minutes and seconds

2. operations using degrees (addition and subtraction)

Procedure:

Find the Greenwich Hour Angle (HG) of the sun and the moon from the table. The difference in their rate of change is the rate in which the moon will back into a stationary shadow.

(figure available in print form)

The sun has an East West motion of 15— per hour.

The moon has an East West motion of 14—26’ per hour.

From this information we know the moon will back into the stationary shadow at an EastWest rate of :

15 14 26’ = 34’ or 2040” per hour

Step VII **: Plot the path of the moon. (Diagram II)**

Prestep work suggestion:

1. scale time measurements to linear measurements.

2. graph points

Procedure:

Plot the EW motion of the moon to the right and the left of the origin. This value is the degree measure found in Step VI. The distance of 34’ or 2040” to the right and left of the origin is the distance in HG one hour before and one hour after opposition.

Plot the NS motion. Plot the degree measurement of 3’ or 180” (Step V) below the western point and above the eastern point.

Connect the NS points. This is the path of the moon through the earth’s shadow.

Step VIII **: Constructing models of the moon on its path through the earth’s shadow. (Diagram III)**

Prestep work suggestions:

1. construct circles of specific radius

2. scale measurements

3. construct circles tangent to Lines

Procedure:

The table provides the Seemdiameter (SD) of the moon. SD 15.8’. Using this value as the radius of 4 circles centered on the line of the moon’s path construct:

Circle l externally tangent to the shadow at the right.

Circle 2 internally tangent to the shadow at the right.

Circle 3 internally tangent to the shadow at the left.

Circle 4 externally tangent to the shadow at the left.

These four circles represent the positions of the moon at the start of the eclipse, the beginning of totality, i.e. the moron is completely hidden, the end of totality, and the end of the eclipse.

(figure available in print form)

Step IX: Calibrate the graph so as to read the time of occurrence of each phase of the eclipse. (Diagram III)

Prestep work suggestions:

1. changing degrees to minutes and seconds

2. constructing circles

Procedure:

The EastWest time is the difference in degrees one hour before and after opposition (our value is 2040”). We may use this value as a scale of time. The origin is the center of the hour time period. We then divide this hour distance into 12 parts, 5 minutes each.

Drop a perpendicular from the center of each circle constructed to the EW axis.

Determine the time where the perpendiculars meet the EW axis.

May 25, 1975

Eclipse Begins	3:40
Totality Begins	4:45
Totality Ends	6:12
Eclipse Ends	7:14

Students should be allowed two to three days to complete each step in the calculations. This will allow students the time to digest the newness of the material and further consider the information they are gathering and recording. The smooth transition from practice drill work to the actual computations can only be achieved when proper emphasis is placed on the skill review work. An example of a skills sheet used for Step I is included in the sample lessons at the end of this unit.

III. The Fabled Eclipse

As a release from the scientific and mathematical aspects of this unit, students can also explore some folk and mythological interpretations of lunar eclipses and of the moon. The remainder of this unit will be devoted to certain folk interpretations with the desire that students read outside sources and ultimately write their own folk interpretation of a lunar eclipse. I have included three tales of eclipses, others may be found in the bibliography.

Students should be given the opportunity to discuss various folklore interpretations. For example, Pythagoras believed eclipses were not only caused by the passage of the moon through the earth's shadow, but "also occasionally by the shadow of the antichthon," the antichthon being a counterearth, a tenth planet that was always invisible to us because "it is between us and the central fire and always keeps pace with the earth".

(figure available in print form)

The Egyptians believed the moon was representative of a boat across the sky. The moon's sequence was depicted by the fable in which the moon had enemies. A sow attacks the moon on the fifteenth day of each month, after a fortnights agony and increasing pallor, the moon dies and is born again. Sometimes the sow manages to swallow it altogether for a short time, causing a lunar eclipse. Finally, Columbus used the eclipse of the moon when he was deserted by mutinous followers. While in Jamaica, the natives refused to deliver food to the Spaniards. Columbus knew that the February 29 eclipse was approaching and told the native chiefs that God was angry with them and would punish them with calamities. A sign of his anger would be the obliteration of the moon as it crossed the sky. Cleverly, Chris told the Jamaicans he would intercede on their behalf if they delivered the goods. Enter Moon. In a similar fashion, students will have an assignment to write a folk tale, written through the eyes of an ancient Indian, to explain the occurrence of a lunar eclipse.

This portion of the unit will allow students to daydream and imagine life in other cultures. I believe it will balance the math and science parts and insure the fact that students will receive a complete picture of the moon and its eclipse activities.

The unit has an adequate blend of mathematics and science. It will give each student an opportunity to view mathematics as a method of problem solving. Hopefully it will enable students to have a broader view of mathematics and its applications to science.

STUDENT INFORMATION SHEET

Eclipse Date: May 25. 1976

Steps	Description	Values	Conversion to secs.
I	Time of Opposition	____	
II	Declination of the sun at opposition	____	
	Declination of the moon at opposition	____	
	Difference between center of the shadow and center of moon	____	____
III	Sun’s Semi-Diameter (SD)	____	
	Solar Parallax (SP)	8.8”	
	Lunar Parallax (LP)	59.5’	
	Radius of Shadow = SP + LP - SD	____	
V	N-S motion of moon	____	____
VI	E-W motion of moon	____	____

Given information in Red.

SAMPLE LESSON

***Worksheet for skills used in Step* I**

(figure available in print form)

SAMPLE LESSON

Lesson: *The Moon in Folklore*

This lesson is based on a New Guinean fable. Read this account of the moon.

1. How would you explain a Lunar Eclipse based on this account of the moon?
2. Write a fable explaining a Lunar Eclipse based on this story.

(figure available in print form)

One day a man from my village had a garden. Every day he used to go and burn the dried leaves, branches, and woods in his garden. After burning this rubbish he swept all of them away. Then he started to grow some food like taros, corn, sugarcane, pawpaw, greens and some other things too. When all the food stuffs in his garden were ripe he started to collect them. He never went home early in the afternoon. He always used to stay in the garden after the sun had gone down. This he did every day. He went home at night when all people were sleeping. When he arrived at the garden he would sit down and have a rest. Sometimes he went there only for a visit to the garden.

One day, when he went to the garden he saw that the greens were stolen. So that night he did not go home. He slept in the garden and watched all night. Then in the middle of the night he saw the moon shining with very bright skin. It was stealing his green vegetables. He got very angry and speared the moon. The moon fell on the ground. The man took the moon home and cooked and ate the moon. He gave half of the moon to the old woman. She didn't eat her share. The moon flew into the sky. So that is why you can see the half moon in the sky.

Bibliography

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This book contains historical interpretations of celestial occurrences and methods of observation.

Fisher, Clyde. The Story of the Moon. Doubleday, Doran and Company, Inc., Garden City, New York, 1943.

A fine book that explains the workings and the behavior of the moon.

Greitzer, Samuel L. "Computing a Lunar Eclipse; An Excercise in Classical Mathematics" Rutgers, The State University, New Brunswick, New Jersey.

Basic source for steps in computing an eclipse. Somewhat technical.

Greitzer, Samuel L.. "Predicting a Lunar Eclipse", *Updating Mathematics* Croft Educational Services, Vision Inc., New London, Conn., Sec 4, Vol.3, No.8; April 1961.

Good reference for a method of studying and predicting an eclipse.

Lieber, Arnold L. M.D.. *Lunar Effect* . Anchor Press, Doubleday, Garden City, New York, 1978.

Interesting thoughts on phases and positions of the moon and mans' behavior, i.e. a full moon can make you crazy

Newton, Robert R.. *Ancient Astronomical Observations* . The John Hopkins Press, Baltimore and London, 1970.

Ancient accounts of the moon and sun and the explanations given for their actions. Interesting explanations.

Resource

Nautical Al *manac 1975* , Published by U.S. Government. Source of the table used in this unit.

Student Bibliography

Abel, E.L.. *Moon Madness* . Fawcett Publications, Inc., Greenwich, Conn., 1976.

Interesting and bizarre accounts of the moon and its effects on man and animals

Beier, Ulli, and Dr. Prithvindra Chakravarti, eds.. *Sun and Moon* . Institute of Papua New Guinea Studies, Port Moresby, 1974.

A fine collection of tales of the origins of the sun and moon from New Guinea, passed orally from one generation to the next.

Jablow, Alta, and Carl Withers. *The Man in the Moon* . Holt Rinehart, and Winston, New York, 1969.

A fascinating collection of sun and moon tales from all over the world. Easy reading and appropriate for all levels of students.

Latham, Jean Lee. *Carry on Mr. Bowditch* . Houghton Mifflin Company, Boston, 1955.

Although it tends to be somewhat chauvinistic, it is an easily readable account of a seaman and his study of navigation and astronomy.