Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
1983 Volume VII: Medical Imaging

Computed Tomography and Nuclear Magnetic Resonance with Mathematical Applications

Curriculum Unit 83.07.04
by Glen A. Hagemann and Joseph R. Cummins

Introduction

We believe secondary students need to be exposed to a complete and comprehensive mathematics education. This is primarily true due to the great technological advancements made in all societies throughout today's world. Most career choices involve specialized knowledge of how, why and where machines and man work. Computers and mathematics have become an international language which generally transcends colloquial dialects. Since this is true, most career choices in this technological age demand a good working knowledge of mathematics.

The function of a secondary mathematics teacher is twofold. First, it is necessary to educate the student in facets of mathematics. The teacher must sharpen both the student's deductive techniques and the student's computational skills. In the process of doing so, the teacher must help expose the student to many possible career choices and specifically, which concepts studied in mathematics apply to what career.

Objective

The work in this unit is specifically for mathematics teachers and students. However, this unit can also be adapted by the science teacher for his students. In our unit we plan to introduce the student to the field of medical imaging. It will specifically help to make the student more aware of computed tomography and the modality of nuclear magnetic resonance. This unit will introduce the student to how each modality operates and to what terminology is used. Mathematics studied in the secondary mathematics curriculum will be developed and how the mathematics applies to both computed tomography and nuclear magnetic resonance will be discussed. Finally, the student will become acquainted with the relative merits of both these modalities and why these modalities are superior to traditional radiological modalities.

In April of 1972, Godfrey Hounsfield a scientist at EMI LTD., England, announced to the world an amazing new imagine technique which he called computerized axial transverse scanning. Today this imaging method is most commonly known as computed tomography or CT. As with most great discoveries CT was the result of many years of work by many different people. The mathematical concept behind CT was laid by an Austrian mathematician J. Radon. Radon, while working with gravitational theory, proved that a two or three-dimensional image could be reproduced from an infinite set of projections. This concept, the basis of CT imaging was established 55 years before a CT scanner was produced.

Computed tomography has successfully overcome many of the limitations of conventional x-ray technology; namely, in showing structures in three-dimensions and in its display of soft tissue. CT achieves a solution to these limitations in a unique and successful way. CT measures the attenuation of x-ray beams passed through the body from hundreds of different angles, and then, from the evidence of these measurements, a computer is able to reconstruct pictures of the body's interior. Attenuation will be explained later on in this paper. These pictures are based upon separate examinations of a series of continuous cross sections. It would be as if we were looking at the body separated into a series of slices or tomograms. It can be compared to taking a slice from a loaf of bread and examining the slice. It is possible to obtain virtually total three-dimensional information about the body when we use CT.

Computed tomography's most important characteristic is its sensitivity. It allows soft tissues such as the liver to be clearly differentiated, which is not the case in conventional radiography. CT can also very accurately measure the amount of x-ray absorption of tissues, thus enabling the nature of the tissue to be studied. CT is not only a diagnostic tool but it also plays an important role in therapy by locating a tumor, indicating the area of the body to be treated, and by monitoring the progress of the treatments afterwards.

Computed tomography does not primarily provide a picture but provides numerical values of tissue density (the attenuation numbers). The data is then provided either in the form of a computer printout or displayed on a cathode ray tube (CRT) from which a permanent photograph can be made (see diagram A).

The pictorial display is produced in a specific range of gray tones which are assigned attenuation values (see diagram B). This gray scale extends from black to white. Black usual represents -500 (air), O represents water, and +500 represents bone. The intervening soft tissues are represented as shades of gray. It is possible for the number of shades to vary depending on the make of scanner being used. Also today's scanners can produce colored representations.

The image reconstruction is based upon the x-ray attenuation data obtained during a scan. As x-rays pass through the various tissues of the body, a portion of the beam is absorbed or scattered. This loss of x-rays from the x-ray beam is called attenuation and differs in various tissues. See section on attenuation which appears later on in the paper. As previously mentioned, graded shades of gray are assigned to different degrees of attenuation. The production of a spatial image is done by the mathematical process called convolution, or filtered back-projection. In this method, a series of attenuation profiles obtained at different angles of view are mathematically back-projected onto the spatial matrix to produce the final image.

The matrix is simply a mathematical grid, rectangular array, that divides the scan into many small individual picture elements. This matrix, grid, is the framework for the computer reconstructed image obtained in CT scanning but it is never visualized. The fineness of the matrix refers to the number of picture elements in the matrix (pixels) and is expressed by giving the number of picture elements along each axis of the grid (examples 80 x 80, 160 x 160). Resolution improves as the size of the matrix increases which in effect produces smaller pixels. However, what needs to be decided is if somewhat better resolution is worth the risk of increasing the doses of radiation necessary to improve resolution.

The original EMI scanner was designed specifically for the brain (see diagram C). The patient remained in one position throughout the scan. The gantry, the portion of the scanner which houses the x-ray source and the detectors, and which provides a mechanism to move these around the patient, moved through two types of motions, one linear and one rotary. The total scan time with the original EMI scanner was 4.5 to 5 minutes (see diagram E).

In the original EMI scanner, the CT image was reconstructed and displayed on an 80 x 80 matrix in two different ways; a paper printout of CT numbers and a visual image on a CRT. The CT numbers were proportional to the linear attenuation coefficient. As mentioned earlier, each square in the image matrix is called a pixel and represented a tiny elongated block of tissue which was called a voxel (diagram F).

The method used in the EMI scanner to produce an image was an iterative method called (ART), the algebraic reconstruction technique, or ray by ray correction. First of all the term iterative refers to a method of successive approximations in which an arbitrary starting image is selected, corrections are applied to bring it into better agreement with the measured projections, and then new corrections are made until satisfactory agreement is obtained.

With ray by ray correction, at the start of each iteration, one ray sum is calculated and corrections are applied to all points that contribute to the ray. This procedure is then repeated for the next succeeding rays always carrying forth previous corrections in each new calculation until all rays in all projections have been treated, thus completing one iteration (see diagram D). This method works best if the corrections are made one projection at a time, with large angles between consecutive projections.

The desire and need to see cross-sectional and transverse images of the human body has been relieved with the development of CT. CT scanning is an excellent diagnostic modality which allows planar slices of the body to be studied. It has anatomic resolution which allows viewing three-dimensional images of the body in superb detail. Also, CT scanning uses short projection times which helps eliminate the fear of voluntary or involuntary bodily movements which interfere with the resolution of subsequent images. Although CT scanning has revolutionized medical imaging in diagnosis and treatment of anatomical disorders, there are areas in which it has proven to be inadequate. These inadequacies have led to the development of Nuclear Magnetic Resonance (NMR) as a new imaging modality.

In computed tomography a cross sectional layer of the body is divided into many tiny blocks like the ones shown in diagram F and then each block is given a number which is proportion to the degree that the block attenuated the x-ray beam. The composition and thickness of the block, in addition to the quality of the beam, determine the degree of attenuation. The linear attenuation coefficient (μ) is used to quantitate attenuation. In the simplest of cases, an isolated block,voxel,the value of u, the linear attenuation coefficient, can be calculated with the following equation; $N = Noe^{-\mu x}$. In this equation,e is the base of the natural logarithm 2.718). The number of initial photons (N0),transmitted photons (N), and the thickness (x) can all be measured. The linear attenuation coefficient (μ) is the only unknown in the equation (see diagram G-1).

If two blocks of tissue with different linear attenuation coefficients are placed in the path of the x-ray beam, the example becomes somewhat more complex. In this situation the equation will have two unknowns, u1and u2, and will be as follows; N = No e-(u,+μ2)x The values of u and u can't be determined without acquiring additional information. At the very least one additional equation is required, and the equation must contain the same two unknowns. Additional equations can be obtained by examining the blocks from different directions. We will increase the number of blocks to four, so that each reading will represent the composite of two blocks (see diagram G-2). We have now gone from two to four unknowns, however,we can now construct four different equations as follows; N1 = Noe-(μ+μ2) x = Noe-(μ3+μ8) N3= Noe-(μ,+μ3x N4= Noe—(μ2+μ4)x. This set of equations looks complicated, but the equations can all be solved for the value of the linear attenuation coefficient with the assistance of a computer (see diagram G-3)

In the original EMI scanner,the matrix in the computer contained 80 x 80 or 6,400 separate picture elements. Each transmission measurement records the composite of 80 separate linear attenuation coefficients (see diagram G-4). The equation is longer and looks very complicated,but it has the exact same format as the preceding examples. N = Noe-(μ,+μ2+μ34 . . . μ 80)x

In the mathematics portion of our unit we chose to deal with matrix algebra because of its relationship to computed tomography image reconstruction. We believe that students too often are taught mathematics in a vacuum,and that by showing how mathematics is used in the field of medical imaging we feel that our students will begin to see the relevancy of the material that they are learning.

NMR imagine differs from CT scanning in that it is able to differentiate to a greater degree between soft tissues. This is particularly true with regard to pathological lesions and tumors. NMR is more tissue sensitive an d is able to view lesions of the brain in their developmental stages, and is also able to show a pronounced image of the lesion. CT, (diagram 10) however, rarely discriminates between lesions and normal tissue until the disparity has grown pronounced. This is also true of tumors. CT images sometimes do not show tumors until they've altered the shape of the organ in which they are located. NMR easily detects tumors and identifies them at their early stages of development. Also, depending upon the technique employed in NMR imaging, tumors will appear lighter or darker in comparison to their surrounding tissue. Finally, NMR, unlike CT, is able to discriminate between gray and white brain matter and ultimately will probably prove superior to CT in the study of the brain (diagram 9).

The primary difference between CT scanning and NMR imaging, however, is technical. CT scanning operates upon the principle of x-ray attenuation and absorption. Therefore, it causes potentially damaging radiation to enter the body. NMR in contrast does not use x-ray attenuation or absorption, it operates upon the use of magnetic fields. The body is placed within an external magnetic field and is pulled with radio-frequency waves. Unlike radiation, these radiofrequency waves have not been shown to produce cellular damage within the human body.

Only nuclei which have an odd number of protons or neutrons are studied in NMR imaging, and those most commonly studied are hydrogen nuclei. This is due to the high density of hydrogen found within the human body. Not only is the body composed of 75 percent water, but it's also composed of lipids and proteins, all of which contain hydrogen. Therefore, hydrogen is easily accessible for the application of the NMR imaging process.

Protons possess innate magnetic spins which causes them to act like tiny bar magnets, an d consequently, due to these magnet-like properties, the protons are called dipoles (diagram # 1). In the absence of an y external magnetic field, the dipoles will spin about their axes and randomly align themselves within their molecular environments (diagram # 2).

In NMR imaging, hydrogen dipoles within the human body are placed into a uniform magnetic field called Bo. At this time, the dipoles will change their random orientation and align themselves parallel or antiparallel (opposite direction) to Bo (diagrams # 3 and # 4). A lower energy state is associated with those protons parallel to Bo, and a higher energy state with those protons an tiparallel to Bo.

Once uniformly aligned, the protons will begin to rotate about Bo which is defined by the z-axis. This rotation about the z-axis is called precession, and results from the interaction between each dipole and the uniform magnetic field. The protons precess at a frequency which is often called the Larmor frequency (diagram 5)

When the body is placed within the external magnetic (diagram 7) field, it's also placed within a coil from which a radiofrequency pulse (RF) is emitted. The RF pulse generates a small magnetic field itself which rotates in an x,y plane and is perpendicular to Bo and the z-axis. When the RF pulse is chosen to exactly match the Larmor frequency, some of the aligned nuclei are displaced from the z-axis.

If the RF pulse is correctly chosen and displacement occurs, the nuclei will absorb energy and either begin to rotate within the x,y plane at right angles with Bo, or they may flip over at 180 $^{\circ}$ angles. During this process, the nuclei's equilibrium will be disturbed, and they will move from their lower energy states to higher energy states (diagram 6).

With the cessation of the RF pulse, the nuclei continue to rotate in the x,y plane. During this process, energy is emitted by the nuclei and is picked up in a receiving coil or antenna in an electrical signal called the free induction decay (FID). The FID signal is generated much like the induced electrical signal in a dynamo by induction when the nuclei begin to return to their equilibrium positions established prior to pulsation of the radiofrequency wave. The FID signal constitutes a complex wave much like the cacophony of sounds heard on a city street. It possesses multiple frequencies analogous to the noise heard during rush hour traffic.

Another concept involved with NMR imaging and one which works in conjunction with the FID signal is relaxation. Two relaxation times, T1 and T2, characterize the nuclei's return to equilibrium. T1 represents the time required for the nuclei to return to their aligned state while T2 is basically the rate of decay of the FID signal. Collectively, T1 and T2 represent the molecular structure in which they are found. The amplitude of the FID signal is presented within a coordinate plane and is measured as a function of relaxation time at each point the FID is recorded. NMR images depict maps of hydrogen density, T1, T2 or complex combinations of these. Images that depend on, for example, T1 strongly may show structures or abnormalities not seen on images that depict hydrogen density alone, but the best choice depends on the clinical problem presented.

The image displayed by the NMR process is produced in much the same way as CT scans. The coils which are used to transmit the radiofrequency waves may also be used to detect the signal dispensed by the nuclei. The signals are sent to the computer which consequently performs complex mathematical transformations and which also produces the NMR images. Those images produced by hydrogen nuclei have excellent resolution and may be displayed according to a gray scale, or, in color. Initially, the NMR images are viewed on a TV monitor but may be transferred to magnetic tape for permanent storage(diagram 8).

The mathematics we will cover in our unit will be matrix algebra. We will attempt to ease the students into matrices by first explaining the relationship between CT and matrices and then by using "magic squares", which themselves can be related to matrix mathematics, as an attention grabber. We find that applicable puzzles and games are enjoyed and attempted by most students. We believe that by introducing our mathematics unit with "magic squares", we can catch and maintain student interest throughout our unit on matrix algebra.

Before beginning the operations which we will cover with matrices, we find it necessary to define some basic terms used in matrix algebra.

Definitions

matrix—A rectangular array of objects. The objects are arranged in rows and columns.
matrices—The plural of matrix.
entry—Each number or variable within the matrix.
dimension or size—The number of rows by the number of columns of the matrix. The number of rows is always given first.
row matrix—A matrix having one row.
column matrix—A matrix having one column.
square matrix—A matrix having the sam e number of rows and columns.
zero matrix—A matrix where each entry is a zero.
identity matrix—A matrix where the entries in the main diagonal are ones and all the other entries are zeros.

A magic square is simply a square that possesses equal amounts of boxes in each row an d column. A three by three magic square, for example, has three boxes per row and three boxes per column. There are nine boxes altogether. The objective of a 3 x 3 magic square is to take the natural numbers 1 through 9, and place them into the magic square in such a way that all rows, columns, and corner to corner diagonals have equivalent sums. This sum is called the constant for the square.

Two basic types of magic squares will be discussed. They are odd-order magic squares and even-order magic squares. Odd order magic squares are 3 x 3 matrices, 5 x 5 matrices, 7 x 7 matrices and so on. They are defined as any magic square with an odd number of boxes in each row and in each column. Even order magic squares are those matrices with an even number of boxes in each row and in each column. Examples of even ordered magic squares are 4 x 4 matrices, 6 x 6 matrices, 8 x 8 matrices and so on.

The first step in solving the magic square is to identify the constant. Once the constant is known, it is possible to complete the puzzle by trial and error. Three methods are presented for determining the constant of a 3 x 3 magic square.These methods may be generalized to all odd-order magic squares. (see lesson #1)

Once a certain amount of frustration has been experienced through trial and error, precise and accurate methods may be learned. They are presented as staircase methods A and B. (see lesson #2)

When working with even-order magic squares, one simple method cannot be developed. The patterns and solutions of each individual magic square are dependent upon which natural numbers the magic squares are multiples of. For example, the solutions of a 4 x 4 matrix and a 6 x 6 matrix are not similar because 4 is a multiple of both 2 and 4 while 6 is a multiple of 2,3 and 6. Although 2 is a factor common to both 4 and 6, all other numbers are not. Their differences are enough to change the solutions of the 4 x 4 and 6 x 6 magic squares.

Lesson #1

---

3 x 3 magic square —methods to determine the constant of the square.

I. (1) Add the first and last numbers of the square.
(2) Multiply the sum by the number of boxes in a row.
(3) Divide the product by two to get the constant.
II. (1) Add all the numbers in the sequence.
(2) Divide the sum by the number of boxes in a row.
III. (1) Make a diagram.
(2) Place the number 1 in the box at the upper left
corner.
(3) Fill in the remaining boxes of the top row with the numbers in their natural sequence.
(4) Continue until the last number is placed at the lower right corner.

(figure available in print form)

Lesson # 2

Method A: Staircase Method

1. Place the number 1 in the middle box of the top row.

(figure available in print form)

2. Place the next natural number in the box diagonally upward to the right. This number will rot be in the magic square.

(figure available in print form)

3. Take the number placed outside the magic square and place it into the corresponding box inside the magic square.

(figure available in print form)

4. When the number to be placed is going to go into a box that is already occupied, place it immediately below its preceding number.

(figure available in print form)

5. Continue filling in the magic squares in this way until all boxes have a natural number between 1 and 9.

(figure available in print form)

Lesson #3

Method B: Variation of the staircase method

1. Place the number 1 in the box horizontally to the right of the center box.

(figure available in print form)

2. Fill in the sequence by advancing diagonally upward to the right until a number is placed outside the magic square. When a number is placed outside the magic square, place it into its corresponding position inside.

(figure available in print form)

3. When a move is blocked, place the number two boxes horizontally to the right of its predecessor.

(figure available in print form)

Although, there are 880 ways to organize the natural numbers 1 through 16 only two lesson plans are included. The other 878 can be explored on your own.

Method A:4 x 4 magic square

1. The natural numbers 1,2,3, . . . ,16 are used in the 4 x 4 magic square.

(figure available in print form)

2. Beginning in the top left hand corner cell, place the numbers in consecutive order until all cells are filled.

(figure available in print form)

3. Leave all the numbers in their current locations except those in the corner to corner diagonals.

(figure available in print form)

4. Reverse the order of the numbers in the corner to corner diagonals.

(figure available in print form)

5. Check the sums of each row, column and corner to corner diagonal. They should all equal 34.

(figure available in print form)

Method B.: 4 x 4 magic square

1. The natural numbers 1,2,3, . . . ,16 are used in the 4 x 4 magic square.
2. A complement is defined as a pair of numbers that are equidistant from the center box. To determine the complement, add the first and last numbers in the sequence. (1 + 16 = 17), therefore, the complement equals seventeen.
3. Place the natural numbers 1 through 16 in consecutive order beginning with the top left-hand corner cell.

(figure available in print form)

4. The numbers in the corner to corner diagonals remain the same but all remaining pairs of complementary numbers are to “switch” locations.

(figure available in print form)

5 Check the sums of each row column, and corner to corner diagonal. They should all equal 34.

(figure available in print form)

Matrix Algebra

Matrix Addition

1. In order to add matrices the matrices must have the same dimensions.
2. When adding matrices we add corresponding entries from the matrices. The sum of these entries listed in a new matrix is the sum of the matrices.
3. The sum of any matrices is a matrix with the same dimension as the dimensions of the matrices being added.

Examples:

Sample #1 Add:

(figure available in print form)

Solution:

(figure available in print form)

Sample #2 Add:

(figure available in print form)

Solution:

(figure available in print form)

In sample #2, the sum is a zero matrix because the matrices being added are opposites (additive inverses) of each other, and when opposites are added zero is always the sum.

Sample #3

Add:

(figure available in print form)

Solution: These matrices cannot be added because they do not have the sam e dimensions.

Matrix subtraction

1. As in addition, in order to find the difference between two matrices, the matrices must have the same dimensions.
2. In order to find the difference between two matrices we find the difference between corresponding elements.
3. Change each entry in the matrix which is the subtrahend to its additive inverse (opposite) and then change the sign of operation to addition and then simply follow the rules for adding matrices.

Examples:

Sample #1 Simplify:

(figure available in print form)

Sample #2 Simplify:

(figure available in print form)

Sample #3 Simplify:

(figure available in print form)

This example cannot be simplified because the matrices do not have the same dimensions.

Matrix multiplication

There are two kinds of products that involve matrices The first is a scaler product which is the product of a real number (a scaler) and a matrix. The product is a matrix in which the entry in any particular address is simply the product of the scaler and the entry in the same address in the given matrix.

Examples of a scaler product:

Sample #1

(figure available in print form)

Sample #2

(figure available in print form)

The other matrix multiplication product is the product of two matrices.

Multiplication of a row matrix by a-column matrix

1. The number of entries in the row matrix must equal the number of entries in the column matrix or it is not possible to multiply.

Examples:

Sample #1

(figure available in print form)

Sample #2

(figure available in print form)

Sample #3

(figure available in print form)

Sample #3 is an example where multiplication cannot take place because the matrices do not have the same number of entries.

Multiplication of a matrix by a column matrix

1. The result of this type of multiplication is always a column matrix with the same number of entries as the number of rows in the non-column matrix. The product's first entry is obtained by multiplying R1 of the first matrix by the column matrix. The second entry of the product is obtained by multiplying R2 of the first matrix by the column matrix. This progression is continued until the final entries are multiplied.
2. In order to do this multiplication, the number of columns must equal the number of entries in the column matrix.

Examples:

Sample #1

(figure available in print form)

The product will be a column matrix. We compute the product as follows.

(figure available in print form)

Therefore, the first entry in the column matrix (product) is 12. We then continue to compute the product.

(figure available in print form)

Thus the answer is:

(figure available in print form)

Sample #2

Multiply:

(figure available in print form)

Since the first matrix has only 3 columns and the column matrix has 4 entries these matrices cannot be multiplied.

Multiplication of one matrix by another

1. First of all, the number of columns in the first matrix must equal the number of rows in the second matrix or the matrices cannot be multiplied.
2. If the requirement above is met, then multiply the first matrix and the first column of the second matrix. List the resulting column matrix. If there are other unused columns in the second matrix, multiply the first matrix and the next column of the second matrix. List the resulting column matrix. Continue until completed, then form a matrix whose columns are the listed columns, written in the order obtained. This matrix is the answer.

Examples:

Sample #1

Simplify:

(figure available in print form)

Sample #2

Simplify:

(figure available in print form)

Sample #3

(figure available in print form)

Since the first matrix has 4 columns an d the second matrix has 3 rows, they cannot be multiplied.

Solving linear systems

1. When you have the system of two linear equations

where A= the system of two linear

(figure available in print form)

equations may be represented by the matrix equation AX=C.

2. When A is invertible, this matrix equation has an unique solution; $X = A^{-1}C$.
3. Therefore, when A is invertible the linear system has a unique solution. The solution may be found by solving the matrix equation.

Example:

Solve: 4x Ðy = 5
3x + 2y = 12

The above system may be represented by the matrix equation:

(figure available in print form)

Since has an inverse. Therefore, the

(figure available in print form)

equation has a unique solution.

(figure available in print form)

Therefore, x=2 and y=3

CT Glossary

---

artifact—A portion of the CT image that does not represent anything really present in the patient.
attenuation—The loss of x-rays from the x-ray beam.
gray scale—Range of gray tones used in a CT scan display.
image reconstruction—The production of a spatial image based on the x-ray attenuation data obtained during a scan.
matrix—A mathematical grid that divides the scan into many small individual picture elements
patient position—The orientation of the subject in space.
a. supine—The dorsal surface of the patient is against the the bed.
b. prone—The ventral surface of the patient is against the bed.
c. Lateral Decubitus—The stated side of the patient is against the bed.
resolution—A measure of the ability of an imaging system to distinguish and discriminate between small structures.
tomogram—An image of a body slice in a specific plane.The tomogram is like a slice pulled from a loaf of bread.

NMR Glossary

angular momentum—A vector quantity expressing the intensity of rotational motion.
lattice—Nuclear environment within which exchange of magnetic energy occurs by spin-lattice relaxation.
magnetic moment—A vector quantity whose magnitude measures the torque exerted on a magnetic system when placed in a magnetic field.
magnetic resonance—The response of nuclei to discrete radiation frequencies and magnetic fields which satisfy the Larmor condition.
precession—A comparatively slow gyration of the rotational axis of a spinning body about another

line intersecting it so as to describe a cone. Caused by the application of a torque tending to change the direction of the rotation axis.
torque—A force that produces or tends to produce rotation.
transmitter—Electronic device generating radio frequency waves.
vector—quantity characterized by a magnitude and a direction.

BIBLIOGRAPHY

1. *An Introduction to the Physics of Diagnostic Radiology;* Edward E. Christensen, Thomas S. Curry, III, James E. Dowdey, Second Edition; Lea an d Febiger; Philadelphia, 1978.

2. *Computerized Axial Tomography* ; by J. Gambarelli, G. Guerinel, L. Chevrot, M. Mattei; Springer-Verlag; Berlin Heidelberg, New York, 1977.

3. *Computerized Tomography 1977;* Edited by David Norman, Melvyn Korobkin, Thomas H. Newton, M.D.S.; Distributed by The C.U. Mosby Company, 11830 Westline Industrial Drive, St. Louis, Missouri 63141.

4. *Computed Tomography, Ultrasound and X-ray* ; An Integrated Approach 1980; Edited by Albert A. Moss, M.D., Henry I. Goldberg, M.Y.; Academic Fress Inc., New York.

5. Doyle FH. Pennock JM. Banks LM., McDonnell MJ, Bydder GM, Steiner RE, Young IR, Clarke GJ, Pasmore T, Gilderdale DJ: *Nuclear Magnetic Resonance (NMR) Imaging of the Liver* ; AJR 138:193, 1982.

6. *Introduction to Practical High Resolution Nuclear Magnetic Resonance Spectroscopy;* by William H. Oldendorf; New York, 1982.

7. Lauterbur PC: *Image Formation by Induced Local Interactions* : examples employing nuclear magnetic resonance; Nature 242: 190, 1973.

8. *Mathematic of Radiology and Nuclear Medicine* ; by Herbert L. Jackson, Ph.D.; Warren H. Green Inc.; St. Louis, Missouri, 1971.

9. Moran, Jim; *The Wonders of Magic Squares* ; New York; Vintage Books, 1982.

10. *NMR Imaging in Medicine* ; I.L. Pykett in Scientific American; Volume 246, No. 5, pages 2-12; May, 1982.

11. *Principles of NMR Imaging* ; I.L. Pykett, J.H. Newhouse, F.S. Buonanna, T.J. Brady, M.R. Goldman, J.F. Kistler and G.M. Fohost in Radiology; Volume 143, pages 157-168; April, 1982.

12. *Shaw, D. Fourier Transform NMR Spectroscopy; Elsevier;* Amsterdam, 1976.

Diagram “A” Numerical value of tissue density of the brain.
(figure available in print form)
Diagram “B” “Gray Scale” For computed Tomography of the Brain
(figure available in print form)
Diagram “C” “Original EMI Scanner”
(figure available in print form)
Diagram “D” An example of an iterative reconstruction technique
(figure available in print form)
Diagram “E” showing the linear and rotary motions of a CT scanner
(figure available in print form)
Diagram “E1” First generation scanner (the original EMI unit)
(figure available in print form)
Diagram “F”
(figure available in print form)
Diagram “G-1”
(figure available in print form)
Diagram “G-2”
(figure available in print form)
Diagram “G-3”
(figure available in print form)
Diagram “G-4”
(figure available in print form)
Diagram #1 Magnetic nuclei behave like small bar magnets.
(figure available in print form)
Reproduced with permission from General Electric, August, 1983.

Diagram #2 Spins are randomly oriented without presentation of an external magnetic field.

(figure available in print form)
Reproduced with permission from General Electric, August, 1983.

Diagram #3 Realignment of the nuclei when placed into a magnetic field.

(figure available in print form)

Reproduced with permission from General Electric, August, 1983.

Diagram #4 Nuclei align themselves with the magnetic lines of induction.

(figure available in print form)

Diagram #5 Precession of a magnetic nucleus.

(figure available in print form)

Diagram #6 The net magnetic moment of each nucleus is displaced when bombarded by the radiofrequency pulse.

(figure available in print form)

Diagram #7 Schematic diagram illustrating NMR apparatus.

(figure available in print form)

Diagram #9—A. CT scan of the normal brain.

(figure available in print form)

B. NMR image of the normal brain.

(figure available in print form)

Diagram #10—A. CT scan of multiple sclerosis showing a brain lesion.

(figure available in print form)

B. NMR image of multiple sclerosis showing more pronounced brain lesions.

(figure available in print form)

Diagram “H” A CT gantry

(figure available in print form)

Diagram “I” A CT computer console

(figure available in print form)