Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
1984 Volume II: Greek Civilization

# The Early Greeks Contribution to Geometry

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by Joseph A. Montagna

## INTRODUCTION

The objectives of this unit are to expose students of 7th and 8th grades to the beginnings of geometry and one man who is credited with a large part of its early development, Pythagoras, and to teach students the basic principles of geometry. The life of Pythagoras is an interesting story, for it gives us a clearer idea of what life could have been like in the ancient world, and what education was like, as well. The history behind the mathematics should motivate students to learn geometry, but it is also important for them to see that each discipline does not stand isolated.

Geometry is one of mankind's tools that has become indispensable. Our world today would be vastly altered had geometry not been developed. Modern society depends on the techniques and methods of geometry to build, to navigate, to design, and to calculate the vast distances of outer space. All around us we can observe evidence of geometry put to practical use; the new skyscraper reaching for the heavens, the jet airliner or ship that arrives precisely at its destination, the tunnels dug through mountains from both sides that meet precisely where they should, or the bridges that span our largest rivers. Geometry is utilized by engineers, builders, astronomers and even do-it-yourselfers.

There is also a great deal of evidence of geometry in nature. Ice crystals that are always hexagonal, the symmetry of living things, the orderly movement of the planets and other heavenly bodies, a snail's spiral shell, or the economical use of simple shapes are each examples of nature's geometry. All around us we can observe some geometrical principles, which is probably what started the ancients on their way to developing this field of study. Nature appears to have a plan, and man seeks to unravel its mysteries.

I think that this unit can be interesting to students. The narrative portion is by no means complete. Much is left to the teacher to fill in on his/her own. The lessons that are included may be altered in any way the teacher deems appropriate. It would have been impossible for me to offer all of the ways that the material in this unit could be taught. This unit represents one way to teach it. The creative teacher will use this unit as a resource and devise other, perhaps better, ways to teach the material. The important thing is to give students more than aspects of it.

The history of mathematics is inspiring. The subject is typically human, that is, it is a body of knowledge that was fathered by man's curiosity. Nature was one obvious topic that gave the ancients their mental workouts.

Look at nature. Observe the perfection in the arrangement of the petals of a daisy, or the seeds that develop in a sunflower. Observe the web of a spider, with its delicate pattern that is perfectly symmetrical. Within the enormous field of the study of natural occurrences man made some mathematical discoveries which, in turn, became stepping stones to further discoveries. Mathematics of this period was also wrapped up in ancient mystical societies whose teachings remained secret except to a small number of their initiates. Numbers had special qualities to those who believed in their magical powers. Even in modern times certain numbers conjure up thoughts of demons, devils, omens and good fortune. Numbers are also referred to in mythology. Aeschylus in *The Prometheus Bound* (line 459), has Prometheus describe, " . . . number . . . which is the most ingenious of all devices," as one of his gifts to mankind, alongside fire and writing. Whether it was religion, superstition, the search for nature's secrets, or the mere practicality of a number system which served as the catalyst, man did create the field of mathematics and will continue to develop it.

The Greeks built upon a solid foundation that was laid by the Egyptians, Babylonians and other ancient civilizations. The Egyptians were the premier architects and builders of ancient civilizations. The pyramids and the buildings they created were tremendous in size, complex in their design and built to last forever. Three thousand years, or so, is hardly forever, yet their existence for so long does make the point. The Egyptians also laid the foundations of surveying and measuring, a skill made necessary by the annual flooding of the plain alongside the Nile River. Every spring the Nile floodwaters would erase all semblance of the markings laid out to distinguish boundaries. They devised a system of measurement that surpassed any other. Indeed, the term geometry is derived from the Greek "geometria", measurement of the earth. It is fair to presume that the Egyptians inspired the coining of this term. Babylonian and Nordic civilizations of ancient times did much in the way of astronomical observation which, in turn, led to the development of dividing time into periods and the prediction of astronomical occurrences. All of this is to point out that the Greeks did not invent mathematics. What they did do is begin to formalize and compile much of the valuable work in the field that preceded them.

Why is it that the Greeks were able to do what no other civilization before them was able to do? I believe that an answer can be found if one takes into account the nature of the Greek culture within the context of the world as it then existed. The Greeks were relentless in their search for "truth", be it in art, politics, or philosophy. Mathematics fit quite nicely into their culture. It was neat. Mathematics was perfection. It made sense. In addition to this, the world of the ancient Greeks existed literally at the edge of knowledge. Situated relatively close to Egypt and Mesopotamia, the Greeks stood on the shoulders of these two giants of the ancient world. The Greeks learned from the Egyptians and the Babylonians and made new discoveries of their own.

It is important for the teacher to be aware of this fact, that mathematics did not begin with the Greeks; that they formalized and refined what developed in earlier periods. How much of this history up to the teacher. However, I would recommend that, at the very least, a capsulized history be presented to aid the students in their understanding of the subject. A few titles for this purpose are recommended in the bibliography.

## PYTHAGORAS

Pythagoras established a school in southern Italy. What we know about this school is through the writings of his students, or his students' students. Pythagoras wrote nothing of a permanent nature. Philolaus is a figure who is generally connected with the beginnings of Pythagorean literature. He was a student of Pythagoras, and probably a teacher himself. According to tradition Philolaus was either commissioned by Plato to write three books on the teachings of Pythagoras, or to buy three books on the subject. The latter is probably closer to the truth since it is said of Philolaus, ejhyeg¸e, meaning "he brought out," i.e., "he published." There is a great deal of conjecture connected with the life of Pythagoras, since the evidence is not complete. Any information that is available about the man amounts to little more than a coloring book outline, the details to be filled in by legend and the imaginations of authors who came later.

Pythagoras of Samos was born around 570 B.C. Samos was an island in the Ionian group, along the coast of what is presently called Turkey. Pythagoras' family was wealthy, his father being a merchant.

Pythagoras' quest for knowledge is legendary. At the time the major sources of knowledge were the temples. Their bodies of knowledge were not told to the general populace, but shared only with those who were their initiates, usually the wealthy. Pythagoras was a student of the local Ionian temples. It is generally accepted that Pythagoras was a student of Thales of Miletus. After absorbing as much knowledge as the local temples could impart, Pythagoras looked elsewhere for new knowledge. He was probably encouraged to do so by a fellow brother of the temples of Ionia, as this was the accepted method of sharing knowledge between temples. Pythagoras is said to have enlisted the help of the tyrant king, Polycrates. Polycrates is said to have contacted the brethren of the Temple of Memphis in Egypt through an ally of his at the time, Amasis, the ruler of Egypt. Pythagoras was accepted to the Temple of Memphis. Egyptian temples had long been regarded as possessing the richest of all knowledge that man possessed. Pythagoras, thirsting for new knowledge, must have been overwhelmed with joy at this occasion.

Being admitted to the Temple of Memphis was the first and the easiest of all obstacles for Pythagoras to overcome in his pursuit of the higher degrees of the Temple of Memphis brotherhood. The customary length of time for an initiate to remain at the Temple to learn the sacred secrets was twenty-two years. Upon completion, the student was free to leave. There was, of course, the oath of silence, wherein a brother swears never to reveal the Temple's secrets. Legend has it that it was in Egypt that Pythagoras developed his understanding of the properties of right triangles and proportions. Supposedly, Pythagoras, while contemplating the inner secrets of his new brotherhood, discovered the concept that the sides of one right triangle are proportional to corresponding sides of another right triangle. This revelation might have come to him as he sat on the Temple grounds contemplating its surroundings, and discovering that objects in the sunlight cast shadows that are proportional to their actual height. The objects, a tree, an obelisk, a statue, standing perpendicular to the earth, formed right triangles; the object being the vertical side, the shadow being the horizontal side, and the sun's rays forming the hypotenuse. Pythagoras learned well.

Having completed his studies in Egypt Pythagoras longed to return to his homeland to impart some of his newly acquired knowledge. It was permissible at the time to teach certain things to outsiders of the temples, as long as one did not reveal the inner secrets. One was also permitted to teach that which was one's own creations and discoveries. It was also customary to share the knowledge with those brothers of one's homeland temple. Pythagoras' departure was delayed by the war with Persia (Ionian Revolt). Pythagoras was taken captive and brought back to Babylon. He and his store of knowledge were considered to be spoils of war, as would be precious metals, or slaves. In Babylon, Pythagoras was well received. The brothers of the temples there were anxious to talk with him and learn from him. These brothers also shared their own secrets with Pythagoras in return. He was given the freedom to move about the temples. Characteristically, Pythagoras delved into this new body of knowledge and absorbed as much of it as they would permit.

Twelve years later he was allowed to leave. He returned to Samos, only to leave again for southern Italy, where he would soon start a school. Supposedly, Pythagoras was accompanied to southern Italy by his mother and a single disciple of his. There is a legend about Pythagoras and a young man who challenged him to explain the utility and benefit of his teachings. Pythagoras responded with an offer to the boy. He would give the boy a quantity of silver for every lesson that he learned, since it was a material gain that he sought from his learning. The boy agreed. When Pythagoras taught the boy a certain number of lessons and doled out the stipulated amount of silver, he was left with none. By now the boy's appetite for learning had grown to a point that he wanted to learn more lessons. He offered to pay Pythagoras an equal amount of silver for each lesson he taught him. This legend illustrates a belief in the intrinsic value of the love of learning. The boy in this tale could very well have been the disciple who accompanied Pythagoras to southern Italy. It is also possible that this person was Philolaus.

Pythagoras formed his secret society, teaching the knowledge he learned in Egypt and Babylon, as well as some of his own discoveries. As mentioned above, Pythagoras wrote nothing of a permanent nature. He probably did all of his teaching by talking to his students arranged in a semi-circle, resorting occasionally to sand drawings as a means of illustrating lessons, as would a modern teacher using a blackboard. One might almost imagine Pythagoras in front of his students. Using little more than a straightedge, two sticks joined by a string as a compass, and perhaps the shadows created by the sun, Pythagoras shared his discoveries with them. Off to the side was an initiate holding a device similar to a casino croupier's stick for the purpose of smoothing out the sand drawing after the Master no longer needed it. Pythagoras was often referred to as the Master by his students. In some of their writings one finds the Greek, ayto§ ewa, meaning "the Master has said."

The teachings of the Pythagorean Society were more than information to be learned or taught, they were a way of life, a philosophy that sought to guide and direct the Pythagoreans and explain nature. According to the Pythagoreans, " . . . all things are number." They believed that number had special qualities that could help to explain the cosmos. They believed that number was the key to unlocking nature's secrets.

No one can say for sure what happened to the Pythagorean meeting place. Some accounts relate that it was destroyed by fire, set by some of the local people. Another account explains that the Pythagorean Society literally fell apart when their philosophy was dealt a serious and fatal attack, when a follower of Pythagoras, Hippasus, illustrated the existence of irrational numbers. Supposedly, he did so by taking the famous Pythagorean Theorem. According to this theorem, if one squared the sides of a right triangle and added the results, this sum would equal the square of the hypotenuse. Hippasus did not attack the theorem's validity. What he did was to assign a measure of one unit to the sides of the right triangle, resulting in the measure of the hypotenuse to be the square root of two (Ã2). This is, of course, an irrational number, for it results in a nonrepeating decimal. Supposedly, Hippasus was drowned at sea for this revelation. Whether this is true is of little importance. One could believe that the Pythagorean Society whose philosophy was inextricably entwined with rational numbers could have fallen apart because of this.

The Pythagorean Society had ceased to exist, but not without first making mathematics part of a liberal education. The Pythagoreans divided the mathematical subjects into four main parts:

Numbers absolute (arithmetic)
Numbers applied (music)
Magnitudes at rest (geometry)
Magnitudes in motion (astronomy)

This so-called “quadrivium”, in the belief of the Pythagoreans, was what constituted the necessary course of studies for a liberal education.

Geometry is the main focus of this unit. It is a field of study that rests squarely on a foundation of axioms, from the Greek “axioma”, meanings things that are worthy. These axioms are premises held to be so self-evident that one does not need a proof. A certain amount of faith is needed to give one a point of departure, since these axioms cannot, in fact, be proved. The entire system of geometry is built upon these simplest of concepts.

The basic concepts of geometry are point, line, angle and surface, or plane surface. The definitions of these concepts that follow are also discussed in the lesson plan portion of this unit. We shall examine them here for the purpose of identifying those aspects that require a measure of faith in order to be accepted.

***POINT***
A point is the location of a point in space, yet it does not occupy space. A point has neither length, nor width, nor thickness, and it is indivisible. It is impossible to see a point, yet we use a dot to represent its existence, even though, by definition, there are an infinite number of points contained in that dot. The dot on this “i” is clearly visible. It has length, width and thickness; therefore, it is composed of a infinite number of points.

A point is an abstract concept. Clearly, a certain amount of faith is needed to support its existence. It is similar to the belief in the existence of atoms before one could actually see them.

***LINE***
If one were to set a point in motion the result would be a line. A line has length, yet it has no width or thickness. It, too, is an abstract concept.

***ANGLE***
Two lines which emanate from the same point produce an angle. The lines are called the sides of the angle, and the common point is called the vertex of the angle.

***SURFACE or PLANE***
If one were to move a line at right angles to its own direction the result is a surface, or plane surface. It is often referred to simply as a plane. Tabletops, walls, a pane of glass, or a floor are all examples of what a plane is, with one important distinction—a plane has no edges. A plane extends indefinitely in all directions. It has length and width, yet no thickness. Planes are two dimensional.

## STARTING OUT: STRATEGIES AND LESSONS

Using the narrative portion concerning Pythagoras' life, the teacher should discuss with students the major points; the major sources of knowledge were temples, their teachings were usually secret, Pythagoras traveled to foreign lands to learn the secrets of other brotherhoods and Pythagoras established a school of his own.

Ask students for some reasons why people would want to keep their teachings and bodies of knowledge secret. Was there some advantage in doing so? Introduce the term "esoteric." Its meaning, "knowledge that is known to a small group," should offer some clues to the reasons behind these secret societies.

In modern times we can find evidence of this practice, although the "secrets" are not so secret. Consider, if you will, the carpenter, the plumber, or any skilled trade. People of these trades have devices and "tricks of the trade" which they are somewhat reluctant to share. The difference between the Egyptian architect, or the Pythagorean student and the skilled trades of modern times being the proliferation of, and the dissemination of knowledge. Education today is not limited to a chosen few, at least not in this country and a number of others.

Discussions of this nature are intended to give students a kind of foothold in understanding the ways things were in ancient times, and to help them relate it to their own period. If nothing else, students should come away with a clear understanding that our civilization provides us with numerous opportunities for self-fulfillment.

The remainder of the lesson and strategies portion is divided into the following sections:

> —immediately following are two "excursions" that take us back in time before Pythagoras. These excursions are intended to give students some information regarding what man's earliest experiences with geometry were. There is much that is hidden by the first excursion, e.g. the scientific explanations behind the movement of the earth and the apparent sunrise and sunset. The teacher should take the time to explain this material. The second excursion takes the circle as being man's primary experiences with shapes and the contemplation of same. For this excursion students will need a compass and straightedge.
> —the basic concepts of geometry.

I would suggest that students make a folder in which they should keep all of their work in this unit. Informing them at the beginning that a grade will given for the completeness of this folder is an incentive for them to get started on this right away. Opportunities for extra credit work are numerous throughout this course, and these assignments should be filed in their folders, as well. Finally, each student will need: a compass, a protractor and a straightedge.

What were man's earliest experiences with geometry? Do we dare to call it geometry? There has been much speculation about the first question and, as we shall see, the answer to the second question is that it was not geometry, at least, not the geometry we know today. Nevertheless, he was engaged in the contemplation of objects he observed, giving rise to the rudimentary aspects of geometry. Consider the following excursion.

Some ancient people observed the sun and moon and regarded them in a mystical manner, i.e. the sun's and moon's movements are controlled by gods. A consequence of this reverence must have been close and painstaking observations of these heavenly bodies, and the recording of these movements. There is more than mere conjecture, since structures such as Stonehenge, the great circles of Avebury, Inca and Egyptian pyramids and other structures from ancient periods are concrete evidence that this occurred.

If a person established a site on high ground, a point from which the sunrise could be clearly observed each day of the year, and if that individual visited this site on a regular basis (every third day) to observe and record the point at which the sun peeked over the horizon what would he observe? Depending on the time of year and his position on the earth, a distinct difference can be discerned.

Let us put this person on top of East Rock on the first day of summer. On this day, the summer solstice, the arc of the sun across the sky is high and long. The sunrise and sunset are at their northernmost points at his time. If our observer continued to visit this site every third day to the first day of fall, the autumnal equinox, he would have observed the sun's arc across the sky grow steadily lower and shorter, and he would have observed that the point at which the sun peeked over the horizon has moved steadily southward. Continuing through the year to the first day of winter, the winter solstice, our observer would notice this apparent movement toward the south, after which this movement would reverse direction through the summer solstice, completing the cycle.

Assuming that this observer recorded these changes, let us say by carving marks in a large slab of stone, perhaps the following would result:

*(figure available in print form)*

What did early man do with this knowledge after observing that this cycle repeats without end. This was his early experience with prediction of seasonal changes, time, calendar development and astronomy. This was also his first experience with angles and sighting along points to make a straight line, though he did not use these terms.

What can we do with this with students? We would find it impractical to visit a particular site at sunrise. There is a great deal of scientific knowledge that students of the seventh and eighth graders should know contained in the preceding page. Students should know why the seasons change, why we have periods of darkness and light, that the length of these periods change daily and the the earth is, indeed, closer to the sun in the winter than in the summer. If your students do not know the scientific explanations behind these and other related questions, then you should take the time to help them understand this. It would be a good opportunity for them to see the interdisciplinary nature of mathematics and science. Later, after they have learned how to measure angles, draw angles and determine what a plane is, it might be valuable to come back to this and learn about the plane of the earth's orbit in relation to the sun, as well as those of the other planets in our solar system. Some valuable insights can develop from this, as well as some interesting lessons. I would also encourage students to research these topics independently.

A natural progression for early man was to contemplate the circle, the shape of the sun and moon. Man's first compass was probably two sticks joined by a length of string, the length of which represented the radius of the circle to be drawn. Placing one stick down on a surface would serve as the center of the circle. The remaining stick stretching the string to its limit is free to pivot about the center, aiding one in drawing a circle. This method is adequate, yet our students can take advantage of commercially produced compasses. Of course, it would be interesting to allow students to experiment with the stick and string method at some point. Students need a lot of practice to become proficient in the use of a compass.

Have students draw a circle whose radius is 5 centimeters. More work on circles is included later in this unit, so it is sufficient, for the time being, that they learn only the terms radius and diameter. They should also know that the diameter is twice the radius, or the radius is one-half the diameter. Allow them the opportunity to discover this and other characteristics of circles, e.g. the boundary of the circle is exactly the same distance from the center. List these discoveries. Try to have students understand that we are attempting to learn about early man's discoveries through our own.

The next step that probably followed was drawing a cross in the circle. Have students do this, making sure that the intersection of the cross is exactly over the center of the circle, and that the cross members touch the boundary of the circle at the 9, 12, 3 and 6 o'clock positions (later they will be introduced to these positions by their respective degrees of arc names). What observations can they make about this figure? List them on the blackboard. The following points should be covered: the circles is divided into four equal parts, the divisions of the boundary of the circle are equal, and the lengths of the cross members are equal. Some might even make the observation that each part of the circle is 90 degrees. Accept this answer but add that early man had no such term.

Now, let us add to this figure by connecting the ends of the line segments which form the cross. Draw straight lines with a straightedge. Label the points as in the figure below.

*(figure available in print form)*

List their observations of this figure. Make sure the following points are covered:

—a square is formed (how do we know it is a square?)
—four equal, or congruent, triangles are formed
—line segments AB, BD, DE and EA are equal to one another (congruent)
—line segments AC, BC, DC, and EC are each 5 centimeters long.

We know these things to be true. In fact, we can measure them to prove it. This is what ancient people did. Geometry, as it developed and exists today, seeks to prove these conclusions without actually measuring them. This is done through a process of logical reasoning called a proof.

The proof in geometry is an oftentimes painstaking process that seeks to arrive at a logical conclusion. We will take up this topic later in the unit, after we have learned some basic definitions. However, at this point it would be wise to introduce reasoning and logical thinking through "IF . . . THEN . . . " statements. They are good exercises for any youngster, and ones which 7th and 8th graders should enjoy doing.

"If . . . then . . . " statements are used by many of us daily. Below are a few examples. Point out that such statements are composed of two parts; the *hypothesis* and the *conclusion* .

If Carol is 28 years old, then she is eligible to vote.
If we win this last game, then we will be in the championship.
If we live in Connecticut, then we must live in the United States.

All of these statements are true and are of the following form:

"IF A . . . ,THEN B".

"A" is the hypothesis and "B" is the conclusion.

Have students to try their hands at writing this kind of statement. Share them. Discuss their validity.

***POINTS***

It is probably desirable to start out by discussing what a point is *not* . This (.) is not a point. Neither is this (¥).

Students have to understand that this concept is abstract, difficult to understand; as difficult as understanding the existence of atoms, even if one has never seen an atom. Perhaps a discussion of what other concepts are abstract will help i.e., justice, the existence of God. Some things have to be taken on faith. This and some other concepts in geometry fall into this category. We must have a point from which to begin.

Have students mark a point on a piece of graph paper. Explain that the dot they made represents a point, but it is not actually the point. The dot is, in fact, covering an amount of space that includes an infinite number of points.

Points are labeled with capital letters. Have students label their points on the graph paper.

***LINES***

Lines are also abstract ideas. A line is an extension of a point. Bring up the idea mentioned in the narrative that a line is formed by a point set into motion.

We sometimes think of lines as being a set of points existing alongside one another. A line contains an infinite number of points. No matter how close one represents two points on a line, by definition, at least one more point exists between them.

There are different kinds of lines:

*(figure available in print form)*

The shortest distance between two points is a *straight* line.

Lines that are in the same plane and maintain an equal distance between them are called *parallel* lines.

*(figure available in print form)*

Lines are named in the following manner:.

*(figure available in print form)*

AB is read "line segment AB"

*(figure available in print form)*

AB is read “ray AB”

*(figure available in print form)*

AB is read “line AB”

Each point on a line can be matched up with a real number:

*(figure available in print form)*

thus, the coordinate of point M is 1, the coordinate of point J is -2.

To find the length of any line segment, one need only to subtract the smaller coordinate from the larger. Thus, MP = 3 units.

***ANGLES***

Angles are formed when two lines have one point in common. The lines are called the sides, or legs, of the angle and the common point is called the vertex. It takes at least three points in the same plane to form an angle. Angles are named by these points, usually. The letter that represents the vertex point is always named in the middle. Thus, in the angle below the correct name for it is ABC, read as “angle ABC.”

Sometimes angles are named by the use of one number or one letter.

*(figure available in print form)*

There are several types of angles. They are classified according to their measure. The next lesson will give practice on how to measure and draw angles using a protractor. For now, students should learn the definitions of the classification of angles.

An *acute* angle is one whose measure is less than 90 degrees. An *obtuse* angle is one which is greater than 90 degrees. An angle that is exactly 90 degrees is called a *right* angle (this has nothing to do with the direction in which it faces, so the teacher should take care in clearing this up).

## THE PROTRACTOR

No matter how large or how small one draws a circle it is divided into 360 equal parts, each of which is called a *degree of arc* . These degrees can be further divided into minutes and seconds; however, it is not appropriate for this grade level to measure an angle with such precision. An angle is measured by a device called a protractor, which measures what portion of a circle the angle is “opened” up to. The protractor is one half of a circle, 180 degrees. It can be read from 0 to 180 degrees from the left side, or the right side. Two scales are drawn on most protractors to make it easy to use.

Demonstrate to students how an angle is measured. Give them practice in measurement of angles. In my experience this is a difficult task, as it is usually the first time they have ever used a protractor. Their precision in measurement should improve with practice.

Demonstrate how one draws an angle of a given measurement using a protractor. This task also requires a lot of practice. A good way to start out is for the student to first locate where he wants the vertex of the angle to be, and take it from there.

Have students measure angles from objects around them, i.e., tables, numbers on a clock, the spaces between their spread out fingers, the angle formed by stretching out their thumb and forefinger.

Give students practice in classifying angles according to the definitions given above.

When two lines intersect they form four angles. Two pairs of angles can be identified as opposite angles, or *vertical angles* .

*(figure available in print form)*

To the right is a figure depicting two pairs of vertical angles. 2, read as “angle 2” , and 4 are vertical angles.

Also, 1 and 3 are vertical angles.

If 2 is 160°, what is the measure of 1? Consider the protractor. How many degrees are there in a half-circle? 1 and 2 together form a 180° angle, a straight line. Do you see the logic in that? What are the measures of the other angles?

Give students practice with this type of angle. Have then pick out vertical angles.

A ray that divides an angle into two equal parts is called the *bisector* of that angle. There is exactly one bisector to any given angle.

Give students plenty of practice with identifying bisectors of various angles. Use protractors a lot.

Some good practice on much of the work so far can be had by having students to make a paper airplane as they usually do, but instead of flying it they would unfold it and notice the angles that are formed Practice can be given with this on many of the skills thus far: labeling points, naming angles, measuring angles, adding and subtracting angles, naming line segments.

## GEOMETRIC CONSTRUCTIONS

These constructions are mathematical games that produce figures, precisely drawn figures, which are accurately done without the aid of any measuring device. The student uses only a compass and a straightedge. These constructions require a certain level of skill with these instruments. Without such a skill the student’s drawing will be poorly done.

With that caveat in mind, and with the presumption that each student doing the following has the necessary skills:

***CONSTRUCTION # 1***

Given: An angle

Construct: A bisector of that angle

Have students draw an angle on their paper. Any size angle will work, but an acute angle is best to tackle as a beginner. The teacher may want to pass out a copy of an angle so that everyone starts with the same size angle.

If the student places the pivot of the compass on the vertex of the angle, and then draws an arc which sweeps across the sides of the angle, he will have identified two points, one on each side where the arc intersects them.

Using these points, the student places the pivot of the compass on one at a time, drawing an arc in the interior of the angle.

The two arcs intersect in the interior producing one point. A line is then drawn from the vertex of the given angle through this new point.

This construction is finished.

***CONSTRUCTION #2***
Given: A line and a point on that line

Construct: A perpendicular line through that point

***CONSTRUCTION #3***
Given: A line and a point not on that line

Construct: A perpendicular line through that point.

To conserve space these constructions are not included here. Each of these and others may be found in any basic geometry book.

## POSTULATES

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*Postulates* are statements which are accepted without proof. They are especially useful in doing geometry proofs. Students must learn these. It is important that they be included in their geometry folders and studied often. Each postulate is simply listed with an example. Obviously, the teacher will have to take the time to discuss each in class.

*Postulates of Equality*

Addition postulate: If a=b and c=d, then a+c=b+d

Subtraction postulate: If a=b and c=d, then a-c=b-d

Multiplication postulate: If a=b, then ac=bc

Division postulate: If a=b and y = 0, then $\frac{a}{y} = \frac{b}{y}$

Substitution postulate: If a=b, then a can be substituted for b in any *equation* .

*Postulates of Geometry*

Before moving on to these postulates we should introduce one more term to those that students already have, *plane* . A plane is suggested by a flat surface. The abstract idea in the definition of a plane is that they have no edge, they continue infinitely. When we draw a plane it appears to have corners, represented by points. Drawings are, therefore, misleading. A term used in discussing planes is *coplanar* . Points that lie in the same plane are coplanar.

Postulate 1: Through any two points there is only one line.
Postulate 2: Through any three *non-collinear* points there is only one plane.
Postulate 3: If two points lie in a plane, then the line connecting them lies in that plane.
Postulate 4: If two planes intersect, then their intersection is a line.
Postulate 5: This postulate is also called the ruler postulate. Each point on a line can be paired with only one real number called its *coordinate* . The distance between these two points is the positive difference between their coordinates.
Postulate 6: This postulate is also called the protractor postulate. If point C is on line ST, then all the rays that have C as an endpoint and lie on the same side of ST can be paired with exactly one real number between 0 and 180.

## COMPLEMENTARY, SUPPLEMENTARY and VERTICAL ANGLES

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*Complementary angles* are two angles whose measure equal 90 degrees. *Supplementary angles* have a sum of their measures equal to 180 degrees. *Vertical angles* are angles that are opposite one another when two lines intersect.

The final lessons of this unit concern geometry proofs. We will look at three theorems of geometry that relate to the three types of angles above. A *theorem* is a statement that has been proved by a logical reasoning process. It is hoped that students will understand how a proof proceeds through its steps, and that they may be able to write a proof of their own. Students should know that it is helpful to remember these and other theorems.

Given Information:

*(figure available in print form)*
Conclusion: 3= 4

To reach the above conclusion we take the following steps:

STATEMENTS REASONS

1. ABC = 90 degrees by definition of lines
2. PQR = 90 degrees by definition of lines
3. 1 + 3 = 2 + 4 each sum = 90 degrees
4. 1 = 2 given
5. 3 = 4 Subtraction Postulate

The conclusion is reached in step 5.

A theorem that results from this proof is:

Theorem 1: If two angles are complements of equal angles, then the two angles are equal.

Given information: 1 and 2 are supplements
3 and 4 are supplements
1 equals 3
Conclusion: 2 = 4

| STATEMENTS | REASONS |
|---|---|
| 1. 1 + 2=180 degrees | by definition of supplements |
| 2. 3 + 4=180 degrees | by definition of supplements |
| 3. 1 + 2 = 3 + 4 | the sums of each = 180 degrees |
| 4. 1 = 3 | given |
| 5. 2 = 4 | Subtraction Postulate |

Theorem 2: If two angles are supplements of equal angles, then the two angles are equal.
Theorem 3: Vertical angles are equal.

Have the class try to prove theorem 3. This is an easy proof, comprised of three steps.

Geometry books are loaded with exercises on writing proofs. Give students an opportunity to work on these kinds of exercises. Below is a list of reasons that are used in proofs.

*REASONS USED IN PROOFS*

* The information is given
* Definitions
* Postulates
* Theorems

The limitation of space makes it necessary to cut these lessons at this point. If one were to continue beyond this point the major topics to be covered should be: triangles and other polygons, areas of polygons, proportions, circles, solid geometry, and the areas and volumes of solids. Some of these topics are covered by objectives in math for 7th and 8th graders, while others are appropriate for high school level.

## TEACHER BIBLIOGRAPHY

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Ball, W.W. Rouse, *A Short Account of the History of Mathematics* , Dover Publications Inc. New York 1960, 5th ed.

This book is especially suited for the teacher who wishes to preface the work in this unit with an account of mathematical knowledge that predates the Greek influence. Also helpful in supplying the details of the work of the Greeks.

Boyer, Carl B., *A History of Mathematics* , Wiley & Sons, Inc., New York, 1968

Each chapter of this book contains essay questions for the reader, as well as some math exercises. This book is appropriate for the teacher with weak or strong background in mathematics. The author writes clearly on mathematics from the ancients to the 20th century.

Brunes, Tons *The Secrets of Ancient Geometry* , International Science Publications, Copenhagen, 1967

A detailed account of the secret geometry of the ancients, which also speculates on the first experiences with geometry and number. This book would be an excellent resource for details regarding man's early experiences with mathematics. Included in this book are drawings of ancient structures and their geometric analysis.

Burkert, Walter, *Lore and Science in Ancient Pythagoreanism* , Harvard University Press, Cambridge, 1972

Every angle of Pythagoreanism and its influence is covered in this book. It is an excellent resource. The author presents the best research available to date on a figure who has been virtually lost to history.

Gorman, Peter, *Pythagoras: A Life* , Routledge & Kegan Paul Ltd., London, 1979

The exhaustive research that went into the writing of this book has produced one of the most complete biographies of Pythagoras. The author presents a number of viewpoints based on this research regarding the philosophies of Pythagoras.

Hammond, N.G.L., & Scullard, H.H., *Oxford Classical Dictionary* , 2nd ed., Oxford University Press, 1970

The editors of this reference source have compiled the articles on nearly any topic or figure of the classical world. This book contains capsulized articles on men who have made their mark on the progress of ancient civilizations.

Heath, Sir Thomas, *A History of Greek Mathematics* , Volume I, Simon & Schuster, New York, 1965

This book traces the history of Greek mathematics from Thales, Pythagoras' teacher, to Euclid, compiler of Greek work on geometry. This work would be an excellent resource for gathering details on important figures of ancient mathematics.

Heath, Sir Thomas, *The Thirteen Books of Euclid's Elements* , Dover Publications, Inc., New York, 1965

This book contains an excellent introduction plus Euclid's Books I & II.

## STUDENT BIBLIOGRAPHY

Diggins, Julia, *String, Straightedge and Shadow* ,Viking Press, 1965

This book was written for students of middle school age. The Egyptians and the Greeks are featured in it, as well as Pythagoras and others. As the title suggests, the methods of geometry, without the aid of measuring devices, are the main idea throughout the book.

Freeman, Mae & Ira, *Fun with Figures* , Random House, New York, 1946

Also written for middle schoolers. Don't let the date of publication fool you. All of the activities included in this book are relevant to the principles of basic geometry that middle school students will enjoy.

Froman, Robert, *Angles Are As Easy As Pie* , Fitzhenry & Whiteside, Ltd., Toronto, 1975

Written for younger children, yet this book contains ideas that coincide with the work of this unit. Recommended for the student who has a reading problem, or the student who needs to see the material presented in a different form.

Froman, Robert, *Rubber Bands, Baseballs and Doughnuts* ,Fitzhenry & Whiteside, Toronto, 1975

This book is on topology, "rubber sheet geometry." The book appears to be "babyish", yet contains some heavyweight concepts.

Muir, Jane, *Of Men and Numbers* , Dodd, Mead & Co., New York, 1961

This book contains the biographies of ancient mathematicians, as well as stories of great mathematicians through the early 20th century.

Phillips, Jo. *Right Angles: Paper Folding Geometry* , Fitzhenry & Whiteside, Ltd, 1972

Contains many fantastic exercises that give students practice with many of the skills of this unit and more.

Ravielli, Anthony, *An Adventure in Geometry* , Viking Press, New York, 1957

This is a good book on the basics of geometry, written for grades of middle school. Every student should read it. The book gives the reader a clear understanding of geometry in our daily lives.

Trivett, Harwood, *Shadow Geometry* , Fitzhenry & Whiteside, Ltd., 1974

Written for younger students, yet contains most of the ideas related to shapes, proportion and size.