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Geometric Systems in Architecture

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Statement of Purpose

The study of architecture may be approached from many directions. We could study architecture as an expression of man’s cultural and social values or as an historical tool by which we may see how buildings describe the attitudes of the time. Architecture may be studied as an extension of man, a visual representation of how man creates his building to suit his environment, as an art form. an expression of man’s ideals, desires, and feelings on beauty or as an expression of geometry as applied to structures. Architecture embodies man s need to be part of his space as well as his need to secure shelter and comfort in that total environment. Architecture is more than just constructing tour walls with a roof to keep out the cold and rain. it is a spiritual extension of man’s unity with nature.

Architecture, as with all man’s constructions, has a set basis upon which the philosophy of the structure and the actual construction grows. Our world is dictated by the surrounding space. More simply, the patterns we find in our buildings, in nature, and in our world are dictated by space and are identifiable by man. Man has taken the patterns evident in nature and space and has used them to create his home, his office and his parks.

The purpose of this unit is to expose students to the bases and patterns of architecture. The unit proposes to identify those patterns that space dictates and show how these patterns are manifested in mathematics, in nature, and in architecture. This unit addresses the middle school student and is to be taught as an extension of the geometry unit. This extension unit will cover a two week period in addition to the existing time allotment for a geometry unit. The unit does not attempt to teach architecture, but only to provide some tangible evidence that mathematics, specifically geometry, has learning value for the children as well as application to their lives. It provides students with an awareness and initial understanding of how geometry cooperates with space.

We live in three dimensional space. Our homes, school, place of business, all have height, depth, and width. Our bodies have height, depth, width, and we understand our world by understanding our relationships in three dimensions. Our orientation is that of three dimensions, we perceive the world in these dimensions and yet many of our students have difficulty taking that intuitive leap, breaking through the limitations of two dimensional construct (their own Flatland) to view the world and space fully. While students exhibit frustration and difficulty in visualizing their three dimensional world in a mathematics class, they are still excited and drawn to manipulating geometrical figures such as cubes. cylinders, pyramids, prisms, and spheres. Though students express difficulty in utilizing formulas to determine area, surface area, and volume of geometric figures, they can easily build these objects and see the relationship of area to surface area, size to volume.

This unit attempts to address the students inability to see the world specially by l)providing students with activities that will help them to see objects specially 2)make students aware of how space dictates the forms around us and 3)point out some distinct patterns that appear in nature and architecture.

Unit Work

If you hand a student 18 cubic units and instruct him to build a rectangular prism using the 18 units, the student will create one of four prisms. (see figure 1) Each prism has the same number of cubic units therefore each prism has the same volume even though the shapes of the prisms are different. Students can easily recognize that the volume *of* each prism is determined by the product of the linear measure of length, width and height. Students can discover all possible combinations of side measurements given a volume by writing the factors of the volume number and determining which three factors can be used to construct a prism with this volume.

(figure available in print form)

Through classroom experience we discover many middle school students have difficulty applying a formula to a given set of information and very few students will be able to explain why this formula is true. By constructing prisms with a given number of cubic units, students can derive the formula and therefore understand why these formulas work. Using the tactile senses of the students will help us reach these students who do not learn well visually or audibly. This simple exercise will help students begin to see volume as a three dimensional measurement.

We may also help students take another step into understanding the space around them through use of this simple exercise. Give each student 3 toothpicks and instruct each student to construct four triangles. Most students will fumble with those toothpicks and build only one triangle in a single plane. How many students will step out of the plane of the desk and build a tetrahedron with the toothpicks? I would venture to guess that those students who are able to describe skew lines or have mastered the Rubick's Cube may quickly complete this exercise. The key to this activity is to step out of the 2-dimensional plane in which we compute our arithmetic and draw our geometrical figures and look at our world specially. (see figure 2)

Hold the 3 sticks at an apex to form the tetrahedron. Consider the table top to contain the fourth triangle.

(figure available in print form)

We can use this simple exercise to make students aware of the many avenues they may explore to solve problems. Children love to manipulate their world, to build and create. We can apply this natural ability and tendency to manipulate to the geometry class by providing exercises that will make sense of the 3-dimensional figures we use in finding volume.

If we secure the apex of the construction in figure 2 and provide more toothpicks to build a full tetrahedron, (a small bit of modeling clay at each apex will do the trick) students can feel the strength of this collection of triangles. Man has determined that the triangle is the most stable system. Triangular braces are used to strengthen bridges, buildings, shelves, and a host of other constructions. Many students may argue that a square base or cubic system is more secure. An interesting exercise is to give half the students toothpicks and clay with instructions to build a 3-dimensional system using squares (a cubic system) while the remaining students construct a 3-dimensional system with triangles (a tetrahedral system). The system constructed with triangles will be stronger and more stable than the cubic system. (see figure 3)

Cubic System Tetrahedral System
(figure available in print form)

Figure 3

Pattern Generation

We have begun to show students how to manipulate space to construct specific figures. It is important for the students to recognize that many of the geometric figures exist in nature and in the world around them. Let us first instruct students to manipulate squares. A very simple pattern develops. (see figure 4). Three dimensional cubes may also be created when squares are joined together. (figure available in print form)

Figure 4

More interesting are the patterns and forms that are generated when equilateral triangles are arranged. Two patterns emerge, a pattern in which the triangles are adjacent to one another to form, more or less, a line, and a pattern emerges when six triangles are arranged around a center point, a hexagon appears. (figure 5)

(figure available in print form)

repeating triangles hexagon
Figure 5

These are clearly 2-dimensional patterns that are identifiable in many tile patterns seen in kitchen floors, tile floors, stain glass windows and in ornamentation on buildings.

If we remove one triangle and arrange those 5 triangles around a central point, the only possible construct is a 3-dimensional cup shape. This in the only pattern that can be constructed. The figure will not lie in an 2-dimensional plane. (see figure 6) Seven triangles centered around the center point would result in a curved or saddled shape. (Lesson 4) No other shapes are available when the triangles are adjoined. The three shapes, the 2-dimensional linear pattern of hexagons, the cupped shape of pentagons, and the undulating saddle with 7 triangles, are the only possible constructions.

(figure available in print form)

5 triangles (pentagon) to form a cup shape

Figure 6

Students can manipulate these triangles to discover whether any other patterns can emerge. All other patterns can be reduced to these cubic, hexagonal, or pentagonal shapes. It is also interesting to note that the same geometric shapes or systems can be derived by looking at other collections of shapes. Take, for example, a set of circles placed in a repeating pattern as in figure 7. If we connect the centers, our square pattern will appear.

(figure available in print form)

square system	hexagonal system
Figure 7	Figure 8

When circles are arranged as in figure 8, the hexagonal pattern will appear. Should we stack spheres instead of circles, and connect the centers as shown in figure 10, the tetrahedron would emerge.

(figure available in print form)

tetrahedral 4 point system

(figure available in print form)

Figure 10

It is amazing to see how the same systems and patterns repeat themselves in nature. Students should play and build models to help them understand how these systems appear by themselves, not by some unnatural application of mathematics or science.

Natural Patterns

These same patterns can be found in nature. For the young scientist, it is known that all living things have a carbon base and the carbon atom itself allows 4 opportunities to bond with other elements. The methane molecule described by a carbon atom at the center and 4 hydrogen atoms can take on the appearance of the tetrahedron with the carbon atom at the center. (see figure 11)

(figure available in print form)

methane molecule > as a tetrahedron
Figure 11

Since all living things are carbon based, all other carbon compounds can be described with tetrahedrons.

The crystal structure of minerals found outdoors or on display in museums also exhibit the same kinds of patterns and systems. These crystals reflects the growth of the mineral based on the atomic structure of the mineral. We have reviewed the basic cubic, tetrahedral, and hexagonal patterns. The structure of crystal are described by these same terms. This is not to say that a *perfect* crystal structure can always be found, but traces of the systems can be identified if we have a good specimen. We can study a specimen of pyrite, or fool’s gold, and see that the crystals are cubic. Halite, or rock salt, also exhibits cubic crystals.

Quartz, a mineral common to our area, can be seen in the form of hexagonal crystals while fluorite, a mineral that possesses the property of glowing in ultra-violet exhibits distinct tetrahedral crystals. These crystal formations are indications of the growth patterns of the minerals. In general tetragonal crystals are often long and slender or needle-like. They are characteristically four sided prisms or pyramids. Hexagonal crystals are generally column or prism-like with hexagonal cross sections, while the cubic system exhibits crystals that are very blocky or ball-like in appearance with many similar, symmetrical faces. Again, these same geometrical patterns appear in all of nature.

Students can walk about the city streets and find many examples of repeating patterns. Ornamentation on windows can take the form of square patterns. Tiling on floors generally take the form of hexagonal patterns or some other combination of squares and hexagons. The same patterns persist. The same patterns are dictated by space to appear in nature, in art, in mathematics and in architecture. There is a sense of cohesiveness, of some master plan, an order to our universe. It is evident in the world around us and is manifested in architecture. It will not take a great deal of effort to open the eyes of our students to the knowledge available to them. Their natural curiosity will take over once they've become aware of the 3-dimensional world around them.

Lesson 1 Volume

Objective:

Students understand volume and can derive the formula for volume of rectangular prisms.

Separate students into groups of four. Give each group of students 24 cubic units and instruct them to build or draw as many rectangular prisms as they can given n cubic units. List the factors of each volume number.

Example: 6 cubic units Formula

(figure available in print form)

A. 1 x l x 6 cubic units

B. 2 x 3 x 1 cubic units

Problems: Drawings Formulas

1) 24 cubic units

2) 9 cubic units

3) 16 cubic units

4) 8 cubic units

5) 13 cubic units

Extra: The area of a rectangle is(base x height) and the area of a triangle is 1/2 x base x height.

If the volume of a rectangular prism is (length x width x height), then what do you suppose is the volume of a triangular prism?

Lesson 2 Geometric Patterns

Objective:

Students can recognize geometric patterns found in their environment.

Floor and ceiling tiles are created through the generation of geometric patterns. Instruct students to observe and make a list of the tile patterns they find in their homes, in school, and-in buildings they visit. Ask students to sketch the pattern, list where it can be found, and then identify the geometric shape.

Questions:

What are the only *regular* shapes of tiles that can fit together to cover a flat surface?

What pattern can be seen in a honeycomb?

(figure available in print form)

Lesson 3 *Geometric systems in Crystals*

Objective:

Students can use resource material to obtain information, study data, and draw conclusions based on the data.

It has been found that the tetrahedron is a more solid, stronger system than the cubic system. Listed below are 14 minerals found in Connecticut. Each mineral has either a tetrahedral, hexagonal, or cubic system of crystal growth.

1) Identify which minerals are cubic, tetrahedral and hexagonal.
2) Find the hardness of each mineral. Based on a scale of 1 to 10, hardness indicates how well the mineral resists scratching.
3) Is there any relationship between the crystal structure and the hardness of the minerals?

List of Minerals:

Bornite, Copper, Fluorite, Apatite, Beryl, Galena, Bismuth, Calcite, Garnet, Magnetite, Quartz, Cassiterite, Pyrite, Tourmaline

Bring in a sample of quartz or any other mineral that exhibits a crystal formation. Can you make a guess as to the classification of the crystal system?

Cubic crystals	hardness	Tetrahedral crystals	hardness	hexagonal crystals	hardness

Lesson 4: Geometric Solids

Objective:

Familiarize students with specific geometric solids; Students will construct a cube, tetrahedron, and octahedron.

Cube : Construct a cube by copying the sketch below. Cut on the solid lines and fold on the dotted lines. Tape tabs to create a cube. (each square is 3” x 3”)

(figure available in print form)

a) A cube has ___________ vertices.

b) A cube has ___________ edges.

c) The edge is the intersection of ______________ faces.

Tetrahedron : Construct an equilateral triangle by;

1) Draw a line segment 3 inches long.
(figure available in print form)
2) Set the compass by putting the point at one end of the line segment (x) and the pencil at the other end (y).
3) Swing the compass from the x vertex and make an arc. Place the point on the y vertex and repeat the motion. Draw segments xz and yz.

(figure available in print form)

Use this model to draw the pattern below. Cut on the solid lines and fold on the dotted lines, then tape the tabs to form a tetrahedron.

(figure available in print form)

Complete: A tetrahedron has ______________ faces.

A tetrahedron has ______________ edges.
A tetrahedron has ______________ vertices.
_____________ faces intersect at each vertex.

Make a model of a octahedron, use the equilateral triangles from the previous exercise.

(figure available in print form)

An octahedron has _______________ faces.

An octahedron has _______________ edges.

An octahedron has _______________ vertices.

Is there any pattern to the number of faces, edges, and vertices of these figures?

Figure sides faces edges
Tetrahedron

Cube

Octahedron

Objective:

Demonstrate the shape created when 5 equilateral triangles are attached around a center point.

Instructions:

Cut along solid lines and fold along the dotted lines, then secure the tabs. The resulting figure will be a pentagonal cup shape.

(figure available in print form)

Objective:

Demonstrate the shape created when 7 equilateral triangles are attached about a center point. The resulting pattern will be an undulating saddle.

(figure available in print form)

Objective:

Construct a triangular prism.

(figure available in print form)

Count the number of vertices _________________.

Count the number of edges ____________________.

Count the number of faces ___________________.

Name the kinds of faces _____________________.

Extra: Build an icosahedron (a 20 sided regular figure)

(figure available in print form)

Does the formula demonstrated with cubes, tetrahedrons, and octahedrons hold true for triangular prisms? for icosahedrons?

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