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Boolean Algebra and its Application to Problem Solving and Logic Circuits

Curriculum Unit 89.07.07
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Purpose

The curriculum unit is designed to introduce a unit of simple logic and have students exposed to the area of Boolean algebra and how it can be used as a tool for problem solving. These mathematical ideas have been left out of the curriculum of many high school students. There is a need for mathematics to become more relevant to today's society. Boolean Algebra with its application to the development of logic circuits will provide some hands on application and to bring to students the importance of technology and the effects it has had on our society.

Rationale of the unit

The guiding principles of this unit is that any area of mathematics can be presented to students at the high school level and that mathematics should be

a) relevant to the existing and growing needs of the society
b) related to the ability of the student
c) should serve as a motivator to careers that will interest the student
d) provide the motivation for further inquiry.

The basic principle of the unit shows how an area of mathematics and some fundamental concepts are applicable to real life and modern Technology. Students should be encouraged to seek out areas in which this technology apply and make some comparisons of the technology now and those that existed in years past.

Objectives of the unit

a) To help students acquire a range of mathematical skills
b) To make mathematics relevant to the experiences of the students, there-fore recognizing mathematical principles in his environment.
c) To apply mathematical knowledge to the solution of problems.
d) To present a unit in the area of Boolean Algebra that is important to the understanding of circuits and how they work.

Audience : This unit is designed for classes at the high school level. It can be a part of a unit in geometry, or could be used by a group of students for independent study.

The Specific Objectives of the Unit

1 Introduction to logic:
 The students will be able to:
 a) Distinguish valid statements from those that are not valid.
 b) To define a statement; To use reasoning; make deductions and implications.
 c) To use simple connectives.
 d) To make truth tables.
2. Introduction to Binary Arithmetic:
 a) Define the numerals in the Binary system.
 b) To find the values of a numeral written in the binary system in base ten.
 c) To perform the basic operations in the binary system.
3. Introduction to Boolean Algebra.
 a) Simple operation with boolean algebra
 b) Make truth tables
 c) Application of Boolean Algebra to
  (1) electrical switches ; (using ON and OFF)
  (2) write truth for adders and half adders
  (3) designing simple circuits.

Introduction

There has been much talk in the media about the expansion of the Japanese technology in the market place. Evidence can be seen in almost all aspect of our daily life. In the last ten years there have been the emergence of new appliances and gadgets that most people find too complicated to understand examples are the video recorder; CD players; cars that are more efficient and telephones that are more intelligent these are only a few. It will be the teachers responsibility to awaken the minds of these students to the great demand that will be placed on them to understand and be aware of these changing technologies and develop in the students a pest for knowledge, and the ability to seek new and different ways to solve problems.

The work world of the next ten years will be demanding workers that are equipped with different basic skills; workers with the ability to think and can understand the operations of these machines developed by the new technology.

The unit attempt to show students how the application of Boolean algebra, and the binary system has spearheaded work in these new technologies. After the unit it is hoped that students will reconsider the options available to them and make more careful and informed decisions as to their career choices.

The unit will begin by discussing the implications of Reasoning and deduction in the formal setting, with extensive work in the binary system and then a simple introduction to boolean algebra.

It is hoped that this unit will find a place with those teachers that are theorists, and those that enjoy working with the hands on experiences that are meaningful for students.

Limitations of the unit

The concepts of Boolean algebra are found in algebra texts designed for higher education students; therefore the language and symbolism used are very technical. In an attempt to make it appropriate most of the proofs will be omitted and only those concepts necessary for understanding will be used. Students should be encouraged to draw diagrams and make tables where necessary.

Because of the limitation of space for the unit there will be the need for the users to research additional problems from the reference given.

Reasoning and Deduction **: Introduction to Logic**

The main ingredient in the study of logic is the principles and method used to distinguish between arguments that are valid and those that are not. Logic deals with reasoning and the ability to deduce or come to some reasonable conclusions. In everyday life we guess what is going to happen on the basis of past experiences; “It looks like its going to rain” we say meaning that it may rain today. If we wait around long enough then it may rain. This is an example of inductive reasoning. In mathematics we can discover whether or not a guess is correct by checking if our conclusions can be deduced from results already known. This is called deductive reasoning.

The starting point of logic is a statement. A statement in the technical sense is declarative and is either true or false, but cannot be both simultaneously.

In logic it is irrelevant whether a statement is true or false, the important thing is that it should be definitely one or the other. Logic statements must be either true or false.

A Statement: is a declarative sentence which is either true or false.

Examples of declarative statements:

(a) New Haven is a city in Connecticut.
(b) The month of June has thirty days.
(c) The moon is made of red cheese.
(d) Tomorrow is Saturday.

The following are not statements:

(a) Come to our party!
(b) Is your homework done?
(c) Close the door when you leave.
(d) Good by dear.

Those are not good statements because they cannot be considered true or false.

The basic type of sentence in logic is called a simple statement. A simple statement is one that has only one thought with no connecting word.

Examples of simple statements

(a) Three is a counting number.
(b) Ann is early for class

If we take a simple statement and join them with a connecting word such as and, or, if . . . then, not, if and only if, we form a new sentence called a complex or compound statement.

Compound Statements: are formed from the combination of two or more simple statements.

Example (a) Ann is early for class and she has her note books.

(b) Three is a counting number and is also a odd number.

Types of Compound statements and their connectives

1. A negation: formed when we negate a simple statement by “not”.
   example : Simple statement: Today is Thursday
   Compound statement: negation: today is not Thursday
   The sentence “today is not Thursday” is a compound statement called a negation.
2. When we connect two simple statements using and, the result is a compound statement called a conjunction.
3. If the simple statements are joined by or the resulting compound statement is called a disjunction.
4. The If . . . then connector is used in compound statements called conditionals.
5. The if and only if connector is used to form compound statements called biconditionals.

We are familiar with using letters as replacements in algebra; in logic we can also use letters to replace statements. The common letters used to replace statements are P,Q, R: but any letters can be used.

Examples. P = Today is Saturday

Q = I passed my test

but P and Q would read Today is Saturday and I passed my test. It is also common practice to use symbols for the connective words (or the connectors)

Connectors	Symbols
(a) not	~
(b) and	^
(c) or	(figure available in print form)
(d) if . . . then	Ð>

(e) if and only if Ð>

TRUTH TABLES: Since a statement in logic is either true or false, we should be able to determine the truth or falsity of a given statement. [Logic is very precise. There should be no worry about ambiguity] Let P be a statement; then ~ P means ‘not P’ or the negation of P. The negation of P is true whenever the statement P is false and false if P is true. These situations are confusing to write, therefore we can record these statements in a truth table.

Example 1: Let P = this is a hard course.
~P= this is not a hard course.
Truth Table

p	~p
T	F
F	T

where T = True and
F = false

In the first column, there are two possibilities of P; P is either True or False. Each line in the table represents a case that must be considered. In this case, there are only two cases. The truth table tells us the truth value of p in every case.

Truth Tables with the Connective ^

The Connective ^ may be placed between any two statements P and Q to form the compound statement p^q
Let P = Today is Monday
Q = I have a Math class.

Truth Table

P	Q	P^Q
T	T	T
T	F	F
F	T	F
F	F	F

In the compound statements, the individual statements are called components. In a compound statement with two components such as p ^ q there are four possibilities. These are called logical possibilities. The possibilities are:

1) p is true and q is true
2) p is true and q is false

3) p is false and q is true
4) p is false and q is false.

The four possibilities are covered in the four rows of the truth table. The last column gives values of p ^ q; This is only true when both p and q are true. Using the examples given, truth tables of a more complicated nature can be built.

Let us consider the situation p v q

Example 2:

P	Q	P v Q	P = Today is Tuesday
T	T	T	Q = I have a Math class
F	T	T	P v Q = Today is Tuesday or I have a math class
F	F	T	
F	F	F	

Find the Truth Table of [(p) ^ (q)]

Example 3:

Truth Table

p	q	~p	~q	(~p) ^ ~ q	~ [(~p) ^ (~q)]
T	T	F	F	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

The given [(~p) ^ (~q)] uses parentheses and brackets to indicate the order in which the connectives apply. Expressions can be simplified by removing some of the parentheses, thus (p) ^ (q) can be written as ~ p ^~ q.

It can be noticed from the Truth Table in examples 2 and 3 that the last columns are the same. Thus we say that these statements are logically equivalent and can be written P = Q and P v Q = (~p ^ ~q).

The conditional and the Biconditionals Statements.

If the connectors Ð> is used between any two statements P and Q to form a compound statement P Ð> Q (reads if P then Q), the statement is called a conditional statement.

Let P = You passed English
Q = You will graduate

Truth Table

P	Q	PĐ>Q
T	T	T
T	F	F
F	T	T
F	F	T

The statement P Đ> Q reads if you pass English then you will graduate. This statement is false only when you pass English (true) but you will not graduate. Therefore the final column will be true in every position but the second.

The connective Đ> is called the biconditional and may be placed between any two statements to form a compound statement P Đ> Q (reads P if and only if Q).

The Truth Table For P Đ> Q

P	Q	P Đ> Q	PĐ> Q	QĐ> P	(P (Đ> Q) ^ (Q Đ> P)
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

From the truth table it can be noticed that P Đ> q = (P Đ> Q) ^ (QĐ>P).

Sample Problems for Students:

1. In these problems English sentences are given. In each case determine whether the sentence is a statement or not.
   (a) On March 8, 1922 snow fell in Atlanta.
   (b) Mary has big feet.
   (c) How much did you pay for that car.
   (d) Keep off the grass.
   (e) Five is a prime Number.
2. If you accept the sentences in column 1 what can you say about the statements in column 2

		Column 1	Column 2
(a) The order of 1, 2, 3, 4, 5, 6 on	The relationship between 5 and 2		
		number line	
(b) That a head and a tail are	The number of heads most likely		
		equally likely on the toss	to be obtained in 600 throws
		of a coin	
(c) That a parallelogram can be	The figure ABCD is a parallelogram		

formed two congruent triangles

(d) If all policemen are over six feet tall. Mr brown is a policeman

(e) A person must be 16 years to drive a car Junior is driving a car

3. Make a truth table for the given statements.

(a) ~(~P) (b) ~PĐ> ~ Q (c) ~ PĐ> Q

(d) P ^ QĐ>P (e) ~ pĐ>Q (f) (PĐ>Q) v PĐ>Q

(g) P ^ Q P v Q (g) (PVQ) ^ R

(h) (P ^ Q) v (P ^ Q).

4. Construct truth table for

(a) ~ (p ^ Q) (b) PV~Q (c) ~ (Pv~Q)

SECTION 2

The Binary System

In the decimal system when a number is read from right to left each digit is multiplied by a progressively higher power of 10. These are commonly referred to as ones, tens, hundreds, thousands, and so on. In the binary system the same concept applies except that the number being raised are powers of two, the digits therefore represents ones, twos, fours, eights and so on. In this counting system the only numbers that are used are zeros and ones.

Example of the comparison of Binary and Decimal Systems.

Decimal system Binary System

$10^3\ 10^2\ 10^1\ 10^0$ $2^4\ 2^3\ 2^2\ 2^1\ 2^0$

The arithmetic in the binary system employs the same operations as the decimal system but may be considered simpler. The addition involves grouping things in groups of twos with carrying to the next higher power.

Example (a) 10 + 10

10

+ 10

100

carrying to the next higher power.

Number conversion

Binary to Decimal conversion: In a binary number each position corresponds to a power of two.

Example (a) 110 means $2^2 + 2^1 + 0$

$=4 + 2 + 0$

$= 6$

(b) 101 means $2^2 + 0 + 2^0 (1)$

$=4 + 0 + 1$

$=5$

(c) 11001 means $2^4 + 2^3 + 0^2 + 0^1 + 2^0$

$=16 + 8 + 0 + 0 + 1$

$= 25$

Multiplication is also a straight forward procedure, since each digit is either 0 Or 1; therefore each potential product is either zero or one.

Example 10 x 01

10	10
x01	x 10
10	00
	10 partial products
	100

The rule for multiplication is simply to write down the multiplicand shifted one place to the left for each of the multiplier that is a one the sum the numbers.

In summary then since binary operations uses the same concepts of value and positions of digits as the decimal system, the associated arithmetic is the same.

In addition we add column by column, carrying where necessary to higher positions. In subtraction we subtract column by column, borrowing where necessary from higher positions, and in division we do repeated subtractions just as in long division.

Examples

1. Addition	2. Subtraction
1110	1101
+1011	Đ1010 with borrowing
11001	0011
Multiplication	Division

(figure available in print form)

Division is done by counting how many times one number can be subtracted from the other; the counts is called the quotient, the quotient in is 0110.

Conversion from decimal to binary .

To convert from binary to decimal we sum the values of the position of each of the digits ; but to convert a decimal number to its binary equivalent progressively divide the decimal number by two and record the remainders. The remainders written in the reverse order forms the binary equivalent.

Example	Change 18 to its binary equivalent

(figure available in printed form)

The importance of the binary system to technology is the convenience of the system to facilitate expressions in symbolic logic. In 1938 Claude Shannon pointed out that switching circuits could be used to evaluate logic statements. He used the dualities of on and off, high and low, and represent these by zero and ones. As a result logic systems like computers are designed to use the binary system.

Section 3

The Boolean algebra provides rigorous procedures for deciding whether a statement is true or false;if the statement can be expressed in two variables. In Boolean algebra true is represented by a 1 and false by a 0. With these two digits (0,1) and the three basic operations called “not”, “and” and “or” digital algebra or switching algebra was developed.

The basic operations and their meaning:

Operations	Meanings	Symbols
or	Determine a single input bit from the values of two or more input.	+ A+B
and	Determines a single input bit from the value of two or more input	. A.B or AB
not	Changes binary bits to its opposite value.	not A; bar over A

Any relationship between logical variables are called logical expressions. These expressions can be written as an equation for example the equation A + B + C = F where F is the name of the output variable. The expression A + B + C = F expresses the action of and/or function. Through Boolean Algebra logical analysis can be performed using these three functions.

The electronic representation of these functions are called logic gates. There are the and gate the not and the or gates. These logic gates are basic functional units for both arithmetic and logic operations; to operate they must accept binary numbers, and should have a carry bit of one or 0, (from the adjacent lower power of two), and should produce as outputs a sum bit and a carry bit for the next higher power of two.

How to design circuits **:**

The first step in the design of a circuit is to establish a truth table that shows the output for all possible inputs.

Truth Tables

(a) A A (b) A B A.B (c) A B A + B

0	1	0	0	0	0	0	0	
1	0	0	1	0	0	1	1	
not		1	0	0	1	0	1	
		1	1	1	1	1	1	
			and			or		

With proper input electronic digital circuits (logic circuits) establish logical manipulate paths. By passing binary signals through various combination of logic circuits, any desired information for computing can be operated on; each signal represents a binary carrying one “bit” of information.

The logic circuits or gates perform the logical operations.

Operations and their Gates:

Examples

1 And gate

(figure available in print form)
(figure available in print form)

Two and three input And gates.
2 Or gate

(figure available in print form)
(figure available in print form)

Two and three input Or gate.
3 Not (the not gate is sometimes called an inverter)

(figure available in print form)
not gate

Some Booleen functions have identical truth tables therefore their logic circuits serves identical purposes; but one may be preferable to the other. To do this more useful logic gates are created. The following gates NAND, and NOR were created for this purpose.

Examples of Truth Tables for Nand and Nor

Nand equation (figure available in print form).

A B F Nand GATE

0 0 1 A

1 0 1 B

0 1 1

1 1 0

(figure available in print form)

Nor equation F = A + B Nor gate

A B F

0 0 1 A

1 0 0 B

0 1 0

1 1 0

(figure available in print form)

Application To Digital Computer Circuits

Computer performs among other things, all kinds of arithmetic operations. The most basic operation is the addition of two binary digits, which consists of 0 + 0 = 0, 0 + 1 = 1, 1 + 1 = 10 and 1 + 0 = 1. The first three operations had sum that is a single digit, but 1 + 1 has a sum of two digits. the higher bit of this sum is called a carry. In adding two multiple digits numbers a carry is to be added to the next higher digit. The circuit that performs the addition of two bits is called a half adder. The circuit that performs the addition of three bits is called a full adder. A half adder needs two binary inputs (A and B) and two binary out puts (S = sum; C= carry).

Truth Table for Half adder.

A B C S

0 0 0 0

0 1 0 1

1 0 0 1

1 1 1 0

The fourth row shows 1 + 1 = 10, here 1 is the carry to the next higher power of two.

S = A + B

C = AB

(figure available in print form)

In a full adder two of the input variable are shown by A and B, the third letter Z represents the carry from the previous lower position. The two outputs is denoted by S and C (S = sum , C = carry)

Truth Table for Full Adder.

A B Z C S

0 0 0 0 0

0 0 1 0 1

0 1 0 0 1

0 1 1 1 0 (figure available in print form)
1 0 0 0 1 (figure available in print form)
1 1 0 1 0

1 1 1 1 1

Implementation of Boolean functions : The translation of the Boolean function to logic circuits is called the implementation. The is the mathematical expression representing a combination of gates. Remember that a basic logic gate performs a single elementary logic operation and their input-output can be expressed as a logic expression.

These logic expression can be represented by a diagram.

GATES, LOGIC EXPRESSIONS AND THEIR DIAGRAMS

AND GATES

(figure available in print form)
(figure available in print form)
OR GATES
(figure available in print form)
(figure available in print form)
NAND NOR
(figure available in print form)
(figure available in print form)
From the previous pages it can be seen that Boolean algebra facilitates the movement from a statement of the function, to truth table, then to a logical expression; these expressions can then be implemented in the form of a diagram using the numbers zero and one (1,0).
(figure available in print form)

Inputs	outputs
A B	F
0 0	0
0 1	1
1 0	1
1 1	1

OR GATE

The truth table explains the result F from the possible values of A and B

AND GATES

(figure available in print form)

Inputs	Outputs
A B C	F
0 0 0	0
1 0 0	1
0 1 1	1
1 1 1	1
0 0 1	1
1 1 0	1
1 0 1	1
0 1 1	1
1 1 1	1

If a small circle is placed at an input or output of a symbol for a logic gate it indicates negation.

1. If the small circle is placed at an input terminal, if the symbol entering is one the symbol leaving the circle and entering the block is 0.
2. If the circle is placed at the out block
   (a) if the symbol leaving the block (and entering the circle) is 0, the symbol leaving the circle is 1.

Truth Table

Input	Output
A B C	F
1 0 0	1
0 0 0	1
1 1 0	1
1 0 1	1
0 1 0	1
0 0 1	1
1 1 1	0
0 1 1	1

(figure available in print form)

Boolean Algebra applied to Electrical problems **.**

This idea can be applied to voltages that are present in a physical circuit. There are high voltages and low voltages. High voltage signifies that current is flowing, low voltage signifies that there is no current. These situations must be given some mathematical significance.

let high voltage	= 1 or	let high = 0	
low voltage	= 0		low = 1

from these we can write truth tables to show the desired operations (or, and).

(figure available in print form)

Input	Output	
A	B	F
low	low	low
low	high	high
high	low	high
high	high	high

or operation

Input	Output	input	output	
A	B	F	A	B F
0	0	0	1	1 1
0	1	1	1	0 0
1	0	1	0	1 0
1	1	1	0	0 0

In conclusion if the dualities of high and low, on and off are given values from the elements of the set of boolean algebra many physical electrical problems can be solved.

Lesson plans 1

Topic: Reasoning and deduction

Objective The student should be able to

Make deductions and draw the necessary conclusions from a given statement.

Development 11,

(a) Discuss real life situations where the outcomes can be predicted on the basis of past experience.

Encourage students to examples of their own experiences

(b) Introduce familiar examples for discussion

Examples It looks like its going to rain.

(c) Discuss reasons that make us believe that it is going to rain.

(d) list these conditions on C.B.

After several examples introduce the word reasoning as used to describe the thought process used.

2. Draw a rectangle on the C.B, cut out a congruent rectangle on cardboard. Fit the cut out rectangle on the one on the C.B in different ways. Have students give the relationship that exist between the opposite sides and opposite angles. Use this as an example to introduce deductive reasoning. Summarize that because the rectangle can fit its outline in four different ways we deduce that opposite sides and angles are congruent.

3. Introduce the idea of a situation that is universally accepted. From these draw conclusions or make deductions.

Introduce the word premise. This word describes situations that are taken for granted, from these premises conclusions can be drawn.

4. Introduce sentences that contain the words such as “if”, “because”, “therefore”, “consequently”, “it follows that”, and “it may be deduced from, it is evident that”; these statements are examples of deductive arguments.

5. Provide sentences for discussion.

example (a) It is accepted that

When it rains the sky is dark. What can we deduce? example (b) Every English. teacher has a good knowledge of English

Mr Brown has a good knowledge of English What deductions can be made. Discuss are all people with good knowledge of English, English teachers.

Evaluation (a) Have students make up statements and discuss the deductions that can be made.

(b) Use accepted premises and draw conclusions.

Lesson Plan 2.

Topic: Logic Statements.

Objectives

(a) students will be able to tell sentences that are statements.

(b) Use connectives to form compound statements

Development

(a) List various different sentences on C. B. have students decide whether each sentence is true or false. (Does the sentences convey some specific idea).

Examples

(1) It is blue.

(2) Is your homework finish yet?

(3) Three is a prime number.

(4) February has 31 days.

(5) George Bush is president.

1. Discuss each sentence indicate that those sentences that cannot be said to be true or false. Define a statement as a sentence that has one idea that can be classified as either true or false.
2. Discuss the criteria of a simple statement. List various examples of these on C.B. Have students identify the one idea of these statements.
3. Introduce the vocabulary “connective”. From simple statement make up compound statements. Give these their names as they are made up.

Example:

Today is sunny and I will take a walk. 28

I will go dancing or I will go riding.
I will go riding if you will come with me. Introduce idea from algebra that letters can be used to replace numbers. Here letters will be used to replace statements.
Let Today is Monday = P
I will go dancing = Q

Make up mathematical sentences

P and Q [use this idea with different connectives] Introduce the symbols for each connectives. Thus P and Q can be written as P ^ Q.

Evaluation: Have student make up their own simple and compound sentences.

Give statements and have students write them using the symbols.

Lesson Plan 3

Topic: Constructing Truth Tables

Objectives The students will be able to construct truth tables representing different compound statements

Development

(a) Review the different types of connectives that can be used to generate compound statements.
(b) Choose the and connective (^) and make a compound statement
Example
P = today is Tuesday
Q = I have a math class
Today is Tuesday and I have a math class.
(c) Examine the possibilities that can result from this statement
(d) Construct truth table on C.B.
(e) Stress the use of the vocabulary
(i) Component
(ii) Logical possibilities
(iii) Conjunction of P and Q
Example the results of the truth table.

Leading questions

(a) In what case is P ^ Q true?

(b) When is the statement false?

(c) How many possibilities will there be?

(f) Choose different statements and use different connectors to form compound statements: Use similar reasoning to form truth tables

Emphasis on “and”, “or” and “not”.

Evaluation

(a) Have students generate their own statements and form truth tables.

(b) Give simple examples for students to complete.

Problems

(a) (P). (b) P ^ Q. (c) P ^ Q (d) (P ^ Q)

Lesson Plan 4

Topic: Binary Numbers.

Objectives

(a) The student will be able to read a base two number

(b) To convert base two to base ten

Development

1. Introduce binary numbers by having students group a number of articles (match sticks, small cubes, pebbles, etc) in groups of twos.
2. Have students re-count the objects but now using only the digits zero and one {0,1}.
3. Introduce the place value chart for binary numbers.
4. Practice changing from base two to base ten by writing the base two digits on the place value chart.
5. Provide many problems for drill and practice.
6. Use re grouping to change base lien numbers to base two.
7. Introduce the method discussed in the content.

Evaluation

(a) Have students count in base two.
(b) Use their place value chart to do conversion
(c) Use re-grouping to change from base ten to base two.

Materials to be Used

1. Make a Binary Counter
   Materials needed: board for sides, wire, cards with zero and ones. A wire runs from end to

end and passes through a number of cards. Each card has a zero or on one side and a one on the other [make as many positions as needed]. By turning them around the cards can be made to indicate a given binary number. This can be used to convert from binary to decimal.
2. Windows can be made in a rectangular piece of card board which is glued to a similar beck board. Slits are made for the tabs to slide in.
The tabs would show either on or off. 1 = on, 0 = off. This can be used to show a binary number.

Problem Solving

(a) A cross word puzzle.

Across	Down
1. 10001 Đ1101	1. 100011 Đ101
10. 1+ 111 +1111	10. 101 X 110
101. a prime number	11. the solution of the equation X+11=1010

Lesson Plan 5

Topic: Boolean Algebra To Building circuits.

Objectives

(a) The students will be able to apply simple ideas from Boolean algebra and write truth tables.
(b) From the truth tables the students will able to implement logic circuits.

Development

(a) Review simple statements and their connectives.
(b) Introduce the basic principles of boolean algebra.
The Elements (0,1)
the operations of addition, multiplication and negation
The elements are used in the truth tables instead of true or false
(c) Discuss dualities that can be considered true or false, on or off, closed or opened
(d) Replace the operations (+, x) by or and and

(e) Review truth tables from the previous section with zero and one

(f) Introduce equations from these truth tables. ex. $F = x + y$. (where x and y are inputs and F the output

Evaluation

(a) Provide drill and practice using zero and one in the truth tables

Application of Boolean Algebra

1. The state of an electrical switch is either on or off. Use zero or one to represent these states.
2. Develop truth tables using the connectives.
3. Discuss the position of switches when opened or closed.

(figure available in print form)

4. Use equations to summarize the desired outcomes. Thus introducing the implementation of boolean algebra with logic gates.

Write equations of desired outcome

ex. $F = X + y$ [use all connectors].

Write truth table table

Draw logic gates. (e) And gates denoted by:

(figure available in print form)

OR gates denoted by:

(figure available in print form)

Suggested problems

1. Write a truth table for the following;

   (a) Nand F = (Xy).

   (b) And F = (X + y)

   (c) F=(X+y)

2. Construct a truth table for the gate shown.

(figure available in print form)

3. Design a truth table for a half subtractor

ans. X y b d

0 0 0 0

0 1 1 1

1 0 0 1

1 1 0 0

4. Design a truth table for a full subtractor.

ans X y z b d

0 0 0 0 0

0 0 1 1 1

0 1 0 1 1

0 1 1 1 0

1 0 0 0 1

1 0 1 0 0

1 1 0 0 0

1 1 1 1 1

4. Construct a truth table for

(figure available in print form)

5. Make a truth table to show that
   CD=C+D
6. Given the following write an equation for
   (a) X (b) Y (c) F
   ans X = AB
   Y=(AC)
   F=X+Y+B

7. Write truth tables for F = AB + AC

   draw the logic gate

8. Design a circuit such that a hall light can be controlled by both an upstairs and a down stairs switch.
9. Design a circuit such that a light can be controlled by each of three switches.
10. What is the Boolean expression for the AND OR logic diagram

(figure available in print form)

   ans; AB + AC = F
11. What is the truth table for the design in 10
   ans; input output input output

A	B	C	F	A	B	C	F
0	0	0	0	1	0	0	0
0	0	1	0	1	0	1	1

0 1 0 1 1 1 0 0
0 1 1 1 1 1 1 1

REFERENCES:

1. Combellack,Wilfred J., *Introduction to Elementary Functions* , John Wilbey and Sons, Inc; Publishers .

2. Lin, Shun Yeng, and You Fen Lin, *Set Theory with Applications* . Book Publishers Inc; Tampa Florida.

3. Oldham, William G., and Schwartz, Steven E. *An Introduction to Electronics* . Holt Reinhart Winston.

4. Smith, Ralph J. *Electronics Circuits and Devices.* Second edition Wilby.