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Mathematics in Architecture

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Introduction

Buildings are used by everyone for a variety of reasons. They serve as dwelling places and shelters for man. They also provide places in which one may conduct a business, care for the sick, teach young people, or spend leisure time. The building that is most familiar to each of us is our home. We all dream of having a home that is functional and beautiful.

To produce structures that are functional as well as models of architectural beauty, designers must apply principles of mathematics in their work. Scale drawings, commonly known as plans are used as patterns in the construction of buildings. Proper ratios and proportions relate each feature of a building with every other one and with the whole structure to obtain a pleasing appearance.

In this unit of study we will try to improve the students' understanding and appreciation of basic mathematical principles used in architecture. The unit will discuss in detail the concepts of measurement, scale drawing, ratio, proportion, and symmetry.

There are several basic objectives for this unit of study. Upon completion of the unit, the student will be able to:

— use a ruler easily and accurately to determine measurements.

— construct and interpret scale drawings.

— understand the principles of ratio, proportion, and symmetry.

— apply these principles in the solution of problems.

The material developed here may be used at the following levels of instruction: (1) in seventh or eighth grade arithmetic classes; (2) in high school algebra or geometry classes; (3) in high school applied mathematics classes; (4) in adult basic education classes.

Measurement

Measurement is essential in every phase of building, from the planning stage to the actual construction work. The instrument used to measure objects is the *ruler* . The architect must use a ruler to produce accurate drawings. Carpenters and other construction workers must use a ruler to obtain measurements for lumber, piping and other necessary materials.

Rulers are divided into inches and fractions of inches. On the ruler shown below every inch is divided into sixteen equal sections, each of which is one sixteenth of an inch long.

(figure available in print form)

We read the designated measurements as follows:

A= 4/16 or 1/4 inch	D = 2 inches
B= 9/16 inch	E = 2 11/16 inches
C=1 5/16 inches	F = 3 12/16 or 3 3/4 inches

Exercises

1. Draw line segments having the following measurements: a) 2 7/9 " b) 15/16 " c) 1 3/4 " d) 5 1/2 ".
2. Give the ruler reading for each of the lettered dimensions.

(figure available in print form)

Scale Drawing

When a new building is being designed, the architect must convert his ideas to drawings. These drawings enable homeowners, contractors, carpenters, and others to know exactly what the architect has in mind. They show the sizes, shapes and arrangements of rooms, structural parts, windows, doors, closets and other important details of construction. The pictures are miniature reproductions of the building and are called *scale drawings* .

Scale drawings which represent the parts of a building must be in exact proportion to the actual structure. Various scales may be used for this purpose. For example, 1/8 inch may be used to represent one foot, that is, instead of drawing an object one foot long, one would draw it 1/8 inch long. One of the most common scales used by architects is 1/4 inch = one foot.

Example 1: Using the scale 1/8 inch = 1 foot, complete the following:
a. 12 feet are represented by inches.
b. 1 7/8 inches represent feet.

Solution : a. foot= 1/8 inch

12 feet = 12 x 1/8 = 1 1/2 inches.

12 feet are represented by 1 1/2 inches.

b. 1/8 inch = 1 foot

1 7/8 $dv 1/8 = 1 7/8 x 8 = 15 feet.

1 7/8 inches represent 15 feet.

Example 2 : Using the scale 1/8 inch = 1 foot, draw a line segment to represent 24 feet.

Solution : 1 foot = 1/8 inch.

24 feet = 24 x 1/8 = 3 inches.

The scale drawing that represents 24 feet must be 3 inches long.

3 “ = 24 feet

Example 3: Measure the length and width of rectangle ABCD. Using the scale 1/4 inch = 1 foot, express in feet the length and width of the actual figure represented.

(figure available in print form)

Solution : The length of the rectangle is 2 inches and its width is 1/2 inch. The dimensions of the actual figure are obtained as follows:

1 foot = 1/4 inch

Length	Width
1/4 inch = 1 foot	1/4 inch = 1 foot
2 $dv 1/4 = 2x4 = 8 feet	1/2 $dv x 4= 2 feet
Actual Length= 8 feet	Actual Width = 2 feet

Exercises:

1. Using the scale 1/4 inch = 1 foot, find the actual length in feet represented by the following lengths on the drawing: (a) 3 in. (b) 2 1/4 in. (c) 4 3/4 in.
2. Using the scale 1/8 inch = 1 foot, how long a segment should be drawn to represent an object whose actual length is (a) 32 ft. (b) 5 yds. (c) 12 ft. (d) 4 ft.?
3. Make a scale drawing of a rectangular shaped room whose dimensions are 14 feet by 24 feet.
4. Make a scale drawing of the floor plan shown, using the scale 1/4 inch = 1 foot.

(figure available in print form)

5. Measure the length and width of your classroom. Choose a convenient scale and make a scale

drawing of the floor plan.

6. Using the scale 1/4 inch = 1 foot, find the actual dimensions of each room in the diagram on the next page.

Suggested Assignment **:**

Measure the rooms in your home. Using the scale 1/4 inch= 1 foot, make a drawing showing the shapes and arrangement of the rooms on each level. Indicate windows by the symbol , fireplaces by the symbol , and doors by a break in the line .

(figure available in print form)

Ratio

From earliest times the Greeks and Romans were preoccupied with building structures that were pleasing to the eye. They were convinced that architectural beauty was obtained by the interrelation of universally valid ratios. Frequently complicated mathematical ratios were used by architects to accomplish their goals.

A *ratio* is a comparison by division of two quantities expressed in the same unit of measure. The ratio may be expressed in words or in symbols. For example, if segment AB is 1 inch long and segment CD is 2 inches long, we say that the ratio of AB to CD is 1 to 2. In symbols, the ratio may be expressed as the fraction 1/2, or it may be written in the form 1:2. The fraction line and the symbol : are taken from the division sign $dv.

Example 1:	The length and width of a room are 22 feet and 14 feet, respectively. Express in three different ways the ratio of the length of the room to the width in simplest form.
Solution :	(1) 22 to 14 or 11 to 7.
	(2) 22/14 or 11/7
	(3) 22:14 or 11:7
Example 2 :	A door is 30 inches wide and 2 3/4 yards high. What is the ratio of the width to the height of the door?
Solution :	Width = 30 inches
	Height=2 3/4 yds. = 2 3/4 x 36=11/4 x 36=99 in.
	The ratio of the width to the height is 30 to 99 or 10 to 33.

Exercises **:**

1. Using a ruler, measure line segments AB, BC, CD, AC, BD, and AD. Evaluate the following ratios:

a.) AB:BC c.) AB:BD e.)BD:BC g.) CD:AB

b.) BC:CD d.) BC:AD f.)AC:CD h.) AD:CD

2. Express each of the following ratios in lowest terms:

a.) 30:35 c.) 4: 1/2 e.).08:.3

b.) 40: 280 d.) 6:.2 f.) 1/5: 7/15

3. Find the ratio of the first quantity to the second:

a.) 3 ft. to 6 yd. c.) 4.5 in. to 3 1/4 yd.

b.) 8 in. to 5 ft. d.) 1/2 ft. to 54 in.

4. In the adjacent figure, find the following measures:

Width of wall_______ Height of door _______

Height of wall _______ Width of window_______

Width of door_______ Height of window_______

Using these dimensions, write all possible ratios.

(figure available in print form)

5. Measure the length, width and height of your classroom. What is the ratio of the (a) length to the width? (b) length to the height? (c) width to the height?

Suggested Assignment **:**

Measure the length, width and height of one room in your house. Find the dimensions of all doors and windows in that room. Using these measurements, write all possible ratios.

Proportion

From earliest times men have recognized the value of good proportions in architecture. The ancient Greeks and Romans followed certain mathematical ratios and proportions to attain order, unity and beauty in their buildings. Using fixed mathematical formulas they were able to establish a pleasing relationship among various parts of buildings that have been admired for generations.

A *proportion* is an equation stating that two ratios are equal. Every proportion has four *terms* . The first and fourth terms are the *extremes* . The second and third terms are the *means* . In every proportion the product of the means equals the product of the extremes. The first equation below shows the order in which the terms of a proportion are written. Equations 2 and 3 demonstrate two ways in which a proportion may be written.

(figure available in print form)

The proportion is read: “2 is to 3 as 6 is to 9”. The means are 3 and 6. The extremes are 2 and 8.

Every proportion may be written in four equivalent forms.

1.) Given	2.) Invert the terms in 1.	3.) Alternate the terms in 2.	4.) Invert the terms in 3.

(figure available in print form)

A new proportion may also be obtained from a given one by adding or subtracting 1 on both sides of the equation in the following manner:

(figure available in print form)

The fourth term of a proportion is called the *fourth proportional* to the other three terms. In 1/2 = 3/6, 6 is the fourth proportional to 1, 2, and 3. When the second and third terms of a proportion are the same, they are called the *geometric mean* or *mean proportional* , and the fourth term is then called the *third proportional* . 1/2 = 2/4, 2 is the mean proportional, and 4 is the third proportional.

Example *1* :	Is 2/3 = 5/7 a true proportion?
Solution :	2/3= 5/7 3(5) = 15 2(7)= 14 3(5)= 2(7) Since the product of the means does not equal the product of the extremes, 2/3 = 5/7 is not a proportion.
Example 2 :	Find the missing term. 4/7 = x/35
Solution :	4/7= x/35 7x = 4(35) 7x = 140 x = 20 The missing term is 20.
Example 3 :	Find the fourth proportional to 1, 2 and 3.
Solution :	1/2= 3/x 1x= 2(3) x= 6
	The fourth proportional is 6.
Example 4 :	Find the mean proportional between 2 and 8.
Solution:	2/x= x/8 x2= 16 x= 4
	The mean proportional between 2 and 8 is 4.
Example 5 :	Given the proportion 3/5 = 8/15, write a new proportion by addition.
Solution :	3/5 = 8/15 3/5+ 1= 8/15 + 1 8/5= 24/15

Exercises

1. Tell whether each pair of ratios given forms a proportion.
(figure available in print form)
2. Find the missing term.
(figure available in print form)
3. Find the fourth proportional to:
a.) 4, 5, 6 b.) 8, 10, 12 c.) 3, 5, 7 d.) 8, 12, 13
4. Find the mean proportional between:
a.) 3 and 27 b.) 4 and 16 c.) 6 and 24 d.) 2 and 50.
5. Write two new proportions using addition and subtraction. 4/7 = 24/42.

Symmetry

One way to attain balance and repose in an architectural structure is by the use of symmetry. *Symmetry* is a distance-preserving transformation of any figure by reflection, translation or rotation. Two points are symmetric about a line if the line is the perpendicular bisector of the segment joining the two points. In figure 1 points A and B are symmetric about line n, since n is the perpendicular bisector of segment AB. B is said to be the mirror image or reflection of A, and n is the *line of symmetry* .

Figure 1: A.

(figure available in print form)

In figure 2, triangle ABC and triangle DEF are symmetric about line m. The corresponding sides and corresponding angles of the triangles are congruent, and m is the perpendicular bisector of segments BE, CF and AD. Triangle DEF is the reflection of triangle ABC, and m is the line of symmetry.

Figure 2: C

(figure available in print form)

Points A and B are symmetric about point M if M is the midpoint of segment AB

(figure available in print form)

Symmetry may be attained by a second distance preserving transformation known as a *translation* . A translation is the composite of two reflections over two parallel lines. In a translation a figure and its image are parallel. In the figure below line p is parallel to line m. Triangle ABC is reflected over line p, then its image triangle DEF is reflected over line m producing triangle GHI congruent to triangle ABC. The same translation may be performed by sliding triangle ABC along the plane to a new position so that the original figure and its image are parallel and congruent.

(figure available in print form)

The third method of attaining symmetry is by *rotation* . A rotation is the composite of two reflections over two intersecting lines. In the figure below, triangle MNP is the reflection of triangle ACB over line p. Triangle GJK is the reflection of triangle MNP over line m. The same image may be obtained by rotating triangle ACB counterclockwise. Triangle GJK is congruent to triangle ACB.

(figure available in print form)

Exercises

1. points A and B have symmetry with respect to the line y= 2. Find point B if point A is (a) (1,3) (b) (0,0) (c) (-4,5)

2.) Points C and D have symmetry with respect to the point (-3,1). Find D if C is (a) (-5,4) (b) (1,2) (c) (0,6).

3. Name the lines of symmetry in the figure shown. Is there a point of symmetry in the figure?
(figure available in print form)

4. Tell whether the following is a reflection, translation or rotation:
(figure available in print form)
5. Which of the following figures illustrate symmetry?

(figure available in print form)

Suggested Assignment **:**

Write a report on architecture of ancient Greece. Sketch a picture of the Parthenon and discuss the proportion and symmetry of its architecture. In the discussion include an explanation of the Golden Ratio.

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