Airplane Mathematics

Curriculum Unit 88.06.11
by Joyce Bryant

This unit is designed for Middle School students. The main goal and objective of this unit is to acquaint the middle school student with the history of Aerodynamics and help to develop some fundamental mathematical skills relating to the field of aviation. Also, to study the history of flight, which will include the first transatlantic flight, the Wright Brothers and supersonic flight (old and new). The math criteria will entail the computation, concepts and problem solving of velocity; acceleration, speed, distance traveled, and time. The middle school student is an entity within himself without goals and objectives as to which route he or she is to travel. Not only will this unit apply to practical day to day situations but it will also aid the student in achieving the challenges that the world of aerodynamics has to offer in this ever changing highly-technological society. Men have always wanted to fly. Watching the free flight of birds, they have been filled with envy and the desire to rise into the air and follow them. Man believe in the possibility of flight. The idea of flying haunted mankind from earliest times, and from the dawn of history man’s attempt to emulate the skill of the birds had been marked by the pathetic, broken bodies of adventures who had tried to flap their way through the air and had crashed, Icarus-like, amidst the wreckage of their artificial wings.

Heavier-than-air flight would not be possible until the early 1900’s, when the technical means had been developed that would bring it within man’s grasp. In the meantime there were other ways of getting into the air. Man’s first incredible escape from the earth, took place well over a century before the Wright Brothers lifted their crude machine from the sands of Kitty Hawk, North Carolina. In November of 1782 Joseph Montgolfier sat by the fire noticing the smoke and hot air rushing upwards and was seized with an inspiration. He asked his landlord for rags of silk, he constructed a sample cloth balloon, filled it with hot air and watched it rise quickly to the ceiling. The lighter-than-air balloon had been invented.

By the spring of 1783 Montgolfier, with his brother Etienne, had put together the world’s first balloon. It was a huge sphere of paper lined with linen and on June 4, filled with hot air from a fire of wool and straw, it rose some 6,000 feet. The event was immediately recognized and at the insistence of the Academy of Sciences at Paris the brothers constructed a new balloon, which was launched at Versailles in September. A sheep, a duck, and a cock were carried aloft on a free flight of some two miles and they managed to survive the ordeal. On the 21st day of November, came Jacques Charles and the Roberts Brothers had harnessed the lifting power, “inflammable air” known today as hydrogen. Their first balloon had flown for fifteen miles on August 27, only to be attacked by terrified peasants on its landing. Mounting public excitement set the stage for human flight. The age of flight really began with the ascension of Charles and the younger Robert in a hydrogen balloon on December 1. That first great flight lasted over two hours, and the balloon landed twenty-seven miles from
Paris (See Diagram 1). By the turn of the century the main lines of free-ballooning had been set.

In the United States John C. Wise and Thaddues C. Love prepared enormous balloons for transatlantic crossings, but both became frustrated. Wise tried again in 1873 but his balloon crashed forty miles outside of New York. Balloons, airships and dirigibles were in the making for the next thirty years. In the 1800’s when electricity was just coming to the fore, the Tissandier Brothers built and flew an electric dirigible. The first fully controllable airship, also powered by electricity was built by Charles Renard and Arthur Krebs with a heavy eight horsepower electric motor fed by storage batteries. The dirigible came of age with Santas-Dumont and Count Von Zeppelin applying internal combustion engines to their ships (See Diagram 2).

It was the airplane, not the balloon, that was destined to give mankind a whole new dimension of flight. The airplane, a heavier-than-air machine could not be held in the sky by hot air or hydrogen. Power was required to lift it up, speed to keep it going, and skill to fly it.

The glider made of cotton-covered bamboo and cane was introduced by Otto Lilenthal (See Diagram 3). The Wright Brothers who belonged to Lilenthal School had been working on flying machines. In 1899 the Wright Brothers were running a bicycle workshop which provided them with a machine shop for their experiments. The Brothers built and tested a biplane glider in 1900 at Kitty Hawk, North Carolina. Encouraged, they built another glider almost twice as large and tested it again at Kitty Hawk, in 1901. In 1902 they tested another glider and this glider proved itself in flight ready for an engine. Provision for a light gasoline engine was set in motion. On December 17, 1903, after months of constructing an efficient propeller, the Wright Brothers returned to Kitty Hawk and flew their machine a distance of one hundred feet which lasted for twelve seconds.

The Wright Brothers moved their operations to Dayton, Ohio in 1904, constructed a new plan, rented sixty-eight acres of land and methodically proceeded to learn to fly. They flew one hundred and five flights. In 1905 with an improved machine the Wrights ended their flying season on October 5, with an incredible flight of nearly twenty-five miles in thirty-eight minutes. The airplane was invented in America, but aviation came of age in France. In 1907 Wilbur Wright went to Europe to negotiate with interested governments, including France, to set up an international flying business based on the Wright Brothers patents. As a result of their business transactions the Brothers began to fly again, practicing at Kitty Hawk. The first official flights were made in Germany, Russia, Italy, and spread from France to the United States. Flying was becoming international.

France along with other countries began to think of military aviation. After the Wrights plane had been accepted a few plans such as the Bleriots, Farmans, Antoinettes and Voisins were placed into service. Airplanes although fragile and unexperienced were used during World War I in 1914. Airpower grew rapidly during the war years, but at the end of the war years the whole elaborate military structure was disbanded. Commercial aviation did not develop in the United States for many years, but in Europe with its shorter distance, the first tentative airline appeared right after the war.

Crudely modified war planes were put into use and covered a route between Berlin and Weimar. There was progress. The Dutch K.L.M., was formed in 1919 and staged its first flight from London to Amsterdam on May 17, 1920. By 1921, ten small airplanes were operating in France. In 1923, the Belgian government organized the airline Sabena. Aviation in the United States was at a standstill until 1925 except for a flight in 1920 which flew some Americans from Florida to Cuba. In 1918 an airmail flight was staged from New York to Washington. A New York to Chicago flight was also staged and in 1920 it was extended to San Francisco. America led in airmail but France took over and they laid the foundation for the international air routes of today.
The greatest challenge was when the Atlantic was crossed four times in 1919. The work goals and objectives that had been accomplished were merely preliminary to the hundreds of great flights that would take place during the next two decades and open up air routes of the world. The United States was spanned nonstop from east to west in a record of twenty hours and fifty minutes. The flights multiplied; France explored North Africa, Britain began to link their empire by air, two Portuguese flew the South Atlantic to Rio, and the first flight from Belgium to the Belgian Congo was accomplished. Looking to the future, the important explorations were those of the polar routes. Lindbergh’s daring flight from New York to Paris in 1927 was a culmination of all previous events and the beginning of a new age of flight.

After the great transoceanic flight the international air routes of the world were opened in the 1930’s. Seaplanes became the aircraft of the expansion. But their engines were not powerful enough for its bulk, and its poor performance and many accidents doomed it to failure. Pan American an airline of today built up a fleet of giant flying boats, began its Atlantic and Pacific service. Finally the lighter-than-air craft lost out to airplanes. The rapidly growing airlines needed a transport capable of carrying a large number of passengers at a maximum speed but at a minimum expense. The United States put together many elements that produced the first modern airplane, the DC-3. It was an all metal, thick-winged, internally braced monoplane with a thick fuselage. It was enough to carry two of the more powerful, radial aircooled engines, their drag reduced by a streamlined cowling.

Aviation played an extremely important role during the war years. After World War II the importance of military aviation could not longer be doubted. War planes had been in action in every theater of the war. Planes had flown over every ocean and penetrated the remotest jungles and snowbound wilderness of the world, leaving behind them serviceable airports wherever they had operated.

Commercial aviation in the postwar era was dominated by the United States. The American manufacturers, with years of experience had learned to build the best aircrafts in the world and at a low cost. Outside the United States civil aviation has had its fastest growth and its greatest in underdeveloped areas such as Latin America and Africa. Air freight grew even faster than passenger-carrying. In the fifties, low fare tourist flights and stiff competition caused international air travel to become commonplace. Farsighted aircraft designers realized that propeller-driven planes had reached their potential speed limits began to study the possibilities of a jet engine. The Germans flew the first successful turbojet aircraft. England’s was jet powered by a Whittle engine two years later.

After studying the Whittle engine, the Americans flew their first experimental jet, the Bell XP-59A. When the British flew their first jet faster than the speed of sound, the jet age was ready to be born and it was. The British, leading in jet engines, decided to leap into the future with a bold program for high-speed jet transports which proved to be unsuccessful.

The entry in the race appeared at London in 1956 astonishing the Western World. It was the twin-jet TU-104 which went into general service. The MIG-15 was unveiled in Korea. Russian introduced the giant turboprop TU-114, the largest commercial plane, powered by four huge engines capable of carrying from 170 to 225 passengers. In the United States in October of 1958 Pan America launched its jet service to Europe, a Boeing 707. It was not only a commercial aviation important but also military. War or the threat of war, so often spells technical progress. Since 1945 military aviation has undergone an astonshing transformation in its history. Today’s military planes are flying faster than the speed of sound and they are armed with missles and other destructive ammunition. One of the important piece’s of aircraft today is the jet. It is a piece of machinery that functions in many ways.
WHAT MAKES AIRPLANES FLY

Lift is what keeps an aircraft in the air. The upward force acting at a right angle to the flight path must be sufficiently large to equal the weight of the plane. The fixed wing of modern airplanes is shaped to produce lift in a most efficient way. Sir George Cayley was the first to propose the idea of a rigid airplane for the generation of lift. He gained his wing to a fuselage carrying a distinctly separate propulsive mechanism and added vertical and horizontal tail surfaces for control.

Laying aside buoyant devices such as balloons and airships, all flights, whether natural or mechanical, depends upon the fact that bodies moving through a fluid experience a force called resistance. The major problem of flight with fixed wings is to find means of turning this force to good account by making part of it sustain the aircraft. The modern flying machine consists of a specially shaped hollow body to which are attached rigid supporting surfaces called ailerons, elevators, and rudders, together with a means of producing a thrust, (a force in the direction of motion) by the employment of an airscrew or a high-speed jet.

In aerodynamics the resistance of air is divided into two parts. One component, called drag, acts along the direction of motion; this is the part of the resistance that directly opposes the motion. The other component, called lift, acts perpendicularly to the direction of motion. If the aircraft is flying straight and level at constant speed its drag balances the thrust and its lift, acting along the vertical, opposes the force of gravity and balances the weight of the machine.

Bodies of special shapes, called aerofoils, produce much more lift than drag when moved rapidly through the air. The purpose of the wings (which are aerofoils) is to develop enough upward directed force to overcome gravity. Some of the sustaining force also comes from the tailplanes, but their main purpose is to balance the machine in flight. Aerofoils can produce lift only when air is driven over them or, what is the same thing, they are moved through the air, and it is found that the magnitude of the lift increases rapidly with the speed of the air sweeping past.

When an aircraft is taking off, it is usually pointed into the wind so that, when it moves along the runway because of the thrust provided by the engines, it experiences a very rapid flow of air over the wings, and this enables it to rise after a relatively short run. When airborne the aircraft is guided by a rudder and controlled for stability mainly by the ailerons and elevator and its own inherent properties.

Issac Newton, who flew kites as a young lad gave the world the theory of lift in his Third Law of Motion which states, “For every action there is an equal and opposite reaction.” The simplest way to understand the theory is to comprehend how a kite flies. The wind pushes the kite and produces lift and the kite pushes the wind which is deflected downward. There is no difference whether its the plane that is thrust through the air to make a wind or the kite standing still on its tether with the wind rushing pass it. Lift is produced by the interaction of the forces.

The amount of lift developed can be determined by using the formula:

\[ c_L = \frac{L}{S \cdot q} \]

\[ L = C_{LS} \cdot S \cdot q \]

\[ L = \text{force} \]
S = area

q = force/area

cL = no dimension, coefficient of lift

The cL is determined by wind-tunnel tests for a particular airfoil at various angles of attack in a range of wind speeds.

Certain features have to be built into the aircraft to maintain stability and control when it passes through disturbances in the air known as turbulence. Lift, gravity, thrust, and drag are important when it comes to flying because they are forces which act on an airplane when it is flying. The aircraft is made up of many different parts each having its own unique function and there are many different aircrafts in use today. (See Diagrams 4-6).

The most aircraft in use today are the jets. There are several kinds of jet engines. The most widely used is the turbojet which pulls in large masses of air through the intake and then compress it. The compressed air is heated and expanded in a combustion chamber. The expanded air rushes against a turbine and spins it. The spinning turbine turns the compressor and keeps fresh air coming in. The old air, reaching the nozzle, is compressed enough and is moving fast enough to provide pressure to push the plane. There is a nozzle exit at the rear, and the force against the front sends the plane shooting forward.

Another type of jet is the ram jet. It depends upon its own forward speed to scoop up air and ram it into the combustion chamber. The compressed air is heated and expanded to increase pressure. A large nozzle creates a pressure imbalance and gives thrust to the forward end of the engine. The third type of jet engine is the turboprop. There are two turbines. One spins the compressor and the other spins a conventional propeller. The engine uses both a propeller and the direct force of gases (See Diagram 7).

The latest and fastest commercial jet engine aircraft of today is the Concorde, built jointly by Britain and France. The Concorde has a crew of three and its fuselage can carry up to 144 passengers, seated four abreast and their luggage. It stands high off the ground and lands at a very steep angle. It carries 26,000 gallons of fuel. The Concorde’s four engines can deliver over 52,000 pounds of thrust. (See Diagram 8).

LESSON PLANS

Invite someone from an airline, through its public relations, to speak to the class about the airlines. Have the individual talk about the different types of jobs and qualifications. If possible plan a field trip.

Have students make their own airplanes. Review measurements and fractions with the class. Materials needed for project; construction paper, scissors, glue and rulers. To accomplish this assignment see Appendix A and B.
ACTIVITIES FOR STUDENTS

When navigating a plane by chart and compass, the pilot frequently determines the compass course from the true course and the true course from the compass course.

The course is the direction in which the aircraft is flying over the earth’s surface. True course is when it is measured clockwise from the true north. The pilot’s correct true course reading is the magnetic course. To find the magnetic course from the true course, one must add west variation, but subtract east variation. To find the compass course from the magnetic course, add west deviation and subtract east deviation.

SAMPLE SOLUTION:

125° true course
+ 7° west variation
132° magnetic course
+ 2° west variation
134° compass course

To find the compass course from the true course, change the true course to magnetic course, then to compass course. To find the magnetic course, from the compass course, subtract west deviation and add east deviation.

SAMPLE SOLUTION:

54° compass course
+5° east deviation
59° magnetic course
Ø11° west deviation
48° true course

1. A pilot is steering a course of 1750. If the variation is 12’ west and deviation is 3’ east, what is the true course?

2. A plane is steered along a compass course of 180. If the variation is 110’ west and deviation is
3’ east, what is the true course?
3. Evaluate the distance formula for the values given.
\[ D = RT \text{ DISTANCE} = \text{RATE OF SPEED} \times \text{NUMBER OF HOURS} \]
A. \( dr = \text{km/hr} \) B. \( r = 60.16 \) C. \( r = 90 \text{ km/min} \)
\[ \begin{align*}
  t &= 5.5h \\
  t &= 6.10 \\
  t &= 6.16 \text{ min} \\
  d &= ___ \text{ km} \\
  d &= ___ \text{ km} \\
  d &= ___ \text{ m}
\end{align*} \]
4. The true course from airport Z to airport Y is 104. If the variation is 6’ west and the deviation is 3’ east what compass course should be steered?
5. During a certain flight, the flight compass course was 268. If variation is 8° east and the navigator found that the deviation is 6° what is the true course?
6. What compass course should a pilot steer if the true course is 63°, variation 9’ east, and deviation is 7° west?
7. Compute the compass course if the true course if 128°, variation 8° east, and deviation 4° east?
8. What compass should be steered if the true course is 235°, variation 12° west, and deviation 4° east?
9. A pilot is determining the compass course, uses the following data: true course 350°, variation 13° west, deviation 3° west. What is the compass course?
10. Find the magnetic course:

<table>
<thead>
<tr>
<th>True Course</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 4°</td>
<td>5° west</td>
</tr>
<tr>
<td>B. 164°</td>
<td>19° east</td>
</tr>
</tbody>
</table>

Find the compass course:

<table>
<thead>
<tr>
<th>Magnetic Course</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>C. 157°</td>
<td>1° east</td>
</tr>
<tr>
<td>D. 65°</td>
<td>2° west</td>
</tr>
</tbody>
</table>
WORD PROBLEMS

1. A plane leaving Washington at 11:20 AM is scheduled to arrive in Philadelphia in 3 hours 28 minutes. After a stop over of 30 minutes the plane is scheduled to arrive at John F. Kennedy Airport at 4:28 PM. How long does it take the plane to fly from Philadelphia to Kennedy?

2. In a series of hops, an aircraft flew for 3 hours 18 minutes, 2 hours 42 minutes, and 4 hours 57 minutes. Find the total flying time.

3. An airplane had 8 hours of fuel at take-off. How many hours of fuel is left after the plane flies for 5 hours 18 minutes?

4. A 747 Boeing plane flew from London to New York City in 5 hours 28 minutes 15.6 seconds. A phantom jet holds the record from New York City to London, 4 hours 47 minutes 56.7 seconds. How much longer is the west bound flight?

5. The contract time schedule for building the Concorde by Britain and France was 60 weeks 15 days. If this was reduced by one fourth, how much time was saved?

6. If unimportant intermediate stops along the air route were eliminated, it would reduce the time of 2 days 12 hours required to transport mail between three cities. How long would it take?

7. The Concorde climbed 5 kilometers in 25 seconds. What was its rate of climb in meters per second?

8. One formation of aircraft flew at an altitude of 12,000 feet, while another flew at an altitude of 5,000 meters. Which formation flew at the higher altitude?

9. The wing area of a model airplane is 64 square centimeters. Express its area in square inches.

10. The world aviation altitude record in 1974 was 95,935.99 meters. Express this altitude record in kilometers.

11. In evaluating the formula for lift, the air speed must be expressed in terms of feet per second.

   a. In finding the lift of a plane flying at an air speed of 380 statute miles per second, what air speed in feet per second must be used in the formula?

12. The top air speed of a foreign plane is 900 kilometers per hour while the top speed of a similar American plane is 500 statute miles per hour. Which plane has the greater top air speed?

13. The James family runs a fleet of planes. To the nearest dime, what is the cost per kilometer if the planes fly a total of 1,240,356 kilometers at a cost of $1,780,542?

14. How many planes will Delta Air Lines need to transport a group of 29,522 persons if each jet carries 509 passengers?

15. The price of jet fuel was $1.40 per gallon. The price increased 10%. How much was the increase?

16. The cost of flying from city A to city B is $147.56, from city B to City C is $186.77, and from City C to City A is $206.00: What does it cost to fly from A to B, to C and back to A?
Lisa can save the communications company 25% of the cost by flying in the evening. How much does the company pay for the trip described in exercise 1 if all the flights are evening flights? A charter flight from B to C and back costs $139.47. How much can Lisa save the company if she takes the charter flight?

17. Find the average ground speed, in miles per hour, of a plane when it travels a distance of: 1,000 miles in 4 hours, 180 miles in 12 minutes, 90 miles in 5 minutes and 20 seconds and 360 miles in 1 hour and 15 seconds.

18. A 747 Boeing Jet flew non stop from Long Beach, California to Dallas Airport in the D.C. and Virginia area, a distance of 2,295 statute miles, in 5 hours 17 minutes and 34 seconds. What was the average speed in knots?

19. Find the closing speed in miles per minute if one aircraft is flying at 180 miles per hour head-on toward a aircraft flying at 220 miles per hour.

20. Justine is a ticket agent for United Airlines. On Friday she sells 3 tickets to three different passengers traveling to London, England. One ticket cost $786.00, another costs $626.00 and the third ticket costs $952.00. On Tuesday the airlines put on a special for all three tickets. The first $600.00, the second $525.00, and the third $837.00. How much would each passenger save if they purchase the tickets on sale?
EXPERIMENTS FOR STUDENTS

KITE FLYING

Launching a kite is easy in a good wind. Fighter kites, the flat dragons and the non-rigged kites are the easiest to get up in the air. When launching a kite try and get it up on the first pull. Walk the kite down and either have it stand itself or have someone stand behind the kite and hold it. Wait for a breeze and shout to your assistant to let go. Pull the string in hand-over-hand in quick yanks, walking backward as you do it. Drop the string on the ground so as not to tangle it. Your kite will tell you when it is ready for more string. Slight pressure means more string. If it drops, a yank will get it climbing. A model plane made from light weight construction can also be used.

A toy balloon can help demonstrate how the force of air molecules can move a jet plane. Blow up a balloon until it is full of air. Hold the end tightly and then suddenly let go. As air shoots out from the nozzle, the balloon spurts away. The force of the air molecules, pressing against the balloon walls is very powerful and there is enough force in this air pressure to send the balloon flying.

Cut two strips of paper approximately twelve inches in length. Turn on an electric fan, place one strip in front of the fan and observe what happens. Later place the other strip in back of the fan and see what happens.

SOME AVIATION WORDS STUDENTS SHOULD KNOW

- ailerons two hinged surface at the back end of the wing that makes the plane slant in one direction or the other. This is called “banking the plane.”
- airborne a plane in the air, free from ground contact.
- airfoil a surface such as a wing, that creates lift from passage or air along it.
- airframe the framework, envelope, and cabin of an aircraft.
- air speed the airplane’s actual speed through the air.
- altimeter an instrument that measures the plane’s altitude above sea level.
- ceiling the altitude at which the sky is covered by clouds, dust or smoke.
- contrails white streaks of condensed moisture in the air that trail behind a jet engine.
- drag the component of the total air force on the body. It is opposite to thrust and parallel to relative wind.
- drift the sideways movement of a plane caused by wind.
- e.t.a. estimated time of arrival at destination or next reporting point.
- elevator a part of the tail that directs the plane up or down.
- fixed-wing a conventional airplane.
- fin airfoil attached to the tail of an airplane to provide directional stability.
- flaps moveable section at rear of wing which when lowered act as brakes and slows down the plane.
- flight plan a written report giving details of a planned flight.
- fuselage the body of the plane.
ground speed the actual speed of a plane over the ground. It is higher than air speed when helped by the wind (tailwind). It is lower than air speed when hindered by wind (headwind).
implement flying the operation of a plane using instruments to navigate.
jetstream bands of high-speed winds 50 to 300 miles wide, found at high altitudes.
lift the nearby vertical reaction which results from passage of an airfoil through the air.
overshoot to fly past a designated mark when attempting to land.
powerplant the complete engine or engines of an aircraft.
speed of sound at sea level in air at moderate temperature, about 750 miles per hour. At high altitudes, where temperature is lower, the speed is less.
thrust the amount of push.
rudder a part of the airplane tail that helps the plane to make right or left turns.
spoilers objects on a wing that help slow the aircraft down when landing.
taxiway a surface area for an aircraft to taxi (move) to or from a runway.
turbulence rough air currents.

Burnet, Charles, *Three Centuries to Concorde* : Mechanical Engineering Publications Limited, London 1979. The Author not only tells what was done but how it was done. It is a detailed study of the British contribution to progress, culminating in the Concorde.


Editors of Air Force Times, *Before The Eagle Landed* : Robert Lucci Inc., Washington, 1970. *Before The Eagle Landed* is the history of aviation told in the words of the men and women who made that history. In chronological order, from Orville Wright to Neil Armstrong, the stories of the actual flights are unfolded with all the excitement in which they were lived.

Smith, Dean, C. *By The Seat of my Pants* : Little, Brown and Company, Baston, 1961. This tells of the experiences of a pilot who flew various missions and his only instruments were the compass, clock and altimeter.

Stewart, Oliver, *Aviation: The Creative Ideas* : Frederick A. Praeger, New York, 1966. The more influential and arresting of achievements in aviation, their origins and developments, and the men who were responsible for them are described in this book.

that have allowed aircraft to fly faster and have recalled the pioneering designs from the wright flyer to the hypersonics of the space shuttle orbiter.


READING LIST FOR STUDENTS


