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Mathematics and Heredity

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Introduction

Students may recall teachers and friends commenting on how much they resemble an older brother or sister. Similar eye coloring, shape of a nose, and special talents are easily identified within a family group. Children readily understand that although family members differ in temperament and attitudes, they are very much alike in particular physical characteristics. This non-verbal understanding of family similarities can help students learn about genetics, and about how their math skills can be applied when studying the science.

A fundamental knowledge of the vocabulary used in the science is needed to begin this mathematical study of genetics. I will explain a few of the basic concepts, how they were derived from the experiments of Gregor Mendel, a 19th century botanist, and how they can be applied in lessons for a mathematics class. I would recommend the instructor become familiar with the information and refer you to a book written by Allen Vegotsky and Cynthia A. White entitled a *Programmed Approach to Human Genetics* (see bibliography). The text is written in a straightforward, sequential manner for the independent reader and gives an understandable explanation of the information needed.

This unit is designed to be presented as a set of extension lessons with practical applications for problem solving dealing with ratios, proportions, percentages, data analysis, and chart and graph reading. The target groups are 7th and 8th grade math students. The information may be used as supplemental lessons showing application of mathematical skills. Upon completion of this study students should:

1) understand the process by which genes of parents are transferred to their offspring.
2) understand the difference between a dominant and recessive trait and understand how the presence of a trait may effect the physical characteristics of an individual.
3) be able to complete a Punnet Square given the genotype of parents (i.e. determine the possible genotypes of their offspring).
4) read and/or construct a pedigree chart mapping a specific trait in a family.
5) determine the probability of a certain phenotype being expressed in an individual.
6) be aware of the integral role of mathematics in science.

The unit is divided into four categories; 1) the effect of Gregor Mendel's research and the laws he developed that govern genetics today. This section will introduce the basic vocabulary and an explanation of the process of gene transference, 2) the development of Punnet Squares to determine the probability of specific genotypes occurring, 3) the reading and construction of pedigree charts mapping specific traits in a family and 4) a glossary of terms and set of sample lessons.

I. *Gregor Mendel*

In 1865 a botanist by the name of Gregor Mendel presented his theories of genetics to a group of prominent scientists. They were unimpressed. It was many years before the genius of Mendel's work was recognized. Through careful study, precise records, and controlled experiments, Mendel discovered that certain characteristics were transmitted from one generation to another in a predictable, mathematical fashion. Through his research and study of plant growth and seed shape, Mendel deduced the following two laws of genetics:

1. Mendel's First Law

> A simple genetic trait is determined by a pair of separable factors (called alleles of a gene). An offspring will receive one allele from each parent.

2. Mendel's Law of Segregation

> Paired alleles of parents segregate during the formation of egg and sperm cells so that only one of the two alleles is included in each egg or sperm.

Knowledge of the process of meiosis is basic to the under. standing of how genetic information is transferred from parent to offspring. A child receives genetic information from each parent. During meiosis, the process in which chromosomes in a cell's nucleus are separated, each egg and sperm cell receives 23 chromosomes or one half of the genetic material needed to create a new organism. A normal child receives 23 chromosomes from each parent, a total of 46 chromosomes. Consider a chromosome as a collection of specific genetic material or genes. All individuals have like chromosomes containing information that will describe one's physical traits. For example, all individuals have a set of sex chromosomes (XX for females, XY for males) and the genetic information that constitutes that set of chromosomes determines the sex of the individual. During meiosis, each parent passes some of their genetic material to their children. Each gene that is transferred from parent to child carries the material that will allow a specific trait to be expressed. For example, a mother may pass a gene that determines a particular hair coloring to her child. The father, in a like manner, passes along a similar gene which expresses hair color. The pairing of these two genes, describing the same trait, hair coloring, is the new genetic material of their offspring, or a child's *genotype* with respect to hair color. (For a more detailed description of meiosis see additional units in this volume.)

Through experimentation and careful analysis, Mendel discovered that genes exist as pairs in the cell. Each gene is represented by 2 *alleles* (an allele is one of several possible forms of a gene). An egg or sperm cell will receive only one of the alleles from each parent during this period of meiosis. When fertilization then occurs, the separate alleles from both parents join to create the new genotype, or genetic material, of the offspring.

Mendel also discovered that if one knew the genotype of the parents, one could accurately predict the probable genotypes of the offspring, and in the proper ratios. He found that certain traits are always expressed (seen in an organism) as long as one allele of this trait is present in the gene pairing. He described this characteristic as *a dominant* trait. The traits expressed only if both the alleles were identical were then called *recessive traits* .

II. *The use of Punnet Squares*

The following is a simple example of dominant and recessive traits. Assume that loving math is passed on genetically and is a dominant trait. The letter 'M' will indicate loving , a dominant trait, while a ,m, will indicate hating math, a recessive trait. The following pairings are possible to describe the genotype of an individual.

MM . . . . . . .a math loving individual

mm . . . . . . .a math hating individual

Mm . . . . . . .a math loving individual

By constructing a table called a Punnet square, we can calculate the possible combinations of alleles and determine the possible genotype of the offspring. Punnet squares work much the same way as a multiplication chart and should be easy for the students to understand. The symbol represents the father and the represents the mother. Listed vertically on the left is the genotype of the mother and horizontally at the top is the genotype of the father. The inner squares are then filled in by finding the "cross-product of the two alleles. See Figure I.

Figure I

Father- mm . . . . .hates math

Mother- MM . . . . .loves math

All of the offspring will be math Lovers.

(figure available in print form)

By crossing each vertical entry with each horizontal entry, the above parings occur. As we can see, 100% of the offspring will love math because the dominant allele M appears in each pairing. Figure II describes the possible outcomes if both father and mother are genotypically Mm.

Figure II

Father—Mm . . . . . . .loves math

Mother—Mm . . . . . . loves math

The ratio of math lovers to math haters is 3:1.

(figure available in print form)

This pairing yields the probability of 75% of the offspring being math lovers as three out of four pairings contain the dominant allele. Students can find the ratios and percentages for all the possible combinations of parents and their possible offspring. (See sample lesson)

Individuals whose alleles are identical. MM or mm, are called *homozygous* , and those whose alleles are different, Mm, are called *heterozygous* . Below is an example of the pairing of a homozygous recessive father (mm, math hater) and a heterozygous dominant mother (Mm math lover).

Figure III

There is a 1:1 ratio of math lovers to math haters.

(figure available in print form)

Offspring of these parents will generally separate into two equal groups of math haters and math lovers. Students should review the outcomes of these standard pairings. This information can be used when studying family pedigree charts, charts used to follow the transference of a genetic trait through a family.

Students can be instructed to develop Punnet squares for various combinations of heterozygous and homozygous parents. Examples can be of a scientific nature. Examples of dominant traits are 1) widow's peak 2) unattached earlobes 3) the ability to curl one's tongue, while some recessive traits are 1) albinism 2) attached earlobes and 3) sickle cell anemia.

The next section will describe how we can use this knowledge of genetic transference to study families and trace inherited traits through a family tree by means of a pedigree chart.

III. *Pedigree Charts*

Pedigree charts are a pictorial description of the expression of a certain genetic trait in a family. The reading and construction of pedigree charts will reinforce students skills in gathering and analyzing data as well as writing information in alternate forms. There are a few basic symbols and guidelines to follow when creating a pedigree chart for analysis. The symbols are as follows:

(figure available in print form)

. . . . female

(figure available in print form)

. . . . male

(figure available in print form)

. . . . shaded symbol indicates that this individual expresses the trait being charted.

(figure available in print form)

. . . . . . .indicates a marriage

(figure available in print form)

. . . . .indicates the three females are offspring of this pairing. The first female child is the oldest (left most) and the order of birth moves from left to right.

(figure available in print form)

. . . . .indicates identical twins

(figure available in print form)

. . . . . . indicates fraternal twins

The location of an individual is described by the tier of the tree on which he or she resides and the numbered placement from left to right. See Figure IV.

Figure IV

(figure available in print form)

Charted above is the pedigree chart of three generations of a family. Tier I represents the grandparents of tier III. Relationships can be found between the dominant trait and the number of individuals expressing this trait. In addition to finding ratios and percentages of the appearance of this trait, the genotype of each member can be deduced by studying the individuals and assuming the genotype of the grandparents.

Pedigree charts for recessive traits may also be examined. Sickle cell anemia disease is a recessive trait. An individual may carry the trait without ever expressing the disease (a genotype of Ss, heterozygous carrier, a genotype of ss, homozygous diseased, a genotype of SS, homozygous, unaffected). Students can study a pedigree chart and determine which individuals may be carriers of the disease, which are affected, and which are unaffected. See Figure V.

Figure V

(figure available in print form)

Students can deduce, using their knowledge of the behavior of recessive traits, the genotype of the parents of the affected individual. Both must be sickle cell carriers i.e. heterozygous, in order for a diseased child to be produced. Students may speculate on whether individuals II1 and II2 are heterozygous or homozygous. Discussions on the practicality of having children when two people are known carriers of a disease will lead to some very interesting discussions on the responsibility of having children especially when a risk of disease is apparent. Students should be encouraged to think out all possible solutions and restrictions before making a decision about the analysis of a pedigree chart. (see sample lesson)

I have developed some lessons in which the information presented here is combined with the reinforcement of mathematics skills. Students will find the presentation of this material informative. They will be able to examine their own families and gather data about gene transmission, and practice analyzing the material they've gathered. It is anticipated that students will become scientifically curious, become open to the information presented and improve their mathematics skills. Most importantly, students will become aware of the role of mathematics in scientific study, completing a circle of learning for them and developing a base of knowledge on which they may nurture their learning curiosity.

Glossary

Allele —an alternate form of a gene.
Chromosome —structure in the cell nucleus that stores and transmits genetic information.
Gene —unit of heredity.
Genotype —Allelic status of an organism for a genetic trait.

Heterozygous —having different alleles of a gene.
Homozygous —having indistinguishable alleles of a gene.
Pedigree —diagram showing the expression of a specific characteristic and the biological relationships among members of a family, often of several generations.
Phenotype —the observable organism, the expression of a genetic trait.

Sample Lesson Plans

One of the many characteristics determined by parental genotype is eye coloring. Basically, individuals can be characterized as being light-eyed (blues, greens, grays) or dark-eyed (various shades of brown). The following two lessons will use eye coloring to:

1) gather data and draw a distribution chart to describe the various eye color shades, and
2) determine the genotype in a family tree as a classroom lab experiment.

I *Eye Color Distribution*

Eye coloring ranges from very light blues and grays to deep browns and blacks.

Objectives:

-Students will collect information about eye color from class members.
-Students will place the eye color of each class member on a scale of 1-10.
-Students will draw a frequency table and distribution chart to describe the information.
-Students will solve problems of ratio, proportion, and percentages based on the information gathered.

Eye color is based on several genetic factor. It is generally acknowledged that dark eyes are a dominant feature while light eye coloring is recessive. The various shadings of blues, greens, and browns indicate a variety of factors determine an individual's eye color. An example of how the lesson can be utilized and a sample work sheet is listed below.

Lesson

Given a class of 20 students, have students;

1) assign a value (1-10) for each student's eye color (1-very light, 10-black)
2) write a frequency table and design a chart or graph to describe the information gathered.
3) answer work sheet problems.

Sample Frequency Table for Eye Color

Class Size: 20 students

Light	1	1
	2	0
	3	2
	4	0
	5	0
	6	10
	7	3
	8	0
	9	3
Dark	10	1

Sample Distribution Chart

(figure available in print form)

Work Sheet Questions

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1. What percentage of students fall in each eye color range?
2. What is the mode of eye coloring? ___

What is the mean eye color? ___
What is the median eye color? ___

3. If colors 1-5 can be categorized as light-eyed people and 6-10 as dark eyed people, what is the ratio of;
   a) light eyes to dark eyes ___
   b) light eyes to class size ___
   c) dark eyes to class size ___

4. What conclusions can be drawn from the data. Discuss dominant and recessive traits. Discuss family eye color.

II. Sample Worksheet: Family *Eye Coloring Chart and Worksheet*

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1. List family members with the following gene pairings.
   A = dominant (brown)
   a = recessive (light eyes)
   —= unknown

Genotypes: Aa AA A- aa

2. *Estimate* the percentage of dark-eyed and light-eyed people. ___

3. What is the ratio of :
   a) dark eyes to total family members? ___
   b) light eyes to dark to total family members ___
   c) light eyes to dark eyes? ___

4. What is the percentage (to the nearest tenth) of:
   a) dark-eyed family members ___
   b light-eyed family members ___

5. What conclusions can be drawn from the data? Does the data satisfy the Mendelian rules of inheritance? ___

6. use Punnett squares to answer the following questions.
   a) If IV5 marries a blue-eyed gent, what are the possible eye colorings of their children? Answer using percentages.
   b) If IV5 marries a brown-eyed gent with a blue-eyed parent, what are the possible genotypes

of their children? A browneyed gent with brown-eyed parents?
7. Discuss inheritance of eye color based on this pedigree chart.

II. Family Eye Coloring

(figure available in print form)

III. Sample Lesson: *Pedigee Charts*

Label the pedigree chart with the information given. Can you determine if:

1) the ability to curl one's tongue is dominant or recessive,
2) having a widow's peak is dominant or recessive, 3 deduce as many genotypes of individuals as possible.

Place a '-' when it is not possible to accurately determine the genotype.

Hint : Test *one* trait at a time, then determine the genotype of the individual. Use Punnett squares to test your hypothesis. Do the percentages match the actual data/

Individual	Trait
II, II4, III7	curls tongue, has a widow's peak
IIi, 3 III6,8	curls tongue, no widow's peak
112, 6 III3	does not curl tongue, has a widow's peak
I2. II5,7	does not curl tongue,
III1,2,4,5	no widow's peak

Label and fill in the pedigree chart

(figure available in print form)

Sample Group Lab Problem

I. Albinism is a recessive trait. Construct a family pedigree chart that depicts the transference of albinism in 4 generations. Include 30 individuals in the chart, label all symbols.

Resource Bibliography

Mange, Arthur P. and Elaine Johansen. *Genetics: Human Aspects* , Saunders College, Philadelphia, 1980.

Informative text. Special attention to Chapters 1-3 discusses rules of inheritance and pedigree.

Olby, Richard. *Origins of Mendelism* . Constable, London, 1966.

Chapters 2,5—historical overview of the origin of Mendelism and events leading to the development of his theory of genetics.

Strong, Leonell Clarence. *Parental Age and Characteristics of the Offspring* . New York Academy of Sciences. Annals. Mol. 57, Art. 5.

Vegotsky, Allen and White, Cynthia. *A Programmed Approach to Human Genetics* , John Wiley & Sons, Inc., 1974 .

Sequential study of genetics. Good text for self-taught genecists.

Children’s Bibliography

Facklam, Howard and Margery. *From Cell to Clone, The Story of Genetic Engineering* . Harcourt Brace Jonanovich, New York, 1979.

Discusses genetic engineering, particularly the history and techniques of cloning. Includes material on genetic research and test-tube babies.

Morrison, Velma Ford. *There’s Only One You* . Julian Messner, New York, 1978.

Introduces fundamental concepts of genetics. Good middle school reading.

Silverstien, Alvin and Virginia. *The Code of Life* , New York: Atheneum, 1972.

Discusses genetic coding, chromosomes, genes, DNA, and the possibility of treating hereditary diseases through gene manipulation.