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Geometric Shapes in Architecture

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Introduction

Man has always needed shelter. In the earliest days men were nomads whose main occupations were hunting and fishing. In order to survive they moved from place to place very frequently. They were content to live in caves and other temporary shelters. With the advent of agriculture, men were able to settle in more permanent locations, and they built lasting structures to use as homes. It was then that architecture came into being.

As years passed, man's knowledge grew and principles of construction improved. No longer were men satisfied to build houses alone. Now they designed tombs in which to be buried, monuments to serve as memorials, palaces to house the rulers, and churches where they could worship their gods. To produce structures that were functional as well as models of architectural beauty, designers had to apply principles of mathematics in their work. Proper ratios and proportions related each feature of a building with every other one and with the whole structure. Various geometric shapes provided maximum use as well as a pleasing appearance in all types of architecture.

At the present time, many school children in New Haven are unaware of the relation between the mathematics studied in their classrooms and the architecture that surrounds them throughout the city. They shop in the Chapel Square Mall without noticing the simple lines and planes that form the pattern of the building. On their way to concerts in the Veterans' Memorial Coliseum, they pass the Supreme Headquarters of the Knights of Columbus and refer to its cylindrical columns as "tootsie rolls". The Ingalls rink, commonly known as "the whale", stirs up lively conversations about ice skating and hockey without any thought that the backbone of "the whale" is a perfect sine curve. Many Saturday afternoons are spent enjoying football in the elliptical stadium known as Yale Bowl. History students, who visit the graves of notable men in Grove Street Cemetery, seem to be oblivious of the fact that the lovely entrance gate is a trapezoid. When they are visiting friends' homes, young people are too busy to see the wide variety of geometric shapes and designs that abound both outside and inside

In this unit of study we will try to improve the students' understanding and appreciation of basic geometric shapes that are used in architecture. me unit will describe various plane geometric figures. It will discuss in detail the properties of several of these figures. Perimeters and areas of polygons and circles will be computed.

There are several basic objectives for this unit of

study. Upon completion of the unit, the student will be

able to:

—appreciate and enjoy the beauty and charm that exist in the architecture that surrounds him.

—identify simple geometric figures.

—understand the properties of polygons and circles.

—compute areas and perimeters of plane figures.

The material developed here may be used at the following levels of instruction: (1) in seventh or eighth grade arithmetic classes; (2) in high school geometry classes; (3) in high school applied mathematics classes; (4) in adult basic education classes.

Polygons

Polygons are evident in all architecture. They provide variation and charm in buildings. When applied to manufactured articles such as printed fabrics, wallpapers, and tile flooring, polygons enhance the beauty of the structure itself.

The word polygon is derived from the Greek words meaning many angles. A *polygon* is a closed plane figure formed by three or more line segments which intersect only at their endpoints. Each endpoint is common to exactly two segments.

Example: The figures below are polygons

(figure available in print form)

The following figures are not polygons.

(figure available in print form)

Segments that form a polygon are called *sides* of the polygon, and an endpoint of any side is a *vertex* of the polygon. If two sides have a common endpoint, they are said to be consecutive. The endpoints of one side are consecutive vertices. The *angles* of a polygon are the interior angles between adjacent sides. A polygon is named by placing a capital letter on each vertex, moving consecutively around the figure in either a clockwise or counterclockwise direction. If a segment Joins two non consecutive vertices, it is called a *diagonal* of the polygon.

Example : This is polygon ABCDE.

(figure available in print form)

Sides	Vertices	Consecutive Sides	Consecutive Vertices	Angles	Diagonals
AB	A	AB and BC	A and B	ABC	AC
BC	B	BC and CD	B and C	BCD	AD
CD	C	CD and DE	C and D	CDE	BD
DE	D	DE and EA	D and E	DEA	BE
EA	E	EA and AB	E and A	EAB	CE

A polygon is *convex* if each interior angle is less than a straight angle, otherwise it is concave. If all sides are equal, the polygon is *equilateral* , and if all angles are equal it is *equiangular* . A *regular polygon* is both equilateral and equiangular.

Example : *Convex Polygon   Concave Polygon   Regular Polygon*

(figure available in print form)

Polygons are classified according to their sides.

No. of Sides	Kind of Polygon	No.of Sides	Kind of Polygon
3	Triangle	7	Heptagon
4	Quadrilateral	8	Octagon
5	Pentagon	9	Nonagon
6	Hexagon	10	Decagon

Suggested Assignment *: For one week keep a log of all the polygons that you observe at home, on the way to school, and in school. Tell the kinds of polygons that you have seen, the places where you have seen them, and their applications in everyday living.*

Exercises:

1. Which of the following are polygons?
(figure available in print form)
2. Using letters name the following polygons.
(figure available in print form)
3. Name the sides, the angles, and the diagonals of each polygon in example 1.
4. Tell whether the following polygons are convex or concave.
(figure available in print form)
5. Classify each of the following polygons:
(figure available in print form)
6. Are the following polygons equilateral, equiangular, or regular?
(figure available in print form)
7. In the following picture identify as many polygons as possible.
(figure available in print form)
The polygons pictured here are:

____

____

____

Triangles

The triangle is the simplest and one of the most familiar of all polygons. It is used in construction and design of every description. We see it in the framework of buildings and bridges. Because it is a rigid figure, the shape of a triangle cannot be changed when pressure is applied to it. For this reason the triangle provides an excellent support for many structures.

A *triangle* is a polygon that has three sides. The symbol used to denote a triangle is ’ . An *altitude* of a triangle is a segment drawn from a vertex perpendicular to the side opposite that vertex, or perpendicular to that side extended. A *median* of a triangle is a segment drawn from a vertex to the midpoint of the side opposite that vertex. Every triangle has three altitudes and three medians.

Example:

(figure available in print form)

Triangle ABC is shown.

CD is an altitude.

CE is a median.

As its name implies, a triangle has three angles. The sum of the three angles of a triangle is 180 degrees.

Example:

(figure available in print form)

Triangles may be classified by their sides. A *scalene triangle* has no equal sides. An *isosceles triangle* has two equal sides. The equal sides of the isosceles triangle are the legs and the third side is the base of the triangle. If three sides of a triangle are equal, the triangle is *equilateral* . In every triangle, the sum of any two sides is greater than the third side.

Scalene Triangle Isosceles Triangle Equilateral Triangle
(figure available in print form)

Triangles also may be classified by their angles. An *acute triangle* is a triangle in which each angle is less than 90¼. A *right triangle* contains one right angle. The sides that form the right angle are called legs, and the side opposite the right angle is the hypotenuse of the triangle. If a triangle contains one obtuse angle, it is an *obtuse triangle* . An *equiangular triangle* has three equal angles.

Acute Triangle Right Triangle Obtuse Triangle
(figure available in print form)

There is a relationship between the number of equal sides and the number of equal angles in a triangle. If all sides of a triangle are unequal, the angles opposite these sides are unequal in the same order, that is, the largest angle is opposite the largest side, the middle angle is opposite the middle side, and the smallest angle is opposite the smallest side.

In an isosceles triangle, the angles opposite the equal sides are equal. They are called *base angles* , and the third angle of the isosceles triangle is the *vertex angle* . An equilateral triangle is always equiangular.

Suggested Assignment: In your home or neighborhood, identify as many types of triangles as you can. Name the places where triangles are used most often. For what purpose are triangles used in architecture?

Exercises

1.) The sides of ’MNP are ___, ___, and ___.
(figure available in print form)
2.) The vertices of ’ MNP are ___, ___, and ___.
3.) If MR = RN, PR is a ___ of ’ MNP.
4.) If MT is perpendicular to PN, MT is an ___ of ’ MNP.
5.) In ’ ABC, A = 67° and B = 36¼ , C = ___¼.
6.) Can the sides of a triangle be (a) 2”, 3”, 7”? (b) 4”, 5”, 6” ?
7.) In right triangle RST, S is the right angle. (a) The legs of ’ RST are ___ and ___. (b) The hypotenuse of ’RST is ___.
8.) In isosceles ’ XYZ, XY = XZ.

(a) The legs of ’ XYZ are ___ and ___.

(b) The base of ’ XYZ is ___.
(c) The base angles of ’ XYZ are ___ and ___.
(d) The vertex angle Of ’ xYz is ___.
9.) In ’ EFG, E=100°, F = 50°, and G = 30°
(a) The largest side of ’ EFG is ___.
(b) The smallest side of ’ EFG is ___.
lO.) Classify each triangle shown as scalene, isosceles, or equilateral:
(figure available in print form)
11.) Classify each triangle shown as right, obtuse, or equiangular

(figure available in print form)

Quadrilaterals

Another very familiar polygon used in architecture is the quadrilateral. Ceilings, floors, walls, windows and doors usually are quadrilaterals. A *quadrilateral* is a polygon with four sides. The most common quadrilaterals are the parallelogram, rectangle, square, rhombus, and trapezoid.

A *parallelogram* is a quadrilateral whose opposite sides are parallel. The symbol used to denote a parallelogram is . A *rhombus* is an equilateral parallelogram. A *rectangle* is a parallelogram with right angles.

A *square* is an equilateral rectangle.

Parallelogram Rhombus Rectangle Square
(figure available in print form)

A *trapezoid* is a quadrilateral with exactly two parallel sides. The parallel sides of the trapezoid are called the *bases* . The nonparallel sides are called the legs of the trapezoid. If the legs are equal, the trapezoid is isosceles. A line segment drawn perpendicular to the bases is an *altitude* of the trapezoid. The line segment Joining the midpoints of the legs is the *median* . The length of the median is equal to one half the sum of the bases.

(figure available in print form)

ABCD is a trapezoid.

AB and DC are the bases of the trapezoid.

AD and BC are the legs of the trapezoid.

DE is an altitude of the trapezoid.

MN is a median of the trapezoid.

Suggested Assignment: Make a bulletin board to display pictures of buildings cut out of magazines. Identity all the geometric shapes that you see in the pictures. Describe how geometric shapes are used as ornaments as well as parts of structural designs in architecture.

Exercises

1.) Identify the following figures:
(figure available in print form)
2.) ABCD is a parallelogram.
(a) The sum of the angles of ’ ABC is ___ degrees.
(b) The sum of the angles of ’ ADC is ___ degrees.
(c) The sum of the angles of ’ ABCD is ___ degrees.
3.) Measure the opposite sides of ABCD. The opposite sides of a parallelogram are ___.
4.) Measure the opposite angles of ABCD. The opposite angles of a parallelogram are ___.
5.) In parallelogram ABCD draw diagonals AC and BD intersecting at X. Measure DX, XB, AX, and XC. DX and XB are ___. AX and XC are ___. The diagonals of a parallelogram ____ each other.
6.) Measure the diagonals of rectangle EFGH. The diagonals of a rectangle are ___.
7.) The bases of trapezoid QRST are 17” and 23” respectively. The length of the median of QRST is ___.
8.) Draw an isosceles trapezoid. Measure the base angles of the trapezoid. The base angles of an isosceles trapezoid are ___.
9.) Draw the diagonals of rhombus EFGH. Measure the angles formed by the intersection of the diagonals. The diagonals of a rhombus are ___ to each other.
10.) Name the polygons illustrated in the adjacent sketch. What quadrilaterals do you see in the

picture?

(figure available in print form)

Perimeter of a Polygon

The *perimeter* of a polygon is the distance around the “rim” or edge of the figure. Linear units such as inches, feet, meters or miles are used to measure perimeter. To find the perimeter of a polygon, add the lengths of all its sides.

Example:

Find the perimeter of a room that is 23 feet long and 15 feet wide.

Solution:

Perimeter 23 23 15 15 76 feet

Exercises:

Find the perimeter of each of the following figures:

1.)

2.)

3.)

4.)

(figure available in print form)

5.) A rectangular swimming pool is 24 2/3 feet long and 15 5/6 feet wide. How many feet of fencing are needed to enclose the pool?
6.) A room is 18.6 feet long and 12.4 feet wide. How many feet of molding are needed to go around the rooms
7.) In the following floor plan of a house, find the perimeter of each room and the perimeter of the entire building.

(figure available in print form)

Area of a Polygon

In designing homes, offices or other buildings, the amount of living area or working area is of primary concern to the architect. The word area is used to indicate the measure of a surface. Square units, such as square feet or square meters are the standard units used in measuring area. One square unit is a square that measures one unit on each side. The *area* of any polygon is the number of square units that are contained within the figure.

_______________ This is one linear inch. It is used to measure the length of a line segment.

(figure available in print form)

This is one square inch. It is a square each of whose sides is one inch long. It is used as a unit of measure of area.

Area of a Rectangle

In a rectangle any side may be called a base. The altitude to the base, also known as the height of the figure, is a segment drawn perpendicular to the base from a point on the opposite side. The measures of the base and altitude are denoted by b and h respectively. Consecutive sides of a rectangle are perpendicular.

(figure available in print form)

In rectangle EFGH, EF is the base, and HE is the altitude drawn to the base. When each side of the rectangle is divided into unit segments, b 3 units and h 2 units. The number of square units contained in the rectangle is six or three times two. Thus, the area of the rectangle equals the product of the base and altitude.

Area of a Rectangle = Base x Altitude or A=bh

Example:

Find the area of a rectangle whose base is 8.4 cm. and whose altitude is 15.6 cm. in length.

Solution:

A= bh A= 8.4 x 15.6 =131.04 sq. ft.

Area of a Parallelogram

Consecutive sides of all parallelograms are not perpendicular. Although any side of a parallelogram may be called a base, the altitude of a parallelogram is not always a side of the figure. In ABCD, AB is the base and DE is an altitude drawn to AB. The formula to find the area of a parallelogram may be derived as follows:

(figure available in print form)

Area of a Parallelogram = Base x Altitude or A=bh

Example:

Find the area of a parallelogram whose base is 8 1/2 inches and whose altitude is 15 1/4 inches in length.

Solution:

Area= 8 1/2 x 15 1/4 = 17/2 x 61/4= 1437/8 = 179 5/8 sq. in.

Area of a Triangle

The diagonal of a parallelogram divides the figure into two equal triangles. The area of each triangle is equal to one half the area of the parallelogram.

(figure available in print form)

Area of ABCD = AB x h

Area of ABC= 1/2 X AB x h

Area of a triangle = 1/2 x base x altitude

A = 1/2 bh

Example:

Find the area of a triangle whose base is 23.9 ft. and whose altitude is 14.8 ft.

Solution:

A =1/2 bh A-1/2 x 23.9 x 14.8 =1/2 x 353.72 =176.86 sq.ft.

Area of a *Trapezoid*

Altitudes drawn from each endpoint of the upper base separate a trapezoid into three non overlapping polygons-two triangles and a rectangle. The area of the trapezoid is equal to the sum of the areas of these three polygons. A formula to find the area of a trapezoid may be derived as follows:

(figure available in print form)

Area of a Trapezoid = 1/2 x Altitude x Sum of the Bases

Example:

Find the area of a trapezoid whose bases are 10 cm. and 14 cm. and whose altitude is 6 cm. long.

Solution:

A = 1/2h(b1+ b2) A= 1/2 x 6(10 + 14)= 3 x 24= 72 sq. cm.

Suggested Assignment:

Using polygons, design a house or other building. Find the areas of the exterior walls, the roof, the doors, and the windows.

Exercises:

Find the areas of the following figures:
1.
2.
3.
(figure available in print form)
4.
5.
6.
(figure available in print form)
7.
8.
(figure available in print form)
9. At $8.50 a square yard, what is the cost of laying a cement floor in a garage that is 6 yards long and 7 yards wide?
10. At $1.85 a square yard, find the cost of painting the front and rear gables of a house 18 ft. wide, the height of the ridge above the eaves being 10 ft.
11. QRST is a rectangle. Find the area of:

(figure available in print form)

Circles:

Not all geometric figures are formed by straight lines. One of the most useful geometric shapes is the circle. It plays a vital part in our lives—in wheels, in all sorts of containers, in machine parts, in design, and in architecture. The circle provides the most economical form of shelter. Round houses are used in the Arctic and at the equator.

A *circle* is the set of points in a plane equidistant from a fixed point in the plane called the center. The circle receives its name from the center. O is the symbol used to denote a circle.

Several important terms are associated with the circle. A *radius* of a circle is a line segment which Joins the center to any point on the circle. AB is a radius of circle A.

(figure available in print form)

A *chord* is a line segment whose endpoints lie on the circle. CD is a chord of circle A. A *diameter* is a chord which passes through the center of the circle. CE is a diameter of circle A. A *secant* is a line which intersects a circle in two points. FG is a secant of circle A.

A *tangent* is a line which lies in the plane of the circle and intersects the circle in exactly one point. EJ is a tangent to circle A. The *point of contact* is the point at which the tangent intersects the circle. ~ is the point of contact of tangent EJ. The *circumference* of a circle is the perimeter or distance around the edge of the circle. An *arc* is a part of the circumference of a circle. EB is an arc of circle A.

If the circumference of any circle is divided by its diameter, the quotient is always approximately 22/7 or 3.14. This special number is represented by the Greek letter pi (). Hence, the circumference of a circle may be expressed as the product of pi and the length of the diameter of the circle. The formula for finding circumference is C = d or C = 2 r. The area of a circle may be expressed as the product of pi and the square of the length of the radius, or A = r2 .

Example:

Find the circumference and area of a circle whose diameter is 14 inches long.

Solution:

C = d	A = r2
C = 22/7 x 14	A = 22/7 x 7 x 7
C = 44 inches	A = 154 sq. in..

Suggested Assignment:

Write a report on circular houses used in Africa and in the Arctic.

Exercises:

1. Find the circumference and area of a circle whose radius is 5 1/4 feet long.

2. In circle X, identify the following parts:

a. RY e. Line ST

b. XZ f. Line UV

c. YW g. Point R

d. ZW h. Point X

(figure available in print form)

3. Illustrated below are floor plans of a round house, a square house, and a rectangular house. The perimeter of each one is 66 feet. Find the area of each figure Which of the houses has the greatest number of square feet of living area?

(figure available in print form)

4. Name the geometric figures that you see in the illustration of the door below at the left.

5. How many square inches of glass are necessary to fill the window shown below at the right?

(figure available in print form)

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