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Global Change for The Basic Mathematics Student

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Outline

This paper will be presented to a group of basic geometry students. Basic geometry is a level III mathematics course. Students who have not done well in mathematics are found here. Few of these students will ever take an Algebra II, College Chemistry or physics course. We try to get the students involved in the work by doing a lot of "hands on" activities, making sketches, charts, graphs, cut outs and constructions. While the custodians complain a lot, we do get a lot of ideas across.

Early on I had thought about collecting a series of books, TV tapes and articles from various papers and magazines on the Earth, Sun and planets. I plan to use these items as my classroom library on global changes, the "greenhouse effect", pollution and other problems which lend themselves to a mathematical examination. Its important to show the students that the mathematics which we learn in the classroom can be applied to things in the outside world. The idea we hope to implant is that our mathematics, once applied will help us understand the events of the past and present. With this understanding we can prepare for the events of tomorrow.

I often hear other teachers ask “where did you get all of these materials”? “Did you have to buy them?”. I am here to say there are a lot of useful items in the newspaper, magazines and in some strange places. Years ago the New England Telephone Company put out its telephone directory and in it was a very nice series of “space facts” which were sprinkled through out the book. One of my students, Elsie Rodriquez, got me a copy of the original print out, and it has occupied a prominent place in my space files.

While we are on the topic of where we can find materials to spice up the classroom I must mention a great source. Local libraries toss out books and magazines, which have not been circulated in the past year or so. Toss out may be a harsh description, many times the books are sold for twenty five cents. At my side are such beauties as Samuel Carter *The Gulf Stream Story* easy to read and filled with amazing facts written for kids. Others are Irene Kiefer’s *Global Jigsaw Puzzle* (a seventy page gem) *Star Flight And Other Improbabilities* by Ben Bova a great little text and finally our friend Isaac Asimov’s *The Collapsing Universe* . These books and others make up my room’s library which is open to all. I suggest we assign a city wide “science summer reading list”. Our students should show up in September prepared to study science and math.

In The Beginning

How do we describe and discuss an event which took place 15 billion years ago? We have to speculate, guess or look at today and work our way back in time from what we have observed. In 1927 Georges Lemaitre, a Belgian astronomer suggested that in the “beginning” there was a “cosmic egg” which exploded, sending matter hurtling across the heavens at incredible speeds. The explosion continues to this day. Our Sun is part of but one galaxy which in itself contains about 100 billion stars, which we here on Earth refer to as the Milky Way. A dull glow in the heavens is all that remains of the original flash.

A swirling mix which was to eventually make up our solar disk laid out in space, gathering unto itself. I have a mental image of a large fried egg spinning in space. It’s estimated that this collection of matter took 100,000 years to cool and collect. Within the hydrogen sphere which lay in the center, great pressures were created and helium was formed. And tremendous energy was sent forth and finally we had ignition. The resulting energy release continues to fill our corner of the heavens with light and warmth. The planets formed in a disk around the now-lit Sun.

Figure available in printed form

In the April 1988 issue of Scientific American under a column titled “Cosmic Forgery” its author made the following comment “Supernova 1987A provided spectacular proof that exploding stars forge light elements into heavy elements and fling them through space”. The article went on to suggest that nucleosyntheses could take place in the accretion disks of black holes. Its estimated that temperatures rise to a billion degrees as the hole gathers mass and density. Intense radiation builds forcing matter out from the disk in a path perpendicular to the disk. Thus explaining the vast jets of energy and particles which are seen pouring out of some galaxies and binary star systems.

A Look At The Earth's Atmosphere

Our Earth's atmosphere is primarily made up of three gases nitrogen (N_2) 78%, oxygen (O_2) 21% and argon (Ar) .93%. Water vapor is the next most plentiful followed by trace amounts of neon, helium, methane, carbon monoxide, sulfur dioxide, ozone and carbon dioxide which logs in at. 03%

This is the same carbon dioxide which bears the name of the "green house gas" (although water vapor is the most important greenhouse) and which is so important in our photosynthesis cycle. As most will recall from their high school days the definition of photosynthesis is the process by which a plant converts CO_2 and H_2O into sugars, starches and O_2. 6 CO_2 + 6 H_2O + Sun light Þ C6Hi2O6 + 6O_2. Animals and man breathe in air for its oxygen content and expel carbon dioxide as part of their exhaling, this couples with other (CO_2) gases which are the bi-product of plant and animal decay. At this point plants absorb this carbon dioxide use it, and then expel their bi-product which turns out to be partially oxygen. And so the cycle goes on. A continuation of this discussion will take place later in this paper in the section on how man can effect some changes on this green house effect.

In our atmosphere we also find small particles of dust, sand, hydrocarbon carbons (from the burning of fossil fuels) as well as amounts of sulfuric acid which spew forth from our ever present and always erupting volcanoes, marine organisims and fossil burning generators.

Three of the most important properties which describe our atmosphere are its pressure, humidity and temperature. Our atmospheric pressure is a mathematical description of the pressure which a column of air exerts on a square unit of area. This column rises from sea level up through to the top of our atmosphere which is called the thermoshpere. While there are several ways to demonstrate the effect of such pressure I think the most graphic way is to collect the following items. A three foot long glass rod, which has been sealed at one end, a dish or a soup bowl, a stand and finally a quantity of mercury. Next we take and partially fill the bowl with mercury and we fill our glass tube with the same. Taking care not to spill any mercury we place a finger over the open end of the glass tube, invert it and place it up right in the bowl, secure it to its stand.

At this point we observe that the mercury has dropped down a bit in its tube, but it has not left the tube. In the top of the tube we have formed a vacuum. The height of the tube can be measured using a metric ruler and hopefully the reading will be close to 760 mm. What keeps the mercury up in the tube? Pressure from the atmosphere should be the reply.

Figures available in print form

Why did we not use water in this experiment? By a simple conversion, based on the densities of the two liquids, we can show that if we had used water we would have needed a 34 foot long tube, this is most impractical for classroom work. As an application of the law of atmospheric pressure I should point out that those of us who rely on wells (shallow) can only draw water from 29 to 30 feet below the surface of the earth. These pumps create a vacuum and the atmosphere "pushes" the water up the pipe. These pumps can create a partial vacuum but they will never be able to "pull" 34 feet of water.

While still on the topic of air pressure it would be interesting to look at a graphical representation of air pressure which is listed as 1000 millibars for the height at sea level. As we examine the greater heights the pressure drops off as seen in the graph below.

Figure available in printed form

Humidity has been described as the amount of water vapor which a given volume of air has in it. Air's ability to hold water vapor depends to a great degree on air temperature, warm air can hold more water vapor than an equal volume of cold air. When we use the word humidity we are giving a percentage which tells us how much water vapor a particular volume of air has in it, also it says just how much more it could hold.

If we had remained planet bound we would have the following impressions. Its warmer at ground level and much colder on a mountain top. These impressions are still valid if we stay on the ground (troposphere). Today we have a wide range of tools which help us "read" our atmosphere. These tools are weather balloons and rockets which probe the outer limits and beyond and send back data. A small part of these data will be seen in the next graph which describes middle latitude atmospheric temperatures. The data found in these tables comes from Goody and Walker's *Atmospheres* (pages 44Ð46).

Figure available in print form

At the troposhpere we find the temperature at 288 kelvin (15°C) which we would have expected but in the far reaches we find that the temperature rises to 1000°K. The molecules at this height are few so the warming effect on a surface is imperceptible. This great temperature is caused by the thermosphere's absorption of solar radiation which bombards the Earth in the form of extreme ultraviolet wave lengths.

In our work it would be good to stop and look at how the positioning of the planets in the solar disk affects their reception of solar energy. If we take a look at some simple ideas we can get a better understanding of that which is to follow. Take and compare the surface area of two spheres. The first has a radius of one and the second a radius of 2. To compare them through a ratio we will get 4X pi X r^2/4X pi X r^2. We see that both contain 4 and pi which will go out as a unity factor, the real thing we must look at are the radii. So we write $1^2/2^2$ = 1/4. This tells us that the area of the second sphere is four time larger, which implies that each unit of its cross section gets but one fourth of the radiation which the first had received. We can look at the next sketch and see how this looks in the form of a picture.

Figure available in print form

Our thoughts now turn to the Earth and the planets which are circling the Sun. How do they compare in amounts of radiation received? Take Earth's distance as one and compare the others to it we would get the following table.

Planet Kilometers From Sun Compared to Earth Decimal

(10^6 km) Equivalent

Mercury 58 $150^2/58^2$ 6.680

Venus 108 $150^2/108^2$ 1.9290123

Earth 150 $150^2/150^2$ 1.0000000

Mars 228 $150^2/228^2$ 0.4328254

Jupiter 778 $150^2/778^2$ 0.0371726

Saturn 1430 $150^2/1430^2$ 0.0110029

Uranus 2870 $150^2/2870^2$ 0.00273161

Neptune 4500 $150^2/4500^2$ 0.00111111

Pluto 5900 $150^2/5900^2$ 0.00064636

A table such as this tells us a lot at a glance. Care to move to Mercury? Their solar radiation is almost 7 times Earth's radiation per square mile. At the other end of the spectrum we find Pluto. Pluto's temperature of -230°C, is connected to its .00064636 on the chart above.

We all have experienced the Sun's radiation in one form or another. At this point it would be interesting to look at the electromagnetic waves and learn something about them. The energy which is generated in the Sun's core needs anywhere from one to ten million years to pass through to the Sun's surface. The next part of the trip runs a little more than 8 minutes.

We take 9.3×10^7 miles (Earth's distance from the Sun) and divide by 1.86×10^5 miles/sec (which is the speed of light) and we get 500 (seconds). This 500 seconds divided by 60 seconds/min. will give us 8 minutes and 20 seconds, this is a very short sprint to Earth.

This flow of radiant energy takes many forms. To distinguish one form from another we have but to study their wavelengths and compare them to a rule or chart. A wavelength is the distance between successive wave crests, the sine waves which we did in high school are a good way to think of this. Mentally picture such a symmetrical wave pattern and measure the distance from the top of one wave to the top of the next and you will have a wave length. Each form of radiation has it peculiar wavelength. The following graph gives typical wavelengths in meters.

Figure available in print form

A major portion of our Sun's radiant energy is seen by us as white light, which when passed through a prism spreads to form a rainbow. All of this is related to a law known as Wien's Law which states that there is a relationship between a body's temperature and the peak wavelength which it emits. The rule is as follows:

Peak Wavelength (m) = 2.88×10^{-3}

Temperature °C + 273

Wien's Law applies to ''black body" objects which absorb all of the energy (radiation) which falls on them and then in turn emits all of the radiation which it should give off at that temperature. It is a law which describes ideal or laboratory conditions but can be used in the class room. Wien's Law has some interesting applications. If we already know that the temperature of the surface of the Sun is about 5700° C. Then we can plug this into the formula and find the wave length of this light.

Peak Wavelength (m)= $2.88 \times 10^{-3}/(5700° + 273) = 4.821697639 \times 10^{-7}$

When we refer back to our spectrum of electromagnetic chart we see that this reading falls between in the visible light range or between the ultraviolet and the infra-red sections. This law can be used to show that our own body temperature falls not in the visible range but rather in the infra-red range of radiation. When we take this law and solve for temperature we get new problems like the following. If we observe the flame on a propane torch and see that it gives off a blue-violet light. We will be able to match that color to the chart and come up with its approximate wave length, which we then plug into the formula to get its temperature.

T(C°) = 2.88 X 10^{-3} x -273

Peak wavelength (m)

T(C°) = 2.88 X 10^{-3} -273 =6927°C

4 X 10^{-7}

Bodies emit many forms of heat and therefore more than one wavelength. We can’t see infra-red in such an experiment but we know its there.

Next our thoughts turn to the examination of the effective temperature of planets. As we follow our path around the Sun we present a side or a face to it (Sun). From a distance the Earth looks like a disk in space, as do all of the other planets. We have some dark areas which readily receive the Sun’s radiation and we have some large areas which reflect incoming radiation (ice caps, desserts, clouds). Thus we only absorb about (1 - .33) or .67% of the incoming radiation. The reflected percentage is referred to as albedo. The albedo rating is different for each planet.

When our planet is seen from the Sun its disk area may be computed using the formula, Area = pi X r^2 (1- albedo). To determine just how much Sun generated energy is hitting the planet we must multiply all of this by our planets solar flux. This flux rating is 1.4 x 10^6 erg. $cm.^{-2}$ sec^{-1} for Earth. This spells out as Energy absorbed (Earth)= 3.14159 x 6371^2 x 10^{10} x 1.4 x 10^6 x .67. Over the year a planet’s energy balance is in steady state, that is the amount of energy (solar flux) which a planet receives is equal to the amount it radiates back out to space.

According to two scientists (Stefan and Boltzmann) who worked on this problem, the outgoing radiation from a surface of unit area in unit time is proportional to the fourth power of the temperature. They computed a constant which is referred to as the Stefan-Boltzmann Constant (s)and it equals 5.67 x 10^{-5} cm^{-2} x sec^{-1}. In this work the planet is treated as a sphere which is emitting radiation from its total surface, so the formula we use to compute the energy which is being radiated to space is. Energy radiated = 4 x pi x r^2 x 5.67 x 10^{-5} x erg x cm^{-2} x T^4. Equate the two formulas and we get following:

Temperature = ^{4} Solar Flux X (1- albedo)

4 x s

Temperature = ^{4} 1.4 X 10^6 X erg X (.67) X 10^5 X cm^2 X deg^4 X sec

4 X 5.67 X cm^2 X sec X erg

Temperature = ^{4} 4135802469deg ^{4} = 253.59455 deg °K

This is the Te of the Earth. (Atmospheres pages 46Ð48)

Aristarchus And Distances In The Solar System

To get a better understanding of the Earth, Moon and Sun spatial relations, the Greeks applied their geometry to this investigation. Aristarchus of Samos (310-230 B.C.) was one of many who were involved in this, and its at his efforts we will now look.

Aristarchus believed that the Earth and all of the planets revolved around the “central fire” (Sun), He thought that the paths which the planets followed were circular. These ideas predate by 1500 years the calculations of Copernicus (1540 A.D.). Aristarchus had a plan. Here I will directly quote Carl B. Boyer who wrote *A History Of Mathematics* “Aristarchus made the observation that when the moon is just half-full, the angle is less than a right angle by one-thirtieth of a quadrant. This would mean in the language of today that the ratio of the distance of the moon to the Sun (the ratio ME to SE) is Sin 3 degrees. From this he derived the conclusion that 1/20 sin 3 degrees 1/18.” What this basically said is that the Sun distance is at least 18 times the distance from the Earth to the Moon but less than 20 times this distance. The diagram, which follows, will show a geometric representation of this idea.

Figure available in printed form

Aristarchus’ “results‘’ were more accurate than the ideas of the day which had put the distance at no less than 9 no more than 12 these distances. His geometry was good but his observations were bad. The angle he thought was 87° should have been 89.5°. Artistarchus did manage to lay the ground work for future observations. He work was designed to measure the relative distances between the Sun, Earth and its moon. Some of today’s techniques mimic these early attempts.

In our attempt to monitor and measure global change we use modern tools. We now have in place a project called LARGOS (laser geophysical satellite). This is a 900 pound satellite with a 2 foot diameter. It has been fitted with 426 corner reflectors. This satellite now circles the globe in a geosynchronous orbit. Space laboratories from all over the globe shoot laser beams at it. These laser “shots‘’ are then timed and measured and plotted and published. When this data is examined we should be able to tell if any movement of continents has taken place.

Figure available in printed form

In another similar way we have targeted a quasar (as our deep space “largos”), which for all practical purposes is a fixed point in space. By taking readings off of it from many points on earth it is hoped that we will be able to detect minute changes which take place between continents. As a point of interest the Moon is moving away from the Earth at the rate of 4 centimeters a year.

Figure available in printed form

There are other attempts to measure great distance which should be immeasurable because of our small base line (the diameter of our planet). Modern man has overcome this detail with a little bit of cleaver thinking. By making observations at the extreme ends of our elliptical orbit we are able to extend our base line and make more accurate readings on distance objects in space.

Figure available in printed form

Archimedes’ Approximation Of Pi

A detailed description of the derivation of pi using Archimedes’ method would not fit into the scope of this paper. It’s an important idea but it would be getting too far from the main topic, that of looking at global change through high school basic geometry.

Early mathematicians had a real problem when it came to measuring things which were circular. The tools with which they worked were linear by nature. For most this was a serious stumbling block. About 300 to 200 B.C. while examining the basic parts of the circle, a circle’s diameter and its circumference, Archimedes thought that there must be some sort of a constant which bound them. He experimented with the idea that maybe if you divided the circumference of a circle by its diameter you would find a constant value. Today we call this constant pi (3. 14159....).

As in real life, if you have difficulty trying to do a particular thing, do you stop and give up? No! You draw on your past experiences.! Archimedes knew how to measure the perimeter of a regular polygon and he knew that when you increase the sides of a regular polygon from 3 to infinity, it goes from being an equilateral triangle to a circle. Archimedes now had the key to his problem.

His approach was simple. He would take a circle and do two things to it. First he constructed a regular polygon of 96 sides around and tangent to it. This polygon’s perimeter would be a bit larger than the circle. Next he constructed another regular 96 sided polygon tangent to but inside of this same circle. True this polygon’s perimeter would a little less than the circle’s perimeter, but in the words of Archimedes “not to worry”. All three figures enjoyed the same “diameter”. Dover books does a fine job detailing Archimede’s calculation of pi, read it.

When you figure the ratio of the major polygon perimeter to its diameter and then the ratio of the minor polygon perimeter to it diameter it would follow that the circle’s ratio must be some where in between. And so it was. Pi as this new number was called showed up as being between 3.14103 > 3.141>3.14271. Today we tend to use pi as 3.14 as our classroom standard, unless we have a calculator which will give us 3.141592654.

Figure available in printed form

In observing a child at play in the dirt or in the sand at a beach we notice that he or she will make constant swirls and circles. This act has been going on for thousands and thousands of years and has lead to some basic questions from inquisitive students. What is the measure of the distance across the circle (diameter)? How do I find the measure of the circumference? And lastly how do we determine the number of square units this circle contains (area)?

The first question can be answered by most school children with a hands on demonstration. First a circle must be drawn. Now pick a random point on the circumference and anchor a string to it at that point. Next keeping the string taught swing an arc over the circle and observe how the points on the circumference “move along” the string to a maximum length and then begin to recede. The diameter's end will be at the maximum length.

Figure available in printed form

Eratosthenes And The Size Of The Earth

In our look at the beginnings of our solar system we see just how involved we are in circles and things which are circular (I have left out ellipses for a good reason) We can turn on the T.V. and view the Earth from the view point of the astronauts or we can take a trip to an IMAX theater (Maritime Center in Norwalk, Connecticut), and see the Earth in all of its beauty and magnitude. The ancients had no such tools (neither space ships nor satellites). How then did they come to know the planet's dimensions.

There was a man who was in the right place at the right time. His name was Eratosthenes (276-194 B.C.) he was an excellent mathematician and he was in charge of the library, in Alexandria, Egypt. At that time this library was the best in the world. At his finger tips were the works of the great minds of the world to that day.

One of the great problems of the day was how large is the Earth, and how do we set about to measure it? Several men were working on this problem. The results of their labors had been published. Eratosthenes was not satisfied with their "results" so he set about to set the record straight. His experiment was a simple one based on a good understanding of basic geometric principles. Erastosthenes' approached the problem this way.

In Egypt there were two well known cities which lay pretty much on the same meridian (imaginary line of longitude) they were Alexandria and Syene (today known as Aswan). The road between these two cities had been well traveled and well measured. These cities were 5000 stadia apart. It was also common knowledge that at certain day at noon, in Syene, when the sun was at its solstice (directly over head), a very deep well was filled with the reflection of the sun. At the same time the Sun was overhead at Syene. Measurements were taken on a gnomon (a column erected perpendicular to the earth) in Alexandria. The angle which the Sun made with the column as well as the lengths of the column and its shadow were recorded.

The Sun's rays were assumed to be traveling in parallel lines so all he had to have was an accurate measure of the angle which was formed on the gnomon at Alexandria. It is here that we must speculate. One thought is that some primitive trigonometry was in play (remember if he was the librarian he was privy to all the new ideas of the day) and he could have quickly computed the angle of depression, and its complement which the Sun made on the pole. Sine Alpha = opposite side/hypotenuse would have done the trick. Or the angle could have been measured with an accurate protractor used in combination with a plumb bob. This is not as far fetched as it first seems. I recall reading about the accuracy of Tycho Brahe's (1546Ð1601) quadrant (half a protractor with sights) which had a radius of 19 feet, not an item to be carried in one's pocket. On this scale one degree was 4 inches on the circumference. One millimeter corresponded to 1/100 of a degree. While a good many years separated these men, the idea of constructing large model was an "in thing".

I suggest that Eratosthenes could have made an accurate large scale drawing on paper and taken an accurate reading from it. I like to tell my classes a story about the great astronomer, Johannes Kepler (1571Ð1630). In Johannes's day the scientific community had assumed that the Earth, and all of the other planets, followed circular paths in their flight around the Sun. This was a problem for the data which had been collected did not support this theory. Kepler spent years working on this incongruity and got no where with the problem until one day he decided to plot the points (given by the data) onto a floor, and it was not until he stepped back to view his work that he saw that the Earth was traveling in an elliptical orbit around the Sun. And the Sun was located at one of the focal points of this ellipse. An accurately drawn picture and great perseverance gave Johannes the incentive he needed to continue. Today every physics book contains this three laws on the planets.

In the “accurately” drawn sketch which follows we see the Earth as it is struck by parallel light rays from the Sun. The well at Syene and the vertical pole and its shadow at Alexandria are shown, along with the angles (alternate interior) which are located at the Earth’s center and at the pole. It should be noted that this angle has been listed as both 7 and 1/2 degrees and as 7 and 1/5 degrees. In my calculations I have chosen to use the latter. A stadia is listed as 516.73 feet which I will round off to 517 feet.

When we set up the following proportion we will get:

Total number of degrees in a circle = Circumference of Earth Angle of shadow of rod at Alexandria Distance between Alexandria and

Syene

360° = X(Circumference of Earth)

7.2° 5000 stadia

250,000 stadia = X

250,000 stadia x 517 feet/stadia = 12950000 feet

12950000 feet/ 5280 feet/mile= 24,479.16667 miles

By anyone’s standards this is a great calculation. The sketch which follows is the geometric representation of his work.

Figure available in printed form

Mathematical And Visual Patterns

Our students should look for patterns in mathematics and in science. With this thought in mind we will look briefly at the work of German astronomer Johann Elert Bode (1747-1826). Bode’s project was to try and determine the underlining pattern which “positioned“ the planets in the Sun’s disc. Given his pattern could we tell just where we would expect to find this or that planet?

Simply put, Bode’s “Law” said if you take the number series 0, 3, 6, 12, 24, 48, and so forth, add 4 to each of the numbers and divide by 10, then you will get a nice approximation of the mean distances (in astronomical units) of the planets from the Sun. The problem is that while some of the results are good there is no hard and fast law. Nor has any law been found. I have included a table which you and your students can use to see how close Bodes table matched with reality. Some would like to talk about the asteroid belt which may contain the mix for a “missing planet”. Its been shown that if all of the material in the asteroid belt did combine to form a planet it would be the size of a very small Moon.

Planet Bode Distance Actual Mean Distance

Mercury 0.4 0.39

Venus 0.7 0.72

Earth 1.0 1.00

Mars 1.6 1.52

Asteroid belt 2.8 ——

Jupiter 5.2 5.20

Saturn 10.0 9.53

Uranus 19.6 19.19

Neptune 38.8 30.07

Pluto 77.2 39.5

Let us think about the Sun and how its parallel rays interact with the surface of the Earth. If we cut a circle from hard paper (5 x 8 card) put a brass paper fastener in the middle and attach this to a lined piece of paper. On our paper disk we should see the equator, north and south poles and our real axis with its 23° tilt. Now what do we see? The solar energy which hits the Earth at its Equator is concentrated over a relatively small area. Now let your eyes move to the top of the “sphere” what do you see? Do you expect more intense radiation here? If you were to compare the two locations how would you rate or compare the intensity of the solar radiation? Use a suitable ratio. Try watching the seasons in the hemisphere (north and south). Let the students make observations.

Figure available in printed form

In the classroom I use this hands on model. A dowel marks a point on its circumference. The removable diameter is a strip of formica shaped like an arrow. Holding the arrow a foot or two from the model I ask the class to “questimate” the length of the model’s diameter, two guesses are marked on the arrow which is then hung on the dowel and allowed to swing in the manner described above. Usually by the third time they can call the diameter on the button.

Figure available in printed form

To solve our next problem, that of determining the circumference of a circle, we must rely on the work of Archimedes (pi) if we wish to get any amount of accuracy. If we are looking for simple ideas for the class we can return to the wooden “lollipop” and its flexible (formica strip) which has now been marked off (diameter). We put a random mark on the perimeter of the circle. Next we take the flexible diameter and wrap it around this circular disc and mark how far it went. We continue in this fashion until we return to our first mark. It will be noted that we were able to wrap three diameters around the circle. There is however a problem. The third wrap does not exactly meet the first mark. We have a space which grows proportionally larger as the circles we are measuring grow larger.

Figure available in printed form

If we disregard the space we can formulate a rule for the circumference of a circle. To find the circumference of a circle take the product of its diameter and three and you will have a good estimate (if the circle is not too large)> This system worked for the Chinese 5000 years ago, but it needed to be refined. The time was 300 to 200 B.C. The Greeks (as well as many others) were trying to measure our space ship and the relative distances to the moon and Sun, and they needed accuracy. As we have just seen they could use the results of Archimedes' computation of pi.

Now to tackle the last of the three questions. How do we determine the area of a circle? I use three approaches. Approach #1. Present each student with a large circle which has been over laid with a square grid. The instructions are simple. Count and record the number of complete squares which appear within the circle. Now count all of the squares which are 90%, 80%, 70%,..., down to 10% complete within the circle. Add the results of each one of these countings and we should have the area of this circle. If you were the "official circle area computing person" in Greece would you look foreword to going to work? If the answer the question is no! Then we should look for a better way.

Figure available in printed form

To prepare for this task we took several large sized wooded disks. The first disk was just a large disk. The next disk was cut through the center to form four pie slices. The next disk was cut through its center 8 times forming 16 equal "pie slices." These were then taken and bound in a strip of leather, screwed and glued in place. After this two fasteners were installed so that this model could be displayed as a circle or as two semi-circles.

The first circle disk was left on the chalk rail for all to see. The second disk, which was now in four parts was show to the class who were then asked if the resulting shapes reminded them of anything. Hopefully they will see these shapes resemble a series of round bottom triangles.

Figure available in printed form

The remaining part of the demonstration may done in the following way. We take the disk which has cut into 16 parts. We unsnap the leather which binds the disk and open up each semi circle, then push the two parts together to form a rough parallelogram. Since the students should know that the area of a parallelogram equals the product of its base times its height. The height is r (radius of original disk) and the base is one half the circumference of the original disk. The formula for the area of a circle is therefore:

Aparallelogram = Base * Height

Area circle = 1/2 *(2 * pi*r) *r

Area circle = pi * r^2

Figure available in print form

What Is Our Role In Saving The Planet?

An Editorial By Paul Cochrane The Earth's man made problems are so enormous that even thinking about them gives us a desperate feeling. All of us in Connecticut are now involved in an aggressive recycling program, hopefully our efforts will lower our energy usage and mineral waste. I can recall that during World War II my father and mother put aside tin cans and metal items to help the "war effort". I thought that like most things the government wanted to involve all of us in some effort so that we could feel the hardships others who were intimately involved in the fighting. The tearing up of wrought iron fences which surrounded the cemetery helped the next fence salesman more than it did the war effort. The removal of trolley car tracks and their non polluting chariots was a big hoax laid on the cities by "Detroit", Standard Oil and the "Goodyear" people. All of our major cities now lie beneath blankets of smog. Our streets, lots and land fills are decorated with old and worn tires, which will serve as markers in some dig ten thousand years from now.

What can you and I do? As individuals a lot. I have switched from burning wood to heat my home (5 cords a year) and this has cleared the air above my "plantation". My furnace is old and not efficient (60%) this summer I will install another more efficient one. Automobiles which get 12 miles to the gallon are not found on my drive way. Four cylinder cars which get 35 miles to the gallon are the rule. My next purchase will better that or I will become re-acquainted with public transportation. My real dream is for solar heat and a solar car within my life time.

The companies which now supply our oil, gas and electrical power are worried that we might be able to make our homes and automobiles interact with the Sun, without *them interceding on our behalf* . Industry wants to work out the "problems" of control and distribution of solar power and when that is firmly in *their* control, we will have our Sun powered automobiles and houses and factories. Industry is not about to give up the sweet deal they have enjoyed for the past hundred years.

In the August 91 issue of "Popular Science" there were some interesting articles. The first was on page 29 entitled "Global" Thermometer and it talked of the work of the Scripps Institute of Oceanography in La Jolla, California. The question which they are addressing is how does one monitor changes in global warming? It is a fact that the speed of sound increases in warm water, so the institute has begun to send sound waves from a base in the southern most part of the Indian ocean. These high decibel low frequency waves will be sent out weekly and will travel up to 11,000 miles across oceans. These sounds and their speeds will be monitored at selected sites over the next ten years and hopefully will give us information on global warming.

Another article entitled "Purifying Water With Sunshine" is a discussion about the efforts of Solar Energy Research Institute (SERI) out of Golden, Colorado, to reclaim polluted ground water. Some water which contains solvents and carcinogenic chemicals which were allowed to seep into the water supply. In general the contaminated water is pumped up from the ground, then fed through glass tubes which contain a photocatalyist. These tubes are suspended in parabolic troughs where the Sun can focus its ultraviolet rays on them. "This detoxification process...breaks organic wastes down into carbon dioxide, water and dilute acid". Any activity such as this which will keep "love canals" from occurring is more than welcome.

Sunlight Chandeliers an article by David Scott (page 28), tells of buildings which are lighted by sunlight. In this case the buildings are equipped with computer controlled heliostats. Heliostats are mirrors which have been fitted with mirrored infra-red filtering glass which focuses the Sun's rays into other fixed mirrors which in turn send this cool light down into "sparkle tubes", which in turn light the interior of the building. At night and on cloudy days the Sun is replaced with spot lights. Such an idea could provide great savings in energy usage for those industries which operated primarily in the daylight hours.

On page 78 of the same issue is, “Reversing The Greenhouse”, which describes some of the ideas put forth by students at Illinois Institute of Technology in Chicago. “Scientists estimate that 225 billion tons of CO_2 are removed annually from the atmosphere by forests and oceans. However 228 billion tons are added, of which 5.5 billion are from the burning of fossil fuels and 2.2 billion are released by deforestation”.

Project Pheonix was organized to address these problems. One idea that they came up with was to make a space platform which would be manufactured on the Moon and placed in a geosynchronous orbit 22,245 miles above the Earth. The proposed platform would span 5.6 miles, and would be designed to collect solar energy, put it into the form of micro wave energy, and through a series of energy bursts beam it down to waiting earth power installations for redistribution into power grids.

The Earth has 3.3 million square miles of deforested land and if these scientists could take control of 772,000 square miles of it for reforestation, special covers could be put in place which would insure optimum plant growth. Then these new plant life could absorb 1.1 billion tons of CO_2 each year. There are a lot of “ifs” in that plan. The second part of their plan is to create series of man made floating vegetation platforms which eventually cover 386,000 square miles of ocean. These “rafts” would be fitted with mangrove trees (which grow in salt water), sargassum (floating algae) and mollusks. It is hoped that fish would flock to such a haven. In return for all of this effort about 0.6 billion tons of C02 could be pulled from the atmosphere. I would worry about one good old hurricane hitting one of these massive man made islands.

Nothing works like not creating the problem in the first place, but its a little too late for that kind of thinking. As an informed group we should do our reading and get to know about the technologies which will run tomorrow’s world. We should support the men and women who push for a cleaner, better and safer environment through donations, letter writing, voting and our own activity within our schools. We should support and encourage those who strive to protect and nourish “mother earth”.

Our students look to us for leadership, so lets become informed and lead. This space ship does not belong to us it belongs to future generations.Let us pass it on in good shape.

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