



Scaling the Natural World Using Dimensional Analysis

Curriculum Unit 91.06.03
by Stephen Kass

I Introduction—Magnitude

Nature's most distant reaches have been probed by many new inventions within the last 25 years. Lasers, rockets, accelerators, fantastic photographic capabilities combined with space telescopes have tremendously expanded our vision of what surrounds us. In twenty years computer scientists have gone from speaking in terms of milliseconds (thousandths of a second), to nanoseconds (billionths of a second). Physicists can detect gravity waves by an amount of a million trillion smaller than an electron. That's smaller than anything on the atomic scales. The launching of a space telescope was supposed to enable astronomers to see ten times farther than can be seen with present instruments!

Scientists are now looking at bigger, smaller, older, farther, and faster orders of magnitudes than have ever been known before in the natural world. Today, they are probing phenomena that are as tiny as $1/1,000,000,000,000$ of a centimeter in an explorable universe whose edge lies at least $100,000,000,000,000,000,000$ miles away. We are studying phenomena so short lived that they occur in $1/10,000,000,000,000,000,000$ of a second. By contrast, astronomers tell us the universe is some $20,000,000,000$ years old by using astrometry to mathematically measure stars and masses in space. For scientists, the frontiers of space hold the key to scientific questions about the existence of life on other planets, the real size of the universe, and Earth's role in it.

The sheer scale of explorable nature has burst beyond our wildest assumptions with incredible proportions. Even as the U.S. surpasses a trillion dollar debt, this amount appears small when compared to our exploding microscopic and telescopic world. Increasingly, students and average people are bombarded with facts and figures of enormous proportions. Such numbers are very awkward to learn and are constantly getting more difficult to comprehend. As our need to deal with more and more zeros increases and decreases, our interest to handle these magnifications is being met by growing indifference. But since we must be prepared to use and manipulate large and small numbers, there must be a way to handle them.

The curriculum *Scaling the Natural World Using Dimensional Analysis* will address the sheer volume of incomprehensible numbers (speed, distance, age) in the natural world. The major goal of this interdisciplinary math and science unit is to enable students to understand the scale of the natural world using the concept of rates, proportions and dimensional analysis. These concepts greatly simplify the learning process for orders of magnitude problem solving in an innovative way. This curriculum should help clarify thoughts about the

magnificent scale of the rapidly changing natural world and human beings place within it.

By the end of this curriculum unit, the student will be able to calculate problems such as the following: Measurements indicate that the continents of Europe and North America are separating (plate tectonics) at the rate of about 2 centimeters per year. If Columbus could repeat his famous voyage of 1492, about how many feet or yards must he travel further?

Curriculum Outline:

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A. Cosmological Time Scale

Event Years Ago

- Big Bang 15,000,000,000
- Solar System/Earth Formed 4,500,000,000
- Oldest Rocks 3,900,000,000
- Oldest Fossils 7,900,000,000
- First Dinosaurs 200,000,000
- First Flowers 135,000,000
- End of Dinosaurs 65,000,000
- Grand Canyon Began Forming 6,000,000

B. The Universe: Galaxies, Stars, and Planets

From the earth's point of view, the most important aspect of the universe is its immensity. It is so large that the size is utterly meaningless to us. We can write it down, and even use the convenience of exponents (where $1,000,000 = 10^6$), but the figures are too large to have significance. We have all stood outdoors on a clear night and looked at the beauty of the stars and wondered with awe about the nothingness of the great space above us. But in terms of actual distance, all we can manage to understand is that the stars are far away. So is China, yet we know they are farther than China. What we cannot intuitively grasp, without great intellectual effort on our part, is how unbelievably far away they are.

Even the units used in astronomical distances seem beyond our immediate comprehension. A light-year is the distance traveled by light in one year (365 days, or 8,760 hours, or 525,600 minutes, or 31,536,000 seconds). Light travels at approximately 186,300 miles per second. To picture this speed, the circumference of the earth is about 24,000 miles; it would take light less than $1/7$ second to go around the world (were it possible to do so). A light-year is approximately 5.9×10^{12} miles, a distance which is already too great to imagine (See related activity about how to calculate a light-year).

If one light-year is too far to imagine, then what possible meaning can we derive from the fact that the farthest galaxy that can be observed with the most powerful telescope is estimated to be over two billion (2×10^9) light years away? Clearly, the diameter of the entire universe must exceed this figure, but the distance is so large that making it any larger seems insignificant.

The universe contains stars. The stars are not uniformly distributed in space, but they form clusters called galaxies, which are about 100,000 light-years in diameter. In the universe there are thought to be some 10^{15} galaxies, and each one of these contains on the average 10^8 stars. Not only are these distances beyond comprehension, but also these numbers.

The galaxies themselves are not evenly spaced throughout the universe; they tend to group together. Therefore, the distance between galaxies varies considerably, and the average distance is about one million light-year!

To continue our size description of the universe, the stars within a galaxy are separated by an average distance of 5 light-years. We are part of a galaxy which we recognize as the Milky Way. It has a diameter of roughly 100,000 light-years and contains about 2×10^{11} stars. Galaxies have different forms, all presumed to be related to their movement, their rotation. They may be round, flattened ellipses, or spirals of different configurations, and in some cases they form rather irregular shapes. Our galaxy is a flattened spiral in which one of the stars, the sun, is about two-thirds away from the center toward the edge. The fact that the Milky Way is a broad band across the sky is an index of the flatness.

The stars themselves vary tremendously in size as well as in their degree of brightness. The sun has a diameter of 864,000 miles. There are stars that are much smaller ($1/10$ the mass of the sun) and some which are much larger (10,000 times the mass of the sun).

Around each star there may be planets. The sun (diameter is 864,000 miles), for instance, has nine such planets, of which the earth is one. Planets are far smaller than stars, and they differ in their distance from their star, their rates of movement and rotation, their density, their chemical composition, their atmospheres, and in the number of satellites or moons that are in turn orbiting around them. In our solar system Jupiter is

the largest planet, with a diameter of 86,000 miles, and Mercury the smallest, with a diameter of 3,100 miles. By comparison, the diameter of the earth is 7,918 miles and the moon 2,160 miles. Mercury is the planet closest to the sun, having a mean distance to the sun of 36×10^6 miles. The earth is the third planet from the sun, with a mean distance of 92.9×10^6 miles. Pluto is the farthest away, on the average $3,671 \times 10^6$ from the sun. Expressed in million of miles, the moon is on the average 0.24×10^6 miles from the earth.

We have in this picture of the universe a whole series of size levels, beginning with the entire universe, then the clusters of galaxies, the galaxies themselves, the stars, and finally the planets. At each level it is striking that the units are not evenly spaced or randomly distributed, but clearly clustered. This applies to groups of galaxies, stars, and planets.

Again the figures have little impact on our imagination. They do not give any appreciation of the immensity involved. A very vivid description is presented by Robert Jastrow in *Red Giants and White Dwarfs*. He says:

“An analogy will help to clarify the meaning of these enormous distances. Let the sun be the size of an orange; on that scale of sizes the earth is a grain of sand circling in orbit around the sun at a distance of 30 feet; the giant planet Jupiter, 11 times larger than the earth, is a cherry pit revolving at a distance of 200 feet, or one city block; Saturn is another cherry pit two blocks from the sun; and pluto, the outermost planet, is still another sand grain at a distance of ten city blocks from the sun.

On the same scale the average distance between the stars is 2000 miles. The sun’s nearest neighbor, a star called Alpha Centauri, is 1300 miles away. In the space between the sun and its neighbors there is nothing but a thin distribution of hydrogen atoms, forming a vacuum far better than any ever achieved on earth. The galaxy, on this scale, is a cluster of oranges separated by an average distance of 2000 miles, the entire cluster being 20 million miles in diameter.

An orange, a few grains of sand some feet away, and then some cherry pits circling slowly around the orange at a distance of a city block. Two thousand miles away is another orange, perhaps with a few specks of planetary matter circling around it. That is the void of space.”

C. Powers of Ten

Measuring Tools Distance (Meters) Systems

Telescopes $> 10^{26}$ Universe

10^{20} Galaxies

10^{16} Light Year

10^{15} Stars

10^{12} Solar System

Eye 10^{10} Sun

10^5 Moon

10^3 Kilometers

10^1 Humans

Microscopes 10^{-8} Molecule

10^{-10} Atom

Accelerators 10^{-14} Nucleus

10^{-15} Proton

10^{-16} Nucleons

10^{-17} Quarks/Leptons

D. Millions, Billions, and Trillions

It used to be that millions was the byword for a large number. The enormously rich were millionaires. The population of the earth at the time of Jesus was perhaps 250 million people. There were almost 4 million Americans at the time of the Constitutional Convention of 1787; by the beginning of World War II, there were 132 million. It is 93 million miles to the Sun. Approximately 40 million people were killed in World War I; 60 million in World War II. The global nuclear arsenals today contain an equivalent explosive power sufficient to destroy 1 million Hiroshimas. For many purposes and for a long time, “million” was the extremely big number.

But times have changed. Now the world has many billionaires—and not just because of inflation. The age of the Earth is well-established at 4.5 billion years. The human population of our planet is 5 billion people and, by the turn of the century, may be between 6 and 7 billion people. The Voyager spacecraft is roughly 2 billion miles from Earth. The U.S. defense budget is around \$300 billion a year. The immediate fatalities in an all-out nuclear war are estimated to be around a billion people. There are billions of stars and galaxies. On the other hand, a few inches contains billions of atoms side by side.

While millions and billions have not faded, these numbers are becoming somewhat small-scale. The new number on the horizon and appearing in everyday language is the trillion.

World military expenditures are now over \$1 trillion a year. The total indebtedness of all developing nations to Western banks is around \$1 trillion. The annual budget of the U.S. government has reach \$1 trillion. The national debt is more than \$2 trillion (amount the U.S. government owes banks, businesses, and other countries). The distance from our solar system to the nearest star, Alpha Centauri, is 25 trillion miles.

Confusion among million, billions, and trillion goes on every day. An easy way to determine what large number is being discussed is simply to count up the zeros after the one. But if there are many zeros, this can get a little tedious. That’s why we put commas, or spaces, after each group of three zeros. So a trillion is 1,000,000,000,000 or 1 000 000 000 000. For numbers bigger than a trillion, you have to count up many triplets of 0’s there are. It would be much easier if, when we name a large number, we could just say how many zeros there are after the one.

E. Exponential Notation

Scientists and mathematicians, being practical people, have created a system to deal with extremely large numbers. It's called scientific notation. You write down the number 10; then a little number, written above and to the right as a superscript, tells how many zeros there are after the one. Thus $10^6 = 1,000,000$. $10^9 = 1,000,000,000$. $10^{12} = 1,000,000,000,000$; and so on. These little superscripts are called exponents or powers; for example, 10^9 is described as "10 to the power 9" or equivalently, "10 to the ninth" (except for 10^2 and 10^3 which are called "10 squared" and "10 cubed," respectively). This phrase, "to the power"—like "parameter" and a number of other scientific and mathematical terms—is creeping into everyday language, but with the meaning distorted.

In addition to clarify, exponential notation has a wonderful side benefit: You can multiply any two numbers just by adding the appropriate exponents. Thus $1000 \times 1,000,000,000$ is $10^3 \times 10^9 = 10^{12}$. Or take some larger numbers: If there are 10^{11} stars in a typical galaxy and 10^{11} galaxies, there are 10^{22} stars in all the galaxies.

But there is still resistance to exponential notation from people a little nervous about mathematics even though it simplifies, not complicates our understanding.

The first six big numbers that have their own name are in the chart on the next page. Each is 1 000 times bigger than the one before. Above a trillion, the names are almost never used. You could count one number every second, day and night, and it would take you more than a week to count from one to a million. A billion would take you half a lifetime. And you couldn't count to a quintillion even if you had the age of the universe to do it in.

Once you've mastered exponential notation, you can deal effortlessly with immense numbers, such as the rough number of microbes in a teaspoon of soil (10^8); of grains of sand on all the beaches of the Earth (10^{20}); of living things on the earth (10^{29}); of atoms in all the living things on Earth (10^{41}); of atomic nuclei in the Sun (10^{57}). This doesn't mean you can picture a billion or a quintillion in your head—nobody can. But with exponential notation, you can think about such numbers when trying to understand the incredible scale of nature

F. Exponential Notation Chart

Number (U.S.)

One

Thousand Million

Billion

Trillion

Quadrillion

Quintillion

Number (written out)

1

1,000

1,000,000

1,000,000,000

1,000,000,000,000

1,000,000,000,000,000

1,000,000,000,000,000,000

Number (Scientific Notation)

10^0

10^3

10^6

10^9

10^{12}

10^{15}

10^{18}

How long it would take to count to this number from 0 (one count per second, night, and day)

1 second

17 minutes

12 days

32 years

32,000 years

32 million years

32 billion years

Larger numbers are called a sextillion (10^{21}), septillion (10^{24}), octillion (10^{27}), Nonillion (10^{30}), Decillion (10^{33}), and 10^{100}).

II. Why Use Dimensional Analysis?

Many research studies conclude that proportions are the most difficult problem-solving mathematical tools to master of any introductory science course. Indeed, many physical science, biology, physics, and chemistry concepts, in effect, are names given to proportional relationships. More specifically, proportional math problems can compose as much as 95% of an introductory chemistry course. Students' ability to comprehend and effectively use proportions, therefore is a major concern of the science and math educator. Yet, little is done within the classroom to increase this proficiency.

Not only is student understanding of proportionality a concern of the science and math educator, it is a major concern of the developmental psychologist. For example, Inhelder and Piaget have studied intellectual development in relationship to students' ability to deal with science concepts. They regard proportionality as a primary acquisition at the stage of formal operations which include subjects from 11 to 15 or 16 years. Unfortunately, there is much evidence that suggests that as much as 50% of some samples of secondary school and college-age students have failed to acquire a working understanding of proportionality.

The concept of proportions is seen as fundamental to understanding many scientific applications as well as consumer problems, advanced science and math courses, and intellectual development in general. Rates can be found in most aspects of life including cooking, navigation, physics, earth science, economics, electronics, business, and industry. Since a large percentage of adolescents are lacking this critical skill, the determination of possible ways of successfully teaching the concept is an important issue.

Sci-Math is an interdisciplinary curriculum designed to address these issues of teaching proportionality in science and math courses while using large or very small numbers. Its development and field testing were funded by the National Science Foundation. Sci-Math was cited by the U.S. Office of Education as an exemplary educational innovation worthy of national dissemination within the National Diffusion Network (N.D.N.).

Sci-Math focuses on the understanding of the concept of proportions and on the use of proportions in word problem-solving. Specifically, Sci-Math uses the rate concept and dimensional analysis used in introductory physics and chemistry courses to solve proportions (see Teaching Dimensional Analysis in the next section). This rate and dimensional analysis method has slowly moved into textbooks and has completely replaced the method of ratio-and-proportions taught exclusively in junior and senior high school mathematics textbooks.

There appears to be good reason for dimensional analysis to have replaced the ratio-and-proportion method in advanced science courses. Dimensional analysis is a simple, problem-solving, error-reducing procedure which seems to require less conceptual reasoning power to understand than does the ratio. Furthermore, it can condense multi-step problems into one orderly extended solution. However, the treatment accorded the method of dimensional analysis by too many advanced science books is confusing, too sketchy, and not logical in the approach to word problem solving.

To support Sci-Math goals, the curriculum uses hands-on activities and experiments. These experiments use simple inexpensive materials already available in schools: spoons, pennies, jars, rulers, string, etc. Proportions are of great use in everyday life as well as an important pre-algebra and physical science tool. While the Sci-Math curriculum deals with the everyday world of measuring, buying, cooking, and driving, the mathematics taught are the mathematics needed for advanced science. A good example of a Sci-Math activity is measuring, then calculating the average rate of 15 pennies to 2.2 centimeters. Using this rate, students find

how many pennies would be necessary to stack in order to reach the moon some 237,000 miles away.

Developing an understanding of the Sci-Math method with its “real life” labels could help algebra students to better understand algebra and see a little more clearly its relationship to everyday life. In addition, a remedial math student, tired of writing all those labels, could easily begin to shortcut his/her work by using letters and therefore naturally begin to use algebra. The rules of adding, subtracting, multiplying, and dividing are the same for both Sci-Math units and algebra variables.

Three research studies on proportional calculations with emphasis on the rate concept, dimensional analysis, and hands-on manipulative experiments were field tested in the ninth and tenth grades. The students showed substantial improvement in proportional problem-solving skills. The studies suggest that any advanced science course is a late point at which to introduce dimensional analysis and the rate concept. It seems that the student needs to learn to understand the logic of the process using familiar experience with the concepts before applying them to the unfamiliar variables of advanced science. Hence, it is important that the rate concept and dimensional analysis be taught prior to advanced science courses. Ideally, these concepts should be taught in the seventh, eighth or ninth grade for college-bound students, in the ninth or tenth grade for non-college bound students. The research indicates both groups can significantly improve their understanding of proportions and problem-solving by using the Sci-Math techniques.

After formally adopting the Sci-Math program from N.D.N., I implemented a team-taught physical science and pre-algebra course at my school. Initially, some teaching problems developed that were particular to lower skills urban students. Despite these problems, the students demonstrated significant improvements on their science and math problem-solving skills. In my many years of teaching, I have never seen such interest in and enthusiasm for word problem-solving and science labs. For example, years later I still find students remembering with excitement how they figured out how many pennies it took to get to the moon, if they stacked one on top of each other. In addition, the algebra and advanced science teacher said that the Sci-Math skills learned in the past year are readily transferable to their courses. It is my personal experience that a firm basis in Sci-Math will also decrease avoidance of advanced science courses by students, and help science teachers who are often forced to teach the mathematics necessary for science.

Although the Sci-Math program is a powerful learning approach, the problem with the curriculum seems to be that it needs to be adapted to meet the particular needs of urban students. Research backs up my intuition. Karplus and Peterson’s 1970 research study found that while successful proportion reasoning is present in half the suburban eleventh and twelfth-grade students tested, **only one-eighth of urban students had this ability.**

The teaching techniques of Sci-Math and dimensional analysis need be modified for the special needs of urban students so they better reflect a logical developmental progression with a great deal more reinforcement and more applications. This is one of the major goals of my curriculum unit.

The National Council of Teachers of Mathematics (N.C.T.M.) has stated that its first goal for the 1980’s was that problem solving must be the focus of school mathematics. According to Shirley Hill, a former president of N.C.T.M., “This means that the ultimate goal in our teaching is the ability to apply the mathematics learned.” The rate concept and dimensional analysis are two excellent tools for this purpose.

A. Teaching Dimensional Analysis

A rate states how much of one quantity per how much of another quantity (a quantity is a number and a

label). It could be any rate like cost per hamburgers. Taking for example 55 miles per hour and the number of miles traveled, 495 miles, you could determine the number of hours traveled as follows:

495 miles 1 hour

$$\frac{495 \text{ miles}}{1 \text{ hour}} \times x = 9 \text{ hours}$$

1 55 miles

If you knew the number of hours traveled, 8 hours, you can use the reciprocal of this rate to find the number of miles covered as follows:

6 hours 55 miles

$$\frac{6 \text{ hours}}{55 \text{ miles}} \times x = 330 \text{ miles}$$

1 1 hour

This method extends to solving multi-step problem such as how many centimeters in 330 miles. First, let's use the old ratio and proportion method still used in most junior and senior high school math book to solve this problem:

330 miles 1 mile

$$\frac{330 \text{ miles}}{1 \text{ mile}} \times x = 3,990 \text{ feet}$$

x feet 5,280 feet

3,990 feet 1 foot

$$\frac{3,990 \text{ feet}}{5,280 \text{ feet}} \times x = 47,880 \text{ inches}$$

x inches 12 inches

47,880 inches 1 inch

$$\frac{47,880 \text{ inches}}{12 \text{ inches}} \times x = 121,615.2 \text{ cm}$$

x centimeters 2.54 centimeters

Now, use the short cut method of dimensional analysis to solve the problem.

330 miles 5280 ft. 12 in. 2.54 cm.

$$\frac{330 \text{ miles}}{1 \text{ mile}} \times \frac{5280 \text{ ft.}}{1 \text{ ft.}} \times \frac{12 \text{ in.}}{1 \text{ in.}} \times \frac{2.54 \text{ cm.}}{1 \text{ cm.}} = 121,615.2 \text{ cm}$$

1 1 mile 1 ft. 1 in.

1. Definitions

MEASUREMENT

Numbers had no meaning in the old days without some unit of measurement such as an arm or foot. The unit of measurement is the measurement label. Without a unit or label, measurement has no meaning. 2 what? 2 arms or 2 feet? Examples—arm, foot, inch, cent, mile. Single and plural labels such as foot or feet are regarded as the same units.

QUANTITY

A quantity tells us how much or how many and always has two parts—a number and label. For example, do we want 2 feet or 3 feet or 3 miles? Every number must have a unit of measurement or label in order to be a quantity. Example—6 students, 30 cents, 8 inches, \$5

RATE

A rate is a comparison of two different quantities. A rate tells us the quantity of one thing per quantity of a second thing. For example, 55 miles per hour, 25 students per teacher, \$5 for 3 gallons of gas, 96 cents for 3 candy bars are all rates. These rates can each be written as fractions:.

55 miles 25 students \$5 96 cents

1 hour 1 teacher 3 gallons 3 candy bars

RECIPROCAL RATE

The reciprocal rate is a rate that is reversed or turned around. For example, the reciprocal rate of

96 cents is 3 candy bars

3 candy bars 96 cents

CONVERSION RATE

A conversion rate is a special rate where the quantities are equal, but the units of measurement are different. In other words, the two quantities are equal and each quantity can be changed into the other quantity in the rate. Examples are

12 inches 1 yard 5 fingers

1 foot 3 feet 1 hand

2. Rules: Single Step Rate Problem

PROBLEM: How many miles will you travel in 4 hours if the speed limit is 55 miles per hour?

Quantity=4 hours

RATE=55 miles/1 hour or 1 hour/55 miles

R Right of equal sign.

= _____

U Units you are look for.

= _____ miles

L Left units are the same as units on the right.

miles = _____ miles

E Each rate must be completed (numerator and denominator with numbers and labels).

55 miles = _____ miles

1 hour

S Solve by multiplying rate times quantity over 1, then cross cancel.

4 hours X 55 miles = 220 miles

1 1 hour

REMEMBER TO SPEND 5 MINUTES ON EACH PROBLEM!!!

3. Rate Rules: Multi-Step Rate Problem

PROBLEM: How many inches in 3 miles, if there are 5,280 feet in one mile.

R Remember to make a list of all rates, reciprocal rates, quantity, and units you are looking for.

QUANTITY= 1 mile RATE= 5,280 feet 1 mile

UNITS = 1 inch 1 mile, 5,280 feet

A Are there missing rates?

YES

T The missing rate(s) or conversion rate(s) must be figured out or use a conversion chart.

12 inches 1 foot

1 foot, 12 inches

E *Equal sign: set up the problem using the RULES*

R = _____ inches

U = _____ inches

L *inches* = _____ inches

E 5,280 feet X 12 inches = _____ inches

1 mile 1 foot

S 3 mile X 5,280 feet X 12 inches = = 190,080 inches

1 1 mile 1 foot

B1. CALCULATE THE CIRCUMFERENCE AND DIAMETER OF THE EARTH

Over 2,000 years ago a Greek scientist, Eratosthenes measured the circumference of the earth. His method involved: 1) measuring the angle between the Sun's rays and the zenith and 2) using parallel lines to find the circumference of a circle.

In the city of Aswan, Egypt, Eratosthenes observed that at noon on the longest day the Sun was directly overhead. It reflected off the bottom of a deep well. At exactly the same time in Alexandria, the Sun was not directly overhead and caused a tall pillar to cast a shadow. By measuring the pillar and shadow, Eratosthenes found the angle between the Sun and the zenith to be 7.2 degrees. Pacers had found the distance from Alexandria to Aswan to be 474 miles. Knowing that light rays from the Sun striking the Earth seem to be parallel, Eratosthenes was able to calculate the size of the Earth.

Examine the diagram below. Using Eratosthenes method, the circumference of the Earth is calculated using a proportion.

360 degrees X 474 miles = 23,700 miles

1 7.2 degrees (circumference of the Earth)

Figure available in printed form

If the earth circumference is 23,700 miles, what is the diameter? (Hint: Circumference equal 3.14 X Diameter); Answer is 7,569 miles. Eratosthenes' method is still used today by surveyors who measure the Earth. They have discovered that you must travel about 66 miles on the earth to make an angle of one degree at the

center of Earth. To find the diameter of the Earth from these measurements, first the circumference must be calculated.

$$360 \text{ degrees} \times 66 \text{ miles} = 23,760 \text{ miles}$$

1 1 degree (circumference of the Earth)

$$23,760 / 3.14 = 7,569 \text{ miles (diameter of the earth)}$$

2. How to Calculate a Light-Year

A light year is a measure of distance. It is the distance light travels in one year at the rate of 186,000 miles per second. The following is how light travels in one year:

$$1 \text{ year} \times 365.25 \text{ days} \times 24 \text{ hours} \times 60 \text{ minutes} \times 60 \text{ seconds} \times 186,000 \text{ miles} = 5,869,713,600,000 \text{ miles}$$

1 1 year 1 day 1 hour 1 minute 1 second

Calculate the distance in miles to the nearest star, Alpha Centauri, is to earth if it is 4.3 light years away.

The Big Dipper: Each star of the big dipper is a different distance from earth. The star closest to the earth, Megrez (last star of the cup), is 63 light-years away. Calculate the distance in miles.

The star farthest away in the big dipper from the earth is Alkaid (the last star of the handle) is 210 light-years away. Calculate the distance in miles from the earth.

Pluto is the last planet in our solar system and the farthest away from earth; some 3,473,000,000 miles away. Calculate the number of light-years away from the earth. Is it farther away from earth than the nearest star?

Figure available in printed form

Background for Solar System Distances Activity

It is difficult to comprehend the tremendous area of space. When measuring the distance between planets, numbers are so large that sometimes students are unable to compare them. The purpose of this worksheet is to have students comprehend the great distances between the planets through using familiar means of transportation for their space travel. Students may choose to take a bicycle trip to the moon, and because the bicycle is a familiar vehicle, they will be able to relate to the time and distance involved in making the trip. The teacher should point out that all years are calculated on a 24 hour day. Thus, riding a bicycle to the moon would take 3 years of peddling 24 hours a day, or converting to an 8 hour day would make the figure three times as great, or 9 years.

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