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1980 Volume VII: Problem Solving

Math Is Everywhere: A Problem Solving Teaching Unit

Curriculum Unit 80.07.10
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This unit on problem solving is intended for use with middle school students. However, the material herein may be appropriate for students in elementary or high school. In the final analysis, it is the teacher who can best judge its usability in his/her classroom.

WHY IS THE DEVELOPMENT OF PROBLEM SOLVING SKILLS IMPORTANT?

In 1979 the National Assessment of Educational Programs was completed. The math component of this assessment was the result of a panel of classroom teachers, mathematicians, and lay citizens who selected objectives and goals for 9 and 13 year old students. The assessment encompassed a wide range of exercises that were deemed appropriate for their age groups. The word problem component consisted of single step and multistep word problems that are typically found in modern textbooks. Included below are a few details concerning the overall performance of students on word problems: [1]

80% of the nine year olds responded correctly to addition word problems.

60% of the nine year olds responded correctly to subtraction word problems.

80% of the thirteen year olds responded correctly to the addition and subtraction word problems.

The results of the multiplication and division word problems were low. This was anticipated for the nine year olds, since it is at this level that these concepts are first introduced. However, when compared to previous assessments, this group performed 22% lower on word problems involving finding the product of a 1digit number by a 2digit number. This same group performed well on exercises which solely involved direct computation. What are the implications of this? Is less time being spent on problem solving? Are the students' incorrect responses due to reading errors? Is more time being spent on isolated computational skill practice?

Only one word problem involving multiplication was presented to the thirteen year olds:

"One rabbit eats 2 pounds of food in a week. There are 52 weeks in a year. How much will 5 rabbits eat in one week?"

*56% of the 13 year olds responded correctly to this item.

*This result was typical of any word problem which contained extraneous information. The results were significantly higher where extraneous information was not included.

Nine year olds had the same difficulty with problems containing extraneous information.

Performance was low overall on problems involving division.

Generally, the results imply that the students did not take the time to think through how to solve the problems. Also, one could reasonably conclude that a large portion of the students did not possess the skills that one needs to systematically figure out *how* to solve the problems.

Further, the Iowa Test of Basic Skills, which has recently been administered to New Haven students, and the proficiency test for ninth graders throughout the State of Connecticut have illustrated the need for developing problem solving skills in our students. These tests have produced nearly the same results that the National Assessment did. Students generally do well in computation, but do poorly on items which involve reading a word problem and solving it

The techniques or methods that a teacher uses in problem solving appear to be of secondary importance to producing success in problem solving. A systematic way of solving problems and a great deal of practice for students is of primary importance in the teaching of problem solving skills. The more practice that a student receives, the sharper his/her skills will become. Also, as outlined later in this paper, discussions between students and teacher, coupled with the proper lead questioning by the teacher, should produce positive results.

To develop competency in problem solving requires the teacher to view problem solving from the perspective of presenting mathematical concepts for the purpose of developing “problem sense” The proverb “I hear, I forget; I see, I remember; I do, I understand” has particular merit in the area of problem solving. Problem solving offers the opportunity for students to make the connection between the mathematical concepts that are taught in school and the real world Therefore; it becomes necessary to make the learning process as concrete as possible. It is also important that students learn a systematic manner in which to accomplish the assigned tasks, to facilitate his/her independence in, and mastery of solving mathematical problems. “The student should acquire as much experience of independent work as possible. But if he is left alone with his problem without any help, he may make no progress at all. If the teacher helps too much, nothing is left to the student.” [2] In the early stages of the learning process of how to tackle word problems, it is absolutely necessary for the teacher to provide a great deal of guidance to the students through discussion.

There are basically two types of word problems which are frequently assigned to students; those which require the direct translation of the verbal languageto mathematical language, and the subsequent application of the math skills (s)he possesses, and those problems which require the student to apply mathematical skills and concepts in a discovery manner. “Solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim that was not immediately attainable.” [3] Both types of problems require the student to draw upon present knowledge and past experiences, but it is the latter which offers the greatest challenge to the student, and which has the greatest potential for helping a student to understand the many ways in which math relates to the other fields of study.

STRATEGIES

Teaching is an art. Therefore; there is no one manner in which it is undertaken that is best. The strategy(ies) that the teacher utilizes should strike a balance between what is comfortable for his/her style of teaching and the students' styles of learning. Also, teacher and student interests play an important role in the subject matter that is presented. The strategies and techniques which are outlined below, and the problem solving activities which follow, will provide the teacher with a number of options for developing student competency in problem solving. Any combination of these teaching/learning modes should be successful in attaining this goal. It is up to the teacher to utilize whatever material that is in concert with his/her style of teaching.

ORAL PRESENTATION OF PROBLEMS

Many students have had relatively little exposure to a systematic approach to problem solving. It is absolutely necessary for the teacher to present such an approach to the students before any independent tasks are assigned. It is suggested that such demonstrations take place whenever a new concept is introduced. The teacher may do so in small or large group settings until (s)he is confident that each student has developed minimal competence to proceed independently. It is further suggested that the teacher repeat these demonstrations as often as (s)he deems necessary.

It is at this initial stage that the teacher lays the foundation for developing problem solving sense. It is here that the teacher seeks to develop a systematic method for students to follow in solving problems. One may find numerous methods for solving mathematical problems, depending on whose work one reads. George Polya's book, HOW TO SOLVE IT, is an excellent resource for teachers. Pages XVI and XVII present a plan which is very useful for teachers. Below is a simplified version of Polya's plan which should be presented to students as their plan for problem solving.

For students:

WHAT FACTS ARE GIVEN? (Do you understand the vocabulary?)

WHAT IS TO BE DONE?

WHAT CAN ONE DO TO FIND THE ANSWER?

IS THE ANSWER I CAME UP WITH REASONABLE?

The presentation of this plan and the subsequent application of it in teacher led activities is designed to lay a firm foundation for students. The teacher and the students should apply this plan to a variety of problems, and openly discuss the procedures involved. This will facilitate their learning of the basics in problem solving. It is suggested that the teacher and students apply this plan to problems involving charts, graphs, etc., as well. Independent tasks should be assigned to those students who have mastered this plan.

EXAMPLE:

It is approximately 3,000 miles from New Haven to San Francisco, California. There are a number of ways available to travel from one city to the other; jet plane, car, or train. Obviously, the jet is the quickest way to reach our destination. How long would it take to arrive at our destination by jet? Car? Train?

The teacher should lead the discussion on how to solve this problem, drawing whateveranswers are volunteered by the students on how to solve it. The teacher should make clear the concept of speed, e.g. distance/time relationship. Discuss the concept of miles per hour. (Car = 55 mph, train = 80 mph, jet = 550 mph)

WHAT ARE THE FACTS?	It is 3,000 miles from New Haven to San Francisco. A car can legally travel at 55 mph, a train at 80 mph, and a jet at 550 mph.
WHAT IS TO BE DONE?	We are to find the number of hours it would take by each of these means of transportation.

WHAT CAN ONE DO TO FIND THE ANSWER? There are at least two way that may be applied to arrive at the correct answer; long division, fractions. We know the speed of each vehicle. We also know the distance. The formula for finding the answer is:

distance/speed= time elapsed

Demonstrate each method. Call the students' attention to the decimal shift from the plane's time to the car's time. Discuss the causes for this shift. Try other examples which involve this type of calculation.

(figure available in print form)

PROBLEM SOLVING IN OTHER FIELDS OF STUDY

This technique is wide open to unlimited uses in the classroom. Virtually every field of study contains material which can be easily adapted for use in problem solving. This technique offers the opportunity to illustrate the importance of mathematics to the very existence of other fields of study, and it is an excellent means for the introduction of career awareness. Mathematics becomes perceived by the student as a tool for discoveries and a necessary part of other fields of study. The teacher who is so inclined will find a wealth of material which may easily be utilized in the classroom.

CAREERS WHICH INVOLVE MATH

Music	Astronomy	Biology	Art
Cartography	Carpentry	Masonry	Banking
Accounting	Navigation	Architecture	Medicine

*See also, Joyce Bryant and Carolyn Kinder's unit

Included below are a number of exercises in science concerning the use of the two most common types of thermometers. These exercises, and others, may be found in a booklet entitled, SCIENCEMATH PROBLEMS. They are good practice for students, and they illustrate the necessity for math in science.

(figure available in print form)

There are two types of thermometers which are commonly used today. They are the Fahrenheit and the Celsius thermometers. The United States is presently in transition in its use of the Celsius thermometer.

It is a simple mathematical process to change from one measurement to the other. For example:

to change from Fahrenheit to Celsius,

F 32 X 5/9 = C

212 32 X 5/9 = C

180 X 5/9 = C

900/9= C

100 = C

to change from Celsius to Fahrenheit

C X 9/5 + 32 = F

100 X 9/5 + 32 = F

900/5 + 32 = F

180 + 32 = F

212 = F

The teacher could assign a number of examples which require the students to apply these simple formulae to convert from one scale to the other. They are good exercises in which to apply their skill

1. The boiling point on the Celsius thermometer is _____.
2. The boiling point on the Fahrenheit thermometer is _____.
3. The freezing point of water is ___ degrees on the Celsius scale, and ___ degrees on the Fahrenheit scale.
4. Outside it is 95 degrees F. Convert that temperature to Celsius. _____
5. How many degrees difference is there between the boiling point of water and the freezing point of water on the Fahrenheit scale? _____
6. How many degrees difference is there between the boiling point of water and the freezing point of water on the Celsius scale? _____
7. When it is O degrees F., how many degrees is this on the Celsius scale?

Below are a number of temperatures that were recorded on a Celsius thermometer. Tell whether you would consider each hot or cold.

50 C ___ 62 C ___ 12 C ___ 52 C ___ 9 C ___

Convert the following Fahrenheit temperatures to the Celsius scale:

18 F ___ 56 F ___ 12 F ___ 34 F ___

These exercises illustrate the obvious relationship that exists between science and math. They are included to demonstrate how a teacher may give his/her class experiences in math during science lessone.

Adapted for use in this unit from SCIENCEMATH PR0BLEMS, ScienceMath Publishers, Tony, Wisconsin, Copy. 1958

PROBLEM SOLVING TO MASTER COMPUTATIONAL SKILLS

After the teacher has determined the student's level of competence in understanding numbers, (s)he proceeds to reinforce those skills which are already learned, and introduces new skills and concepts. The use of problem solving is an effective and less boring technique for reinforcing skills. Problems can be an exciting alternative to the relatively boring, isolated drill activities that cause many students to respond negatively. There is a built in motivator in problem solving, the challenge, to which the underachiever and the accelerated student should respond positively. As each student is working independently at problem solving, the teacher then becomes a facilitator of learning, spending his/her time in the classroom in a more efficient manner.

Instead of drill cards with isolated number facts, students can work in pairs to attain mastery of number facts. Students will also get more practice in solving word problems. A set of 5" X 8" index cards can be assembled for students' use. The material on these cards can be made up by the teacher, or word problems from old textbooks could be cut out and glued to the index cards. Below are some suggested types of activity cards.

Direct translation of verbal language to mathematical language

Cards which require the student to perform more than one operation:

ESTIMATION

The final step in the suggested plan for students to use in problem solving states, "Is the answer I came up with reasonable?" Estimation is an important skill in the solution of problems, for it focuses the student's attention on the solution of the problem before (s)he attempts it. The critical thinking involved in this type of activity gives the student a frame of reference, against which the calculated answer may be compared.

Some worthwhile activities in estimation can take place with materials which may be readily obtainable around school. Collections of various sizes and kinds of beans which are stored in various types of jars could lead to some interesting activities in estimation. Students are asked to "guesstimate" the number of beans in a jar based on a sample. For example; a student may fill a one ounce glass with the beans to find out how many fit into this container. The student may then judge how many beans are in the larger container. Macaroni or pebbles may also be used.

UTILIZING STUDENT INITIATED PROBLEMS

To further involve the students in their development of problem solving skills, teachers may use this technique: It gives the students sense that the teacher respects their input into their own learning activities, and illustrates the fact that not all learning comes from the teacher, that learning can be selfdirected. The teacher may set aside a particular day of the week for this type of activity, in which a student may challenge the rest of the class to solve his/her problem.

PUZZLES AND GAMES

Puzzles and games which may or may not involve computation are an excellent means for developing problem solving sense in students, and in getting them to appreciate the challenge. Puzzles and games are an excellent means for motivating even the most reluctant student. A number of games and puzzles are available for use with this unit. Other books, such as THE MOSCOW PUZZLES, may be obtained at any library. Puzzles such as the one below are fun for the students to attempt, and help students to sharpen their math skills.

Place the numbers; 1,2,3,4,5,6,7 in the circles in such a way as to yield a sum of twelve in the indicated directions.

(figure available in print form)

UTILIZATION OF VISUAL MATERIALS **(Charts, tables, schedules)**

Visual materials play an important role in our understanding of a number of activities in our daily lives. Often these materials are overlooked in the context of problem solving. Visual materials present information in an efficient and organized manner. Reading an airline or bus schedule, a movie or television timetable, a thermometer, or even a calendar requires a particular skill. Interpreting the information on a graph, a diagram, or a map requires certain skills, also. These materials allow students to perceive ideas through the use of other senses.

A good method to use when first beginning this kind of activity is to select some good examples of charts and tables and discuss the nature of their organization. Students can learn how to collect information on a particular topic, and organize it into tables of their own. Below are some examples of activities which offer students practice in constructing visual materials. Future assignments should require them to collecttheir own information.

#1 For the month of September, 1979, the 5th, 12th, 19th and 26th fell on Tuesday. Make a calendar which illustrates this.

#2 The students of Mr. Jackson’s gym class were weighed and measured. The following heights and weights were recorded. Organize a chart to show this information. Put the students’ first names in alphabetical order.

Jeff 5’2” tall, 112 pounds; Inez 5’0” tall, 97 pounds; Carol 5’6” tall, 115 pounds; Anthony 5’8” tall, 136 pounds, Ronnie 5’1” tall, 124 pounds; Selma 5’2” tall, 100 pounds; Yvonne 5’4” tall, 111 pounds, David 5’6” tall, 131 pounds; Kenneth 4’11” tall, 94 pounds; Tonya 5’3” tall, 99 pounds.

#3 The following information concerns a student’s schedule for classes. Organize this information into a usable table. Reading/Language Arts is held every day of the week from 8:4510:15, Math is every day of the week from 10:1711:03, from 11:0511:50 the following activities are held: Mondayart, Tuesdaygym, Wednesdaycooking, Thursdayindustrial arts, Fridaystudent government, from 11:5212:35lunch, science classes are held on Mondays, Wednesdays and Fridays from 12:371:30, social studies is held from 12:371:30 on Tuesdays and Thursdays, Monday through Thursday, from 1:322:15 is when enrichment activities take place, and on Friday at that time is class meeting.

Students can be assigned the task of collecting some sort of information on their own to organize a chart. It is a good idea for the first few assignments to assign information collecting for homework. Students could construct their charts at home, to discuss in class the next day.

GRAPHS

There are many ways in which to show information Graphs are an important way to show certain kinds of information because they can clearly picture the information one wishes to illustrate. There are three kinds of graphs with which the students should become familiar; bar graphs, line graphs and picture graphs. Each Of these will be discussed below. The construction of each should be discussed in class, and students should be givenexercises which will give them practice in collecting information and constructing graphs. Also, activities which require their interpretation of the information contained in graphs should be assigned.

Picture graphs (picto graphs)

This type of graph utilizes a particular symbol to represent a particular quantity. Demonstrate the construction and use of picture graphs, such as the one below. Students will have very little difficulty with graphs of this type.

(figure available in print form)

Bar graphs

This type of graph is more abstract than the picture graph, but it is utilized in the same manner. Bars of varying length are used to represent particular quantities. By reading across the bar and down on the vertical lines, the reader can determine what the graph is depicting. Consider the information that is included on the bar graph on the following page. Discuss the construction of the bar graph with the class.

(figure available in print form)

1. How many students chose “Happy Days” as their favorite TV program?

2. Which program was the least favorite program?

3. How many students chose “The Muppets” as their favorite program?

4. Which program was chose by students as their favorite program?

Line graphs

Line graphs are probably the most difficult type of graph for students to read. It is important that the teacher spend a great deal of time explaining the construction of the line graph. The teacher should relate the grid system on the line graph to the grid system that is superimposed on world maps to locate points on the surface of the world. The line graph clearly illustrates highs and lows. Consider the following:

(figure available in print form)

The line graph, which contains exactly the same information as the chart, graphically demonstrates the sun’s earliest rise on the first day of summer, and its later arrival on the days past that date.

MAPS AND CHARTS

We receive a great deal of information in our daily lives through the use of maps and charts. Some of us use them more than others. It is important for you to get a lot of practice in how to use these maps and charts to

get information contained in them.

DIRECTIONS: Read each problem below carefully. Solve each, using the maps and charts provided.

(figure available in print form)

The map at the left depicts a region of the United States that was recently visited by the Keely family.

In their travels they went from: Salt Lake City to Denver

Denver to Sante Fe

Santa Fe to Phoenix

Phoenix back to Salt Lake City

1. What is the distance from Denver to Santa Fe?
2. What is the distance from Santa Fe to Phoenix?
3. What is the distance from Phoenix to Salt Lake City?
4. How many miles did the Keely family log during this trip?
5. If the family car gets 18 miles per gallon, then how much gasoline did they use for the entire trip?
6. Gasoline cost the Keely family $1.18 per gallon. How much did they spend on gasoline?

***THAT'S LOGICAL* !**

The process of elimination is a way that can be used to find the correct answer. We try to eliminate the answers that we know are *not* correct. For example:

The baker, the plumber and the shoemaker are all residents of the town of Rosewood City. They are named Harold Baker, Janet Plumber and Fred Shoemaker, but not necessarily in that order. Match the person with the correct occupation. You are given the following clues:

1. No person holds a job that is his/her last name.

2. Janet is not the baker.

HOW TO SOLVE IT:

1. Organize the information in a table that suits the problem.

(figure available in print form)
2. Clue #1 tells us that Harold is not the baker. Put an “X” in box across from Harold and under the column headed baker to show that we have eliminated Harold for that job. Proceed in the same manner for Fred and Janet.
(figure available in print form)
3. Clue #2 tells us that Janet is not the baker. Put an “X” in the proper box to show that you have eliminated her for that job.
(figure available in print form)
4. Now it should be easy for you to complete the table and decide which person performs which job.

NOT ENOUGH !

Sometimes a word problem doesn’t include all the information that is necessary to solve it. For example:

> Terri was saving her earnings from a paper route to buy a new bicycle. She saved $15.00 each week for three weeks. How much more does she need to buy the bicycle?
>
> We know that Terri saved $45.00 (3 X $15.00 = $45.00), but we don’t know how much the bicycle costs, so we cannot compute how much more Terri needs to buy the bike.

Each problem below omits information that is needed to solve them. Do each with the class to give the students practice with this type of activity.

1. George and Sylvia Cannon attended a church dinner with their three children. The admission price was $2.75 for adults and $2.00 for children. How much change did Mr. Cannon receive?

WHAT FACTS ARE GIVEN? There are 2 adults and 3 children.

WHAT IS TO BE DONE? So, 2 X $2.75 = $5.50 and 3 X $2.00 = $6.00.

The total amount of money paid is $11.50

How much change did Mr. Cannon get back?

We would have to subtract $11.50 from the dollar amount Mr. Cannon presented for payment, but we are not given this information.

2. Gasoline sells for $1.23 a gallon at Fred’s Service Station. Wilbert filled his tank. How much did Wilbert spend?

WHAT FACTS ARE GIVEN?	Gasoline cost Wilbert $1.23 per gallon.
WHAT IS TO BE DONE?	We are to compute how much is cost Wilbert to fill his tank. We should multiply the number of gallons of gasoline by the cost of each gallon ($1.23). We are not given the number of gallons that Wilbert's gas tank holds. so we cannot solve it

DO THE SAME FOR EACH OF THE FOLLOWING:

3. Henry drives to work with his friend Joe. Joe takes his car on odd numbered days, and Henry takes his car on even numbered days How many miles do Joe and Henry drive each week?

4. Kathy likes to jog. Using a stopwatch, Kathy figured that she jogs one mile in 10 1/2 minutes. How many miles did Kathy jog in one week?

MORE THAN WHAT'S NEEDED

Often a word problem contains more information than is needed to solve it. This is know as *extraneous* information. The student who is a careful reader and knows how to solve word problems should have no trouble finding out which information is not needed. A great deal of practice with problems that contain extraneous information is needed, for it is on this type of problem that the National Assessment proved that students did poorly.

1. The circus was in town for 5 days. Gary and his friend George went on the first day. They spent $3.00 each to get into the Coliseum. Also, each boy bought a soda for $.50, popcorn for $.75, a balloon for $1.00, and a souvenir book for $2.25. Clown makeup costs $3.25. How much did each boy spend?

WHAT FACTS ARE GIVEN?	Each boy bought exactly the same things;			
		admission ticket	$3.00	
		soda	,	$ .50
		popcorn	$ .75	
		balloon		$1.00
		souvenir book.,	$2 25	
WHAT IS TO BE DONE?	We are to find out how much each boy spent, We can do this by adding the amounts of money for the items listed above. We don't include the price of the clown makeup because it didn't tell us that they bought it. This information was not needed.			

2. Oscar and Felix drove to Washington, DC The trip was 325 miles. Their car gets 15 miles to the gallon. If they drove at a steady speed of 55 m.p.h., how long did the trip take?

WHAT ARE THE FACTS ?	Both men took a trip to Washington, DC	The trip was 325 miles. Their speed was a constant 55 m.p.h.,
WHAT IS TO BE DONE?	How long did the trip take them? We would compute this by dividing the mileage by the speed. We don't need to know the miles per gallon that the car gets to solve this problem.	

3. A horse eats one acre of grass in one week. There are 52 weeks in a year. How much does one horse eat in 6 weeks?
4. Chicken sells for $1.56 per pound. There are 50 pounds of chicken in the meat case at the supermarket. How much does 3 pounds of chicken cost?

The teaching of problem solving does not have to occur solely during the time that the teacher has set aside for math instruction. The teacher who uses his/her insights and creativity will always find the opportunity to incorporate problem solving into the normal activity of the classroom. The teacher should make a conscious effort to recognize the opportunity when it arises. For example:

> During a social studies lesson, a class is taking a look at the thirteen original states that became united under the Constitution of 1787. The teacher presents the class with the populations of each state in 1790. The students could learn several points by comparing the population of 1790 with the most current figures on those states. This could turn into a lesson which gives the students practice in information gathering, constructing tables, and making graphs to illustrate the information. By assigning small groups various tasks, the teacher could incorporate math experiences into the social studies, and probably make the total experience more exciting for the students.

It is important that teachers encourage their students to investigate problem solving that is related to other fields. This practice will help them to make the connection between subjects that are often compartmentalized. Also, many learning experiences that occur spontaneously are often successful, for it was the student interest that caused the teacher to pursue it. It is not a waste of time for the class, or a small part of it, were to go off on a tangent to research a particular topic. Rather, it is drawing the energy of the students together in a cooperative: effort. Theteacher is merely trading off a planned activity for one thatis unplanned; unplanned in the sense that the teacher is not sure how far the students will pursue the topic. Both can be quite valuable to the students.

The kit of materials that is available for use with this unit is designed to provide independent experiences for students in math problem solving. The kit contains two types of materials; those that are commercially produced, and task cards that were written by the author. The task cards are grouped according to their level of difficulty and type of activity. Students are to proceed in an independent manner through these sets of task

cards. The commercially produced materials include; teacher resource books, student activity books, Soma Blocks, Tangram Activity Cards, and other types of puzzles and games. These materials should be assigned to the students as the teacher determines that each is ready.

A student is to progress at a rate that is comfortable for him/her. The student that finds the tasks to be easy will work at an accelerated rate, while the slower, more deliberate student will work at a pace that is comfortable for him/her. This allows the teacher to take on the role of facilitator of learning, and could circulate more freely from individual to individual. Further, the teacher may choose to agree on work contracts for individual students. The amount of work and the number of days in the contract period could be agreed upon beforehand. This is beneficial for students, for it focuses his/her attention on a long range goal.

An important aspect of this kit of materials is the record keeping of student progress and performance. The teacher should run off a number of grids with the students' names on them. These should be prominently displayed in an accessible spot. It is on these charts and graphs that the students and teacher will keep track of completed work by individuals. Each student is to record his/her performance on each task that is assigned. The students will receive practice in charts and graphs in this manner. Further, graphs should be kept solely by the teacher on each student's performance on periodic tests. I've found that the most attractive aspect of this type of record keeping is that these records are readily available for students, teacher, and parents. The records practically speak for themselves.

The materials for this unit are available through the Teachers Institute for use by New Haven teachers. These materials should be kept in a place in the classroom that is readily accessible to students for their use. It is necessary to include other equipment such as; rulers. scales, tape measures, and other materials in this area. These are available through Mr. Levitin, math supervisor.

FOOTNOTES

1. ARITHMETIC, April 1980 2. Polya, G., HOW TO SOLVE IT, Princeton Press, NJ, 1973, p. 1 3. Polya, G., MATHEMATICAL DISCOVERY: ON UNDERSTANDING, LEARNING, AND TEACHING PROBLEM SOLVING Vol. 1, Wiley & Sons, NY, 1962 p. V

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