Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
1988 Volume VI: An Introduction to Aerodynamics

Aerodynamics: The Mathematical Implications

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Table of Content

1. Rationale for the Unit

The study of aerodynamics has provided the modern world with one of the most efficient modes of transportation. It is important to recognize that it is mainly through its applicability and technology that man will be able to explore the new frontier space. The application of aerodynamics is embodied in various mathematical concepts. It is this relationship between aerodynamics and mathematics that this unit will explore.

The purpose of this unit will be to find these mathematical concepts that are embodied in the study of aerodynamics, and to present these to a group of average and below average students.

The development of this unit will be justified in the emphasis placed on making mathematics relevant and practical, and probably meaningful to students, providing them with a “peek’’ into a topic whose concepts are not available in their school environment.

Aims This unit is designed to help students to:

(a) acquire the technical and scientific vocabulary;
(b) understand the concepts of aerodynamics;
(c) to develop the ability to apply their mathematical skills to the concepts of aerodynamics;
(d) to gather information and present it in tabular form;
(e) to design and perform experiments based on their knowledge and concepts of the topic.

1. General Objectives

1. The ability to group the meaning of the underlying concepts in the topic of aerodynamics.
2. The ability to integrate these concepts with their mathematical skills;
3. The ability to do basic mathematical computations
4. The ability to use facts, principles and concepts to solve problems
5. To analyze and organize information so that they may be used selectively.

Experimental skills

a. Plan experiments
b. Manipulate the experiment
c. Work effectively with each other
d. To make observations

Limitations of This Unit

The unit makes an attempt to develop a mathematical course outline that will be suitable for students in a math application course. These students are usually those who have difficulty in Mathematics, caused either by lack of motivation in the subject or students who have failed to grasp the basic concepts. The main aim of the unit is to teach the basic mathematical concepts with an emphasis of its usability in the aircraft trade.

To make the unit more realistic students will make visits to aircraft plants to get a first hand look of how the concepts learned can be used. It would be more applicable if the unit could be taught in conjunction with the vocational trade, where students can actually use the materials.

To teach must of the concepts it will be necessary to use pictures and designs of present day aircraft.

Because of the limitation of space for the unit the content cannot be provided for all the objects stated. The references will provide the teacher with more content area.

3. Content and Specific Objectives for the Unit

Content Specific Objectives

1. History of Aerodynamics
    a. The students will be able to tell the main developments in the history of flight.
2. How airplanes fly
    a. The student will become familiar Forces in Flight the vocabulary of aerodynamical terms
        i. airfoil
        ii. Leading edge
        iii. trailing edge
    b. Students will be able to calculate resultant force
    c. To understand how airfoil produce lift
    d. To calculate the ratio of lift and drag
    e. To calculate (C.F.) centrifugal force

3. The Steel Rule
    a. Students will be able to use the steel rule to measure length
    b. To measure different degrees of accuracy
    c. To perform the basic operations on ruler fractions.
4. Decimal in Aviation. Students will be able to
    a. Reed decimal numbers
    b. Check dimensions with decimals
    c. Perform the basic operation on decimals
    d. Read a decimal equivalent chart
    e. Determine tolerance and limits
5. Measuring Lengths. Students will be able to
    a. Determine the units of length
    b. Calculate perimeter of different shapes
    c. Calculate the circumference of a circle
6. The Areas of Simple Figures. The Student will be able to
    a. Determine units of area
    b. Calculate the area of rectangle and squares
    c. Calculate the area of a triangle
    d. Calculate the area of a trapezoid
7. Volume and Weight. Students will be able to:
    a. Determine units of volume
    b. To use formulae for volume
    c. To calculate weights of Material
    d. To calculate Board feet
8. Angles and Construction. Students will be able to:
    a. Use a protractor
    b. Construct angles
    c. Determine the units of measure of an angle
    d. Calculate angles in aviation
    e. Bisect angles and lines
    f. Construct perpendicular and parallel lines
9. Graphic Representation of Airplane Data. Students will be able to
    a. Construct and read a Bar chart
    b. Construct and read a pictograph
    c. To construct a broken line graph
    d. Construct a line graph
10. Spherical Geometry. Students will be able to:
    a. Use the globe to measure distances
    b. calculate the distances between two airports
    c. To read polar coordinates
    d. To represent direction using vectors
    e. Pilot the path of flight between cities
11. Other Topics—The Weight of the Airplane. Students will be able to

a. Calculate the wing area
b. Find the mean chord of a tapered wing
c. Calculate Aspect Ratio
d. Calculate the gross weight of an airplane
e. Calculate pay load
f. Calculate wing loading
g. Calculate Power loading

12. Airfoils and Wing Ribs. The student will be able to
a. Draw a scale of the upper camber
b. Draw a scale of the lower camber
c. Draw upper and lower camber when data is given as per cent of chord
d. Draw the nosepiece and foil sections
e. To calculate the thickness of airfoils
f. To draw airfoils with negative numbers

13. Mathematics of Materials. Students will be able to
a. Define and demonstrate the tension of different materials
b. Calculate compressive strength
c. Calculate shear strength of a material
d. Calculate Bearing strength
e. Define and calculate cross-sectional area
f. Calculate bend allowance

14. Aircraft Engine Mathematics. Students will be able to
a. Define horse power and pressure
b. Calculate the area of piston head
c. Calculate different horse power
d. Calculate fuel cost
e. Calculate cruising time

15. Scale Drawing. Students will be able to
a. Read information from a blue print
b. Reproduce a diagram using a scale
c. Calculate length and distance using a scale

Forces in Flight

The performance of an airplane depends on five forces that act up on it during flight. In some maneuvers all five of these forces are acting on the airplane. When we see an airplane overhead we cannot help but wonder how it is possible to sustain so much weight in a medium as light as air. The airplane wings must produce a force equal to its weight. This sustaining force that opposes the weight is called lift.

(figure available in print form)

Lift is affected by

1. Density of air due to
    a. High or low barometric pressure
    b. Temperature
    c. Varying moisture content of the air

2. Ascending or descending current

How an Airfoil produces Lift

The action of the air on the wing of an airplane during flight is similar to the action of the air on the kite. The air exerts an upward and backward force on the wing. This resultant force can be resolved into the horizontal drag force and the vertical lift force.

(figure available in print form)

Airfoil : This term refers to a surface designed to produce an efficient lifting force; it is also used in reference to the shape of a cross section of a wing.

Angle of Attack : The angle between the chord of the airfoil and the direction in which the airfoil is moving through the air. This direction is often referred to as the flight path of the airplane. The angle of attack can also be defined as the angle between the chord and the direction in which the air is flowing over the wing.

Leading edge : The front or forward edge of an airfoil

Trailing edge : The rear edge of an airfoil. A typical wing tapers off towards the rear with the edge as sharp as possible.

Chord : The straight line from the leading to the trailing edge of an airfoil.

Air flow : The fuselage of the airplane is designed so that it will pass through the air with the lease possible resistance. It should be shaped so that the air will follow its surface and flow over it smoothly. The shape of the airfoil must be such that the air will not only flow over it, but will develop a definite force in a given direction. This force is called lift. An airplane can stay in the air only as long as its maintains sufficient relative motion between these two pressures.

Drag : Air resists the movement of any object through it. This resistance is called drag. The drag set up by an airplane in steady level flight absorbs all the power development by the engine.

The drag of an airfoil can be attributed to two main causes. The part of the total drag caused by eddying currents and turbulence set up in the air is called *profile drag* . The skin friction is causes by the friction between the object and the air.

When airfoil is adjusted so that the chord is parallel to the airfoil or at zero angle of attack considerable lift *is produced* .

(figure available in print form)

The lift force of a wing is derived from the independent action of its upper and lower surface.

Lower Surface Lift : The leading edge of the wing is slightly higher than the trailing edge so that the air staking the under surface is deflated downward.

(figure available in print form)

Newton's first and third Laws of Motion explain why deflecting the air downwards must result in a corresponding upward force. The Law states "that a body in motion will continue to move in a straight path unless it is acted upon by some exterior force. The under surface of the wing supplies the force to deflect or change the direction of the air.

Newton's Third Law states that for every action there must be an equal and opposite reaction thus the force required to deflect the air downwards imparts and equal and opposite force that pushes the wing upward.

The lift action of an airfoil can also be explained by Bernoulli's theorem which states that the pressure on any fluid is least where the velocity is greatest and the pressure is greatest where the velocity is least.

The air moving over the upper surface of an airfoil is forced too travel farther, therefore velocity is increased. The increase in velocity caused a decrease in pressure. This causes lift in the upper surfaces. The air that passes beneath the airfoil has less distance to travel, this results in increase in air pressure on the lower surfaces and in decrease in air pressure on the upper surfaces. The total lift produced by the airfoil is equal to the difference downwards. The wing is said to have a relative angle of attack.

(figure available in print form)

The lowering of the leading edge produces no lift therefore called zero angle of lift.

The lift drops off at high angles of attack because the air instead of flowing smoothly over the upper surface breaks away from it and forms eddying currents.

When the lift and drag values of an airfoil are plotted on the same graph, the relationship ratio between them at the different angles of attack is shown. The angle of attack that results in the lowest drag is not the same angle that produces the least lift but is usually near the zero angle of attack. When high angles of attacks are reached.

Discussed were forces that affected the airfoil in flight lift and drag. The forces acting on the wings must be considerable in conjunction with the other forces that affect the airplane.

Gravity

Gravity is the force that plays an important part in the performance of flight maneuver. The direction of gravity is always constant. It is exerted directly towards the center of the earth or downwards.

In straight and level flight the direction of the lift force is exactly opposite to the pull of gravity. The resultant of the lift force and gravity can be determined by completing a vector diagram.

(figure available in print form)

Thrust

The forward force that the propeller develops is called thrust. It pulls or pushes the airplane forward through the air, overcoming the drag or the resultant of lift and gravity plus the drag.

The direction of the thrust is always in line with the crankshaft of the engine which is generally parallel to the longitudinal axis of the plane.

The thrust force developed by a propeller depends on the propeller design, the density of the air; and the speed at which it is turning. When the engine is idling by the thrust is negligible. The maximum thrust is limited by the highest power output of the engine.

(figure available in print form)

During climbing flight, the resultant of lift and a portion of the thrust is equal to the pull of gravity. Lift is less than the resultant and is therefore less than gravity.

The total thrust (t, plus t2) must equal the Drag d1 plus the left gravity resultant (d2).

When the airplane is flying at a constant rate of speed in level flight, the thrust pulling it forward must be exactly equal to the drag holding it back.

Centrifugal Force

Newton’s first Law; a moving object will travel in a straight line indefinitely unless acted upon by an outside force. The law indicates that a force is required to compel any object to travel in a curved path. Centrifugal force can be mathematically determined by the application *of C.F.= mv2/f*

(figure available in print form)

Mathematical concepts underlying the force in flight

The lift and the drag of an airfoil section have a definite relation for any angle of attack. In addition the lift and drag depend on

a. the angle of attack
b. the contour of the wing
c. the density of the air
d. the area of the wing
e. the square of the airspeed

1 = c1P/2 s.v2

D = c3P/2 s.v2

Where 1 and d are the left and drag in pounds c1 and cd are the lift and drag coefficient which depends on the wing contour used and the angle of attack, is the air density in slugs per cubit foot is, the area of the wing in square feet, and V is velocity or airspeed in ft per second.

Vectors and Forces

A vector has both magnitude and direction. In describing force, not only the magnitude but also the direction must be stated. Force is therefore a vector quantity.

Speed is the distance travelled in unit time, it is not a vector, but velocity in a vector, since it refers to a speed in a given direction. Therefore velocity signifies both speed and a given direction.

Addition of Forces. Vectors can be drawn graphically by representing each force by a line, the direction of the line being the line along which the force acts. The length of the line represents the magnitude of the force. The arrow head is always placed at the end of the line to indicate the direction in which the force acts.

Any action produced on a body by two forces at the same time will be exactly the same as if a single force whose magnitude is the sum of the other two force. This single force is called the *resultant* of the other two forces. The two forces are called components.

A diagram is used to show the components of a vector.

If two forces act on a point but their lines of action are not identical, then a diagram is constructed so that each of the two forces is acting away from the point representing the point on which the forces are acting. A line parallel to the other force is drawn through the opposite end of each line, forming a parallelogram. The diagonal line is called the resultant. This resultant force has the same effect on a pivot as the two original forces.

Navigation and Spherical Geometry

Methods of Navigation

a. Pilotage is the method of conducting an aircraft from one point to another by observation of land marks either previously known or recognized from a map. This method has limitations for example when the flight is over poorly mapped country, over bodies of water, at night or when visibility is poor. Pilotage should always be used in conjunction with other methods of navigation. b. Dead Reckoning. This is the basic method of navigation. It uses known or estimated factors such as wind direction, wind velocity, and airspeed are used to compute a position from a known position. Examples of head-reckoning is Lindbergs New York to Paris flight.

c. Radio Navigation is the method of conducting an aircraft from one point to another by radio wave. Its major features is that one does not have to see the ground to make instrument approaches and landings. Celestial Navigation is the method of determining the geographical position of an aircraft by observation of the celestial bodies to determine position.

The Earth's Surface and Mapping

Using the globe is the only accurate means of representing the spherical surface of the earth, but a globe is impractical to use in the cockpit of an aircraft. It is therefore necessary to use charts to represent the earth surface. The method of representing the earth surface on a chart is called projection. How to measure distances on the globe. If a string is stretched between two points on the globe, the length will represent the shortest distance between the points.

If the string is extended around the globe then the circle is divided into two halves, the line is called the great circle: The shortest distance between any two points will be a portion of the great circle.

Locating a point on the earth's surface.

To form a system of reference, meridians running in straight lines are draw from the pole meridian drawn in degrees from east to west of Greenich (England). The prime meridian is called longitude zero.

Lines of latitude are drawn at 90 Degree angles to the meridians starting at the equator, these lines are called latitude lines.

To locate a point the coordinates of the point is given usually latitude followed by the longitude.

How a pilot plans a trip.

(a) draw a straight line between the two points (b) measure the course in degrees from true north from a point about midway on the course. This is called True Course (T.C). The effect of winds. When an airplane moves through the air mass at a given airspeed, the airspeed remains constant, but the aircraft's speed measured by the distance it travels over ground is affected by the movement of the air.

(figure available in print form)
The effect of the wind speed causes the aircraft to drift. The wind triangle.

A course is the direction towards it's destination. If there is no wind the aircraft will fly towards it's direction. If the wind is blowing it will affect the aircraft's tract. In order to establish a heading direction in which the plane is going a parallelogram of forces is constructed.

(figure available in print form)

The wind triangle is one half of the parallelogram of forces representing the interaction between the airplane's airspeed and wind velocity.

problem. T.C = A to ,90 degrees
Distance A to = 150 miles
Wind Velocity = 35 knots (40 m.p.h.)
Airspeed = 120 m.p.h
Variation = 7 degrees East.

Constructing a wind triangle.

1) draw a north-south line, locate T.C at the 90 degree point, and the direction from which the air is blowing at 45 degree point.

(figure available in print form)

2) Draw the wind vector in the direction of its force.

(figure available in print form)

3) Using the same scale draw a line from the end of the wind arrow.
4.) The true heading () can be measured directly with the protractor.
5) The wind correction angle can also be measured with a protractor
6) ground speed can be measured along T.C. lines from point E to the vector at point P.

Completed Diagram.
(figure available in print form)

Calculating Time of a Trip and Fuel Consumption.

After the pilot has determined his path, he needs to calculate the time and fuel required for the trip. He needs to have fuel for the trip and enough reserve for 45 minutes of extra flying time.

Calculations Required.

Calculating minutes to hours.

To find the time in flight when groundspeed is known

Time = distance/groundspeed.

To find the distance flown for a given time

groundspeed x time = distance.

The groundspeed in flight is often not the same as calculated, this variation is due to wind velocity. To calculate groundspeed accurately use

groundspeed = distance between check points/time.

To calculate fuel consumption:

Fuel = fuel consumed/time.

4. SYMBOLS, DIAGRAMS and DEFINITIONS USED:

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1) What makes airplane fly:
C.F = m.v2 f; centrifugal force.
1= c p/2 s v.2; lift
d= c dp/2 s.v3; drag
cp = p1 2/p2/q; pressure coefficient
2) Vector
AB: position vector.
3) Area of simple figures.
Length = A/W
width = A/1
The circle Area = ¹r2
The Triangle Area = bh/2.
The trapezoid = 1/2h (b1+b2)
Volume V = A x h
4) The Weight of Material.
Weight = V x unit weight.
boardfeet = Area x t (t = thickness of material)
5) The Weight of the Airplane:
Area = span x chord.
Aspect Ratio = span/chord
Mean chord = area/span.

Aspect Ratio is the relationship between the span and the chord. It is important to the flying characteristics of the plane. The mean chord: is the average chord of a tapered wing. It is found by dividing the wing area by the span.

(figure available in print form)

Empty Weight: The weight of the plane without gas.
Useful Weight: the weight of the plane when passengers, baggage, oil and gasoline are added.
Gross Weight: The Maximum weight that an aircraft can carry.
Gross weight = empty weight + useful load.
Payload: The weight of all the things that can be carried for pay.
percent payload = payload/gross weight x 100.
Wing loading: The number of pounds of gross weight that each square foot of wing must support in flight.
Wing loading = gross weight/wing area
Power loading: The ratio between weight and engine power.

6) Airfoil and Wing Ribs:

Datum line: is the base line or horizontal axes
(figure available in print form)

Vertical Axis: The line running through the leading edge of the airfoil section and perpendicular to the datum line.
Stations: are points on the datum line from which measurements are taken.
Upper camber: is the curved line running from the leading edge to the trailing edge along the upper surface of the airfoil.
Lower camber: is the line from the leading edge to the trailing edge along the lower surface.
Strength of Material:
Ultimate tensile strength: The amount of weight a bar one square inch in cross-sectional area will support in tension before it fails. Tensile strength = A x u.t.s (ultimate tensile strength)
Ultimate compression strength: is the number of pounds one square of material will support in compression before it breaks.
Compression strength = area x u.c.s (ultimate compression strength)

5. SAMPLE ITEMS ILLUSTRATING SOME OF THE SPECIFIC OBJECTIVES:

Measurement using a steel ruler:

1) In the figure showing an airfoil section measure the thickness of the airfoil section at each station to the nearest 16th of an inch.

(figure available in print form)

2) Find the overall length of the figure

(figure available in print form)

3) The army plane has a wing area of 375.O sq. ft. If each square foot of the wing can carry an average weight of 25.3 lb, how many pounds can the whole plane carry.

4) If the aircraft shown has a length of 346.5 inches. what is the length in feet.

(figure available in print form)

5) Find the area of the aircraft wing.

(figure available in print form)

6) Approximately how many cubic feet of baggage can be stored in the plane wing compartment shown.

(figure available in print form)

7) Make a full scale drawing of the airplane shown.

(figure available in print form)

Presentation of Data

9) The drag of an aircraft increases as the angle of attack is increased, Represent the following data graphically.

angle of attack	drag lb
0	100
2	100
4	200
6	250
8	370
10	430

12	600
14	800

10) Draw a bar chart to represent the following information.

(figure available in print form)

Weight of an airplane:

11) Calculate the gross weight of the following planes.

Airplane	Empty weight	Useful Load	Gross weight
Cessna	1,380	970	
Boeing 627	48,545	33,955	
D,C 10	30,500	25,900	

12) The gross weight is 1,530 lbs and the payload is 350 lbs. What percent of the gross weight is payload?

13) Find (a) the gross weight (b) percent payload (c) mean chord (d) the wing loading (e) the power loading (f) the aspect ratio of this aircraft with the following specifications.

(figure available in print form)

Weight capacity = 4,440 lbs

Useful Load = 3,160 lbs

Payload = 1,000 lbs

Wing area = 460 sq. ft.

Engine: Pratt & 650 h.p

Span = 60 ft 5 ins.

Fuel and Oil Consumption.

14) An aircraft powered by a : Pratt & Whitney engine with specific fuel of 0.60 lb per hp. hr, and specific oil consumption is 0.025 lb per hp.hr when operating at 735 hp.

a) How many gallons of gasoline will be used in 2 hrs?

b) How many quarts of oil would be consumed in 1 hr?

c) If the tank holds 206 gals how long can the airplane stay up before the tank is empty?

Navigation and Spherical Geometry:

15) Construct a wind triangle using the following information
Course 110 degrees
Wind 20 m.p.h from 225 degrees
Air speed 100 m.p.h.

Airfoil

16) Design an original airfoil section on graph paper and complete a table of data

6. SAMPLE LESSON PLANS: #1

Topic How an airfoil produces lift.

Objectives The students will be able to demonstrate the principles that produces lift.

Rationale The action of the air on the wing of an airplane during flight is similar to the action of the air on a The air exerts both a upward and a downward force on the wings. This resultant force can be resolved into the horizontal drag force and the vertical lift force.

Demonstration of one principle of flight Bernoulle's Principle. Place a small model aircraft on a float board. Blow a stream of air over the airfoil at high velocity.

Result The airflow will rise away from the board although the air stream is pushing it downwards.

Conclusions The air flowing over the upper surface of an airfoil is forced to travel farther and its velocity causes a decrease in pressure this accounts for the lift of the upper surfaces. The total lift produced by the airfoil is equal to the difference between the two pressures.

Materials

a) paper airfoil

b) flat board

c) tape

d) air hose

Lesson plan #2

Topic Forces in Flight: Centrifugal force.

Objective The student will be able to measure centrifugal force. Students will be able to calculate centrifugal from given information.

Rational If the pilot moves his control to change the flight path of the airplane centrifugal force begins to act. Centrifugal force plays an important part in every day activities. For example when riding a bicycle one must lean towards the center to get around a corner. Centrifugal force makes it necessary for a car to turn a corner at high speeds without turning over.
Centrifugal force can be studied by experimenting with a string and a ball.

Demonstration

a) have a student whirl a ball attached to an elastic string around in a circle.
b) change the weight of the ball, and the speed at which the ball is whirled.
c) change the length of the elastic band and whirl at the same speed.
d) have students observe the stretch of the elastic band when the speed, weight of ball and length of elastic band changes.

Conclusions The magnitude of centrifugal force depends on the speed at which the object is moving and the size of the circle it inscribes.
To determine centrifugal force use;

c.f = m.v/f

Example of problem An airplane weighing 3,000 lbs is making a turn with radius of 500 ft at a speed of 150 ft per second.

Materials:
a) elastic

b) balls of different weights.

Lesson plan #3

Topic Scale Drawing: Constructing the Upper Camber of an Airfoil.

Objective The students will be able to construct the upper camber of an airfoil, given specific data.

Rationale The airfoil's surface is designed to produce an efficient lifting force. The term airfoil section refers to the shape of a cross section of a wing, and also to the outline of a lifting surface. Experiments have shown that the shape of the airfoil can affect the performance of the plane. Specifications for constructing an airfoil is supplied by the manufactures, any inaccuracy will mean a change in the plane's performance.

Demonstration Teacher provides example.
Step 1: Draw the datum line and vertical axis

(figure available in print form)

Step 2: Mark all sections as given in data.
Step 3: Mark the distance of the upper camber from the datum line.
Step 4: At the first station, mark the distance of the upper camber above the datum line.
Step 5: Mark off the other points, then connect with a smooth line.

Example

(figure available in print form)
Station

Upper camber

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