Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
1985 Volume VIII: The Measurement of Adolescents

An Introduction to Elementary Statistics

Curriculum Unit 85.08.02
by Lauretta J. Fox

Introduction

Statistics play a role in all our lives. From time to time we provide data or collect data to be organized, analyzed, and used in making decisions. For example, the registrar of vital statistics records our date of birth and our date of death. Periodically we are counted in the population census taken by the federal government. School children are always concerned about their scholastic averages. When planning vacations we are interested in data provided by the weather bureau. We try to predict the outcome of elections by analyzing the results of political polls that have been taken. Business people conduct surveys to find out what products are most frequently sought by consumers. Sports fans compare batting averages, wins and losses of various teams. Although our list could continue indefinitely, it is sufficient to say that we encounter statistics in many facets of our lives, and all school children should be introduced to basic statistical terms and concepts. In this unit of study we will try to improve the students' understanding of the elementary topics included in statistics. The unit will clearly define the arithmetic mean, the median, the mode and the range of a group of numbers. Methods for computing these measures of central tendency will be discussed. Frequency tables, histograms and frequency polygons will be explained and constructed. Following the explanation of each topic, a set of practice exercises will be included.

There are several basic objectives for this unit of study. Upon completion of the unit, the student will be able to:

— understand and appreciate the use of statistics in everyday life.
— define basic terms used in statistics.
— compute simple measures of central tendency.
— construct tables, bar graphs and line graphs that display these measures of central tendency.

The material developed here may be used at the following levels of instruction: (1) in seventh or eighth grade

arithmetic classes; (2) in high school applied mathematics classes; (3) in high school consumer mathematics classes; (4) in adult basic education classes.

Arithmetic Mean

The *arithmetic mean* is another name for the average of a set of measures. To calculate the arithmetic mean, add the members of the set and divide the sum by the number of items in the set.

> *Example 1* : Find the arithmetic mean of the following set of numbers: 15, 10, 12, 18, 20.
> *Solution* : (15+ 10+ 12+18+20) dv 5 = 75 dv 5 = 15 The arithmetic mean is 15.
> *Example 2* : On four English tests John received grades of 92,80, 88, 94. What grade must he receive on the next test if he wishes to maintain an average of 90?
> *Solution* : If John wishes to have an average of 90 on five tests, the sum of his scores on the five tests must be 5 x 90 or 450. His total score on the first four tests is 92+80+88 + 94 or 354. He must receive 450 354 or 96 on the fifth test to maintain an average of 90 on the five tests.

Sometimes an item appears in a set of measures more than once. The number of times any item occurs in the set is its *frequency* . To find the arithmetic mean of a set of measures when some items occur several times, multiply each item in the set by its frequency and divide the sum of these products by the total number of items in the set.

> *Example* : Find the arithmetic mean of the following numbers: 8, 6, 10, 11, 12, 6, 6, 8, 14, 15, 14, 14, 10, 12, 6, 11.

(figure available in print form)
Sum of Products:

24+ 16+ 20+ 22+24+ 42+30+20 = 198

Total Number of Items:

4+2+2+2+2+3+2+1 = 18

Sum of Products dv Total Number of Items:

198 dv 18 = 11

The arithmetic mean is 11.

***Class Assignment* :**

1.) Measure the height of each student in the class. Find the average height of the students in the class.
2.) Keep a record of your test scores for one marking period. At the end of the marking period find the arithmetic mean of the test scores.

Exercises Solve the following set of problems.

1.) Five employees of Brad's Department Store earned the following hourly wages: $4.35, $3.67, $3.36, $5.00 and $4.82. Find the average hourly rate of pay.
2.) During a six week period Mary worked the following number of hours per week: 40, 42 1/2, 37 3/4, 48, 45, 44 3/4. Find the average number of hours that Mary worked per week.
3.) Eight automobiles were priced at 10,499; $11,988; $7,444; $5,995; $14,999; $6,492; 10,750; and $7,937. What is the arithmetic mean of the prices?
4.) Last winter a homeowner purchased 504 gallons of heating oil at an average cost of $1.23 per gallon. If he paid $1.19 per gallon for the first 354 gallons, what was the total cost of the remaining oil purchased?
5.) Lou allows himself an average of $2.50 a day for lunch at work. If he spent $2.00 on Monday, $2.75 on Tuesday, $2.25 on Wednesday, and $2.50 on Thursday, how much may he spend for lunch on Friday?
6.) Mrs. Smith purchased ten dozen rolls as follows: 2 dozen $1.19, 1 dozen $1.88, 3 dozen $.94, and 4 dozen $1.28. What average price per dozen did she pay for the ten dozen rolls?
7.) On a vacation, Jack bought gasoline as follows: 10 gallons $1.29 per gallon, 15 gallons $1.19, and 12 gallons $1.23. What was the average price per gallon?

Median

When the elements of a set of numbers have been arranged in ascending order, the number located in the middle of the set is the *median* of the set. The median divides the set of data into two equal parts. To determine which element of a set is the middle number, use the following formula: Middle Number (Total Number of Elements + 1). Ö $ 2 If the set contains an even number of elements, the median is the average of the two middle numbers.

Example 1 : Seven students received the following scores on an examination: 100, 04, 62, 95, 73, 71, 88. Find the median score.
Solution : Arrange the scores in order from lowest to highest: 62, 71, 73, 84, 88, 95, 100.
(7+ 1) dv 2 = 0 dv 2 = 4. The fourth number of the set is the middle number. The median of the set is 84.
Example 2 : Eight workers received the following hourly wages: $5.50, $3.75, $6.00, $4.25, $8.50, $3.90, $7.85, $4.80. What is the median wage?
Solution: Arrange the wages in ascending order: $3.75, $3.90, $4.25, $4.80, $5.50 , $6.00, $7.85, $8.50.
(8+ 1) dv 2 = 9 = 2 = 4.5 The two middle numbers of the set are the fourth and fifth numbers. $4.80 and $5.50.
($4.80 + $5.50) dv 2 = $10.30 dv 2 = $5.15
The median wage is $5.15.

***Class Assignment* :**

1.) Find the median height of the students in your class.
2.) Keep a record of your test scores for one marking period. At the end of the marking period, find the median test score.

Exercises : Solve the following problems.

1.) Find the median of the given data:
   a.) 18.2, 16.8, 13.3, 19.4, 17.6
   b.) 68, 62, 64, 60, 63, 61, 59
   c.) 237, 225, 230, 228, 236, 232
   d.) 70, 74, 71, 72, 80, 78, 75, 69
   e.) $18.42, $16.74, $19.88, $15.64, $24.38, $14.76
2.) Write mean or median to complete the sentence for the following data.
   a.) 5, 6, 2, 9, 8. The is 6.
   b.) 11, 2, 3, 4, 6, 10. The is 6.
   c.) 9.7, 4.2, 6.3, 8.5, 7.4. The is 7.4.

d.) 85, 83, 86, 90, 88, 91, 93. The is 88.

e.) $3.60, $2.85, $4.90, $4.50, $5.00, $2.75. The is $4.05.

3.) Are the following statements true or false?

a.) The median of 12, 5, 6, 13, 10, 9, 7, 8, 14 is 10.

b.) The median of 6.5, 4.0, 8.5, 2.5, 5.0, 3.5 is 4.5.

c.) The mean of 2.8, 7.6, 5.4, 8.2, 3.9 is 5.4.

d.) The mean of 12.6, 6.8, 5.9, 10.7 is 9.

e.) The median of 184, 200, 150, 148, 178 is 178.

Mode and Range

The *mode* of a set of numbers is the element that appears most frequently in the set. There can be more than one mode in a set of numbers. A set that has two modes is *bimodal* . One that has three modes is *trimodal* . If no element of a set appears more often than any other element, the set has no mode. The mode is an important measure for business people. It tells them what items are most popular with consumers.

Example 1 : Find the mode of the following set of numbers: 2, 5, 4, 2, 3, 7, 9, 5, 2, 4, 8.

Solution: Element Frequency

(figure available in print form)

The number 2 occurs most frequently, hence 2 is the mode of the set.

Example 2 : Find the mode of the following set of numbers: 10, 11, 15, 10, 9, 11, 10, 12, 11, 12.

Solution :

(figure available in print form)

The numbers 10 and 11 each appear three times. The set has two modes: 10 and 11.
Example 3 : Find the mode of the following set of numbers: 1, 2, 3, 4, 5.
Solution : No number appears more than any other one in the set. The set has no mode.
The *range* of a set of numbers is the difference between the highest and lowest numbers of the set. To find the range of a set of numbers, use the following formula:
Range = Highest Number Lowest Number
Example : What is the range of the following set of numbers? 5, 8, 10, 15, 7, 6, 20, 9
Solution : The highest number of the set is 20. The lowest number of the set is 5.
20 5 = 15. The range of the set is 15.

***Class Assignment* :**

1.) Find the range in height of the students in your class.
2.) Find the range of test scores obtained by students in the class for one marking period.

Exercises : Solve the following problems.

1.) Find the mode of the given data.
   a.) 3, 5, 6, 7, 6, 2.
   b.) 14, 15, 14, 17, 15, 15, 18, 19.
   c.) 231, 237, 248, 244.
   d.) 84, 86, 87, 84, 86, 89, 90, 87, 87, 84, 86.
   e.) 29, 30, 31, 28, 29, 35.
2.) Find the range of the given data.

a.) 3.6, 9.2, 5.8, 7.4, 12.1.
b.) $22.54, $19.82, $50.00, $35.60, $42.78, $15.63.
c.) 70 64, 98, 69, 82, 85, 59.
d.) 12, 6 1/2, 13, 8 1/2, 11, 15 1/4.
e.) 26.3, 9.27, 15.7, 28.9, 18.8.

3.) Write mean, mode, median or range to complete the sentence for the given data.
a.) 17, 19, 17, 15, 20, 16, 18. The is 17.
b.) 78, 68, 82, 96, 84, 90, 76. The is 82.
c.) 42, 69, 53, 75, 97, 88, 38. The is 69.
d.) 22, 10, 42, 39, 27, 32, 49. The is 39.

4.) Complete the table for the given data.

(figure available in print form)

Frequency Table

Large amounts of information can easily be organized, read and understood by listing data in a frequency table. A *frequency table* is a chart in which the members of a set are tallied, and the total count for each item is recorded. If the set has a wide range of elements, it may be divided into equal intervals to make the frequency table shorter.

Example 1 : The frequency table below shows data about favorite types of music. Use the table to answer the following questions.:
a.) How many people are included in the survey?
b.) What percent of the people surveyed preferred rock and roll music?

c.) What is the ratio of people who prefer waltz music to those who prefer country and western music?

d.) If the number of people who prefer waltz music were increased by 6, What would the percent of increase be?

(figure available in print form)

Solution :

a.) 17+ 14 + 18 +11 + 12 = 72 72 people are included in the survey.

b.) 18 dv 72 = .25 = 25% 25% of the people surveyed preferred rock and roll music.

c.) 12:14 = 6:7. The ratio of people who prefer waltz music to those who prefer country and western music is 6:7.

d.) 6/12 = x/100 12x = 600 x = 50

The percent of increase is 50%.

Example 2 : The prices of 25 different television sets are listed below. Show the data in a frequency table. Determine the median price and the range of prices.

Prices of Television Sets

$219 $399 $400 $359 $318
$360 $200 $480 $247 $430
$475 $250 $260 $278 $397
$499 $480 $427 $387 $435
$314 $425 $450 $287 $498

Solution : Establish six intervals of $50 each. Tally the number of items in each interval.

(figure available in print form)

The median may be determined as follows:

(25 + 1) dv 2 = 26 dv 2 = 13

The l3th item of the set is the median price. The l3th item is the fourth item in the interval $350 $399. The items in this interval are: $359, $360, $387, $397, $399. The median price is $397.

The range may be determined as follows:

Range = $498 $200 = $298.

Exercises : Solve the following problems.

1.) Ask your classmates which of the following flavors of ice cream they prefer-vanilla, chocolate, butter pecan, or chocolate chip. Construct a frequency table to display the results of your survey.

a.) How many people are included in this survey?

b.) What percent of the people surveyed prefer chocolate ice cream?

c.) What is the most popular flavor of ice cream among this group of people?

d.) What is the least popular flavor of ice cream among this group of people?

e.) What is the ratio of people who prefer butter pecan ice cream to those who prefer chocolate chip ice cream?

f.) If the number of people who prefer vanilla ice cream were increased by 8, what would the percent of increase be?

2.) The high temperatures in twenty-five cities on June 29 were as follows:

69 64 61 87 87
58 71 58 87 72
70 52 82 107 112
73 68 78 87 68
71 73 90 96 83

a.) Arrange the temperatures in intervals and make a frequency table for the set of data.

b.) What is the range of the temperatures?
c.) What is the mode of the temperatures?
d.) What is the median temperature?
e.) What is the mean temperature?

3.) The salaries of thirty people are listed below.

$12,500 $23,900 $18,750 $24,000 $14,000
$18,750 $11,570 $25,000 $9,200 $15,000
$24,000 $22,000 $20,500 $12,500 $17,300
$10,980 $15,550 $18,750 $18,000 $16,200
$32,000 $13,000 $22,000 $35,000 $21,000

a.) Arrange the salaries in intervals and make a frequency table for the set of data.
b.) What is the range of the salaries?
c.) What is the median salary?
d.) What is the mean salary?

Histogram

The frequency of a set of numbers can be represented graphically on a histogram. A *histogram* is a bar graph on which the bars are adjacent to each other with no space between them. To construct a histogram, arrange the data in equal intervals. Represent each interval on the horizontal axis of the graph. Represent the frequency of items in the interval on the vertical axis of the graph.

Example: The number of students in twenty-five classes of a high school are as follows: 15, 27, 22, 18, 10, 25, 27, 12, 19, 26, 14, 12, 22, 20 21, 17, 21, 20, 13, 12, 22, 27, 21, 17, 27. Arrange the class sizes in intervals and construct a histogram depicting the number of classes in each interval.

(figure available in print form)
(figure available in print form)

Exercises : A test was scored on the basis of 1 to 20 points. The scores obtained by thirty students are as follows: 20, 12, 18, 11, 20, 8, 1, 5, 10, 12, 15, 15, 18, 13, 19, 20, 18, 15, 16, 18, 17, 14, 20, 19, 13, 16, 12, 18, 20, 8.

1.) Arrange the scores in intervals and construct a histogram depicting the number of students who scored in each interval.
2.) In which interval do most test scores lie?
3.) Which interval contains the least number of test scores?
4.) What is the average number of points scored?
5.) What is the range of test scores?
6.) What is the median test score?
7.) What percent of all scores lie in an interval from eleven to fifteen inclusive?
8.) What is the mode of the test scores?
9.) How many students received a grade of 75% or better?
10.) If 60% of the total possible points represents a passing grade, how many students passed the test?
11.) How many more students scored in the interval 16 to 20 than in the interval 11 to 15?
12.) Did any student receive the median score?
13.) How many students scored higher than the mean?
14.) How many students scored below the median?
15.) What percent of the students failed the test?

Frequency Polygon

A frequency *polygon* is a line graph which can be used to represent the frequency of a set of numbers. It is formed by connecting a series of points. The abscissa of each point is the midpoint of the interval in which the point lies. The ordinate of each point is the frequency for the interval. The polygon is closed at each end by

drawing a line from the endpoints to the horizontal axis at the midpoint of the next interval.

Example : The number of students in twenty-five classes of a high school are as follows: 15, 27, 22, 18, 10, 25, 27, 12, 19, 26, 14, 12, 22, 20, 21, 17, 21, 20, 13, 12, 22, 27, 21, 17, 27. Arrange the class sizes in intervals and construct a frequency polygon depicting the number of classes in each interval.

(figure available in print form)
Number of Students
(figure available in print form)

Exercises : The students in a certain class received the following marks on a test:

94 96 72 90 87 84
72 90 60 68 93 78
68 80 94 75 87 90
87 78 81 85 70 87

1.) Group the data in intervals and construct a frequency polygon to show the number of students in each interval.
2.) What is the mode?
3.) What is the mean score?
4.) What is the range?

Bibliography for Teachers

Coxford, Arthur F., and Payne, Joseph N. *HBJ Algebra 2 with Trigonometry* . New York: Harcourt Brace Jovanovich, Incorporated, 1983.

Algebraic skills and concepts are applied in each of eight “Using Statistics” lessons. The problem solving techniques illustrated involve organizing data in a table and graphing the data in order to draw a conclusion.

Downing, Douglas, and Clark, Jeff. *Statistics the Easy Way* . Woodbury, New York: Barron’s Educational Series, Incorporated, 1983.

This book is clearly organized and contains practical information written simply for rapid learning. It is a good overview of the subject

with numerous examples and exercises.

Kline, William E., et al. *Foundations of Advanced Mathematics* . Second Edition. New York: American Book Company, 1965.

A fine textbook for high school students who are studying advanced algebra and trigonometry. A chapter on statistics and probability is included.

Mendenhall, William. *Introduction to Probability and Statistics* . Fourth Edition. North Scituate, Massachusetts: Duxbury Press, 1975.

The author provides a cohesive, connected presentation of statistics that identifies inference as its objective and stresses the relevance of statistics in learning about the world in which we live.

Runyon, Richard P., and Haber, Audrey. *Fundamentals of Behavioral Statistics* . Fifth Edition. Reading, Massachusetts: Addison-Wesley Publishing Company, 1884.

This text provides excellent resource material on statistics for teachers.

Stein, Edwin. *Fundamentals of Mathematics* . Modern Edition. Boston: Allyn and Bacon, Incorporated, 1960.

A comprehensive textbook on contemporary general mathematics for the junior and senior high schools. It contains all the basic topics of mathematics and includes computational practice and related enrichment materials. It is ideal for use in consumer mathematics and shop mathematics classes in the high school.

White, Myron R. *Advanced Algebra* . Boston: Allyn and Bacon, Incorporated, 1961.

A good text for twelfth year mathematics students. The subject matter is flexible and easily adapted to individual and group needs. Exercises are divided into two groups: 1) those that represent minimum essentials and should be required of all students, and 2) those that present an additional challenge.

Willoughby, Stephen S., and Vogel, Bruce R. *Probability and Statistics* . Morristown, New Jersey: Silver Burdett Company, 1968.

An excellent reference book for teachers. It is meant to be used for a one semester, pre-calculus course in probability and statistics.

Reading List for Students

Bolster, L. Carey, and Woodburn, H. Douglas. *Mathematics in Life* . Second Edition. Palo Alto, California: Scott Foresman and Company, 1982.

Special features of this text include a pretest of each skill to be presented, lessons on the skills, and a posttest after the skill has been studied. Recreational puzzles are provided to capture the interest of students.

Clark, Gerlena R., et al. *Holt General Mathematics* . New York: Holt, Rinehart and Winston Publishers, 1982.

A general mathematics book in which emphasis is placed upon basic skill development and practical applications. Worked out examples guide students through the solution process. Exercise practice organized according to skill and level of ability.

Gerardi, William J., Jones, Wilmer L., and Foster, Thomas R. *Essentials of Mathematics* . New York: Harcourt Brace Jovanovich, Incorporated, 1983.

A basic mathematics textbook stressing consumer and career skills. Many practical applications of these skills are included.

Goozner, Colman. *Business Mathematics the Easy Way* . Woodbury, New York: Barron's Educational Series, Incorporated, 1984.

An ideal source for brushing up on basic mathematics. The book is clearly organized and filled with practical, straight forward information.

Keedy, Marvin L., Smith, Stanley A., and Anderson, Paul A. *Applying Mathematics A Consumer , Career Approach* . Menlo Park, California: Addison-Wesley Company, 1983.

A nicely illustrated textbook with an ample supply of exercises to give students the opportunity to check their understanding of the principles studied.

Nichols, Eugene D., et al. *Holt Pre-Algebra* . New York: Holt, Rinehart and Winston, Publishers, 1980.

This book is designed to aid students in making the transition from elementary mathematics to algebra. Chapter 10 includes a section on elementary statistics.

Piper, Edwin 8., and Gruber, Joseph. *Applied Business Mathematics* . Cincinnati, Ohio: South-Western Publishing Company, 1965.

A textbook designed to have value for consumer use as well as for business use. It develops the fundamental principles and operations through a study of problems faced by every individual daily. A section on finding simple and weighted averages is included.

Price, Jack, Rath, James N., and Leschensky, William. *Pre-Algebra* . Columbus, Ohio: Charles E. Merrill Publishing Company, 1982.

The textbook provides an in-depth presentation of the skills, concepts, and problem-solving techniques needed to help students become comfortable with and successful in algebra.

Willcutt, Robert E., Fraze, Patricia R., and Gardilla, Francis J. *Essentials for Algebra Concepts and skills* . Boston. Houghton Mifflin Company, 1984.

Chapter 10 presents a nice introduction to statistics.