Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
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# Mathematics and Architecture Designs

Curriculum Unit 93.01.07
by Winnifred Morgan

It is well known that virtually all disciplines e.g.. Music, and Language, Mathematics or Science are introduced to children at a very early age as distinct, isolated subjects that have no connection (inter-relationship) among them. Consequently, although children may have a ready grasp of architectural ideas before fourth grade, they cannot see the relationship between the elements of design and symbols and the Mathematics of the design. From as early as first grade, steeples, columns, special window shapes, colors, and figurative ornaments have being singled out and illustrated by students. Such drawings litter the walls of classrooms; and people, stars, trees, animals, houses, and buildings are symbolically sized, simplified, and composed together to tell a story. Here the "golden" opportunity for integrating architectural concepts with Science and Mathematics is ignored, leaving students at a disadvantage.

The introduction phase of Architectural understanding is excluded from most, if not all modern educational programs at both the elementary and high school levels, despite the overwhelming presence of buildings in the everyday life, of cities and suburbs and despite the unique role of architecture in orienting people to personal and public space and history. Indeed, Architectural orientation is also excluded from college and university curriculum except for those students who choose to pursue a career in the discipline. And yet we spend most of our lives in and around buildings, knowing very little about them or how we are influenced by them. We make daily decisions related to architecture such as selecting color for a room, re-arranging the furniture, plan gardens or even buy a house. How much easier these decision making processes would have been if we were exposed to some basic fundamental architectural knowledge.

Symbols remain the visual principle in architectural design, serving as the key elements in the design of memorials. Their presence is indispensable to our understanding of the history of communities. But the Mathematics of such symbols must not be ignored for without mathematics we would have many lopsided buildings, and poorly articulated designs and patterns.

Because Mathematics and Architecture have traditionally been treated by the curriculum as an either/or polarity, students are denied the opportunity to see the unique inter-relationship between the two disciplines. If the gap is to be bridged between ideas from Architecture and Mathematics, the teacher must pose tasks that will engage the students' intellect and foster a better understanding of the concepts and procedures of both disciplines that will stimulate them to make the necessary connection between the disciplines.

This unit seeks to bridge the existing deficiency of the curriculum and aim at highlighting the importance of pure Geometry that is utilized in Architectural plans, designs and facades. Briefly stated the general objectives of this unit are:

1. To provide teachers with hands-on-activities to experiment with their Math class,
2. To establish the link between topics in Mathematics and Architecture,
3. To develop students awareness of the interdependence of the mathematical concept of ratio, proportion and geometry in Architectural designs and constructions.
4. To develop students ability to apply mathematical knowledge to problem solve architectural procedures, plans and pattern forming processes.
5. To facilitate and encourage a greater appreciation for Architecture by students, teachers and parents.

This unit is being organized to be presented to tenth and eleventh grade students who have been exposed to Algebra and Geometry. They meet for one forty-five minutes period per day, five days per week. The unit hopes to utilize between ten to fifteen classes depending on the ability grouping of the students.

Architecture clearly articulates the principles of harmony, order and similarity. Therefore, the activities will incorporate mathematical principles such as symmetry, similarity, Pythagorean Theorem, congruence, ratio, proportion and measurement. These basic elements of geometry reinforces the concepts of order and continuity. To design or build a structure we must use simple geometry. To create order or patterns we much use symmetry. Symmetry is defined in Math as a rotation for translation of a plane figure which leaves the figure unchanged but alters its position. Architecture defines the concept as the interchange of two elements A and B. The four basic forms of symmetry used in architecture to reinforce the concepts of grouping, order and patterning are:

1. Translation: defined as the parallel movement of a plane-figure from one position to another.

*(figure available in print form)*

2. Rotation: defined as the movement of a plane figure or object around an axis.

*(figure available in print form)*

3. Reflection: defined as the bending or folding back of an object upon itself.

*(figure available in print form)*

4. Glide reflection which is a combination of translation and reflection.

*(figure available in print form)*

These principles are demonstrated in various ways in architectural designs and buildings. Below are a few examples of the ideas of symmetry that are found in buildings and pattern designs:

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The use of similarity and repetition (congruent elements) are also utilized to create a sense of coherence in buildings. Similarity of details evident in the dimensions of the walls, corners, cornices, windows and doors counterbalances individual differences of size and proportions of houses.

*(figure available in print form)*

*(figure available in print form)*

The concept ratio is used in Mathematics to describe a comparative relationship between two quantities. Proportion is used to equate ratios. In architecture these concepts are used in designs and buildings to create balance, patterns and beauty. There are in buildings an amazing ratios of equivalent dimensions such as window ratio to door or the ratio of columns to the wall space between the columns, etc. Architects utilized proportion to express rationally and beauty. Pythagoras used numbers to demonstrate basic order in space. He believed that all things could be denominated as a whole integer and was therefore beautiful by definition. He used the magic numbers 1, 2, 3, 4 to explain order in space: point, line, plane, solid

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He utilized the system of proportion which is commensurate with sizes to develop his Tetra chord or "Perfect Golden Triangle." The principle combines ratios into larger units which in turn forms clear ratios. This allows architects to choose any preferential relationships they desire. Usually the ratios used are 1:1, 2:3, or 4:5 since they are more apparent to the eye than a ratio of say 7:8.

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If proportions are not observed perfectly, the visual image of architecture will be defective. Proportion by itself does not create beauty in architecture. The constitutive principles of architecture derive from regularity, symmetry, and a combination of all the other elements.

One of the basic concepts in mathematics that students utilize everyday is that of measurement. The concept is used in architecture to denote scale. Size and relationships of size are important qualities that are most likely to be distorted in designing a structure or a pattern or making a plan. The chances of such errors are greatly minimized when there is a scale to relate the size of the sketch or plan to the actual. For example, the house floor plan below makes no sense unless it is accompanied with a scale. Measurement therefore become a vital link to good architectural designing.

*(figure available in print form)*

Constructing a model of a public building would be great, but that would require a longer time span and mechanical skill than is available for the duration of this unit, so we will concentrate our efforts on analyzing the proportions of selected buildings, including the Center Church on the New Haven Green and other public buildings and if time permits build motifs for a facade for a public building.

"Architecture is at once a Science and an Art. As a Science it requires knowledge; as an ART is demands talent. To learn a Science requires listening, understanding and assimilating." [1] Accordingly, the purpose of Architecture is not just for pleasure or decoration for public and private utility but the happiness and preservation of individuals and society. The study of Architecture will not only stimulate historical investigations but will furnish valuable data on climate, structure of places, nature of materials, growth of technology and industrialization, and civilization of different peoples and epochs.

"Building a shelter against sun, wind, or rain is a fundamental human need, but it goes beyond functional necessity." [2] It is a creative partnership with the natural world through which man develops and utilizes his intellect and skills. Architecture, like history, is a process in which many circumstances combine to produce certain results. The availability of material, the existing technology and the particular cultural, economic and political circumstances, influence the Architectural "forms" evident in Western Architecture. Material used include glass, marble, timber, ceramics, a variety of stone including limestone, sandstone, granite, flint and brick, concrete, tiles, and mosaic.

The Architectural achievement of the Greeks has brought a distinct and exquisite refinement of detail and proportion to the existing structural system. Briefly stated, Classical Architecture is a combination of the temple architecture of the Greek and the religious, military and civil architecture of the Romans. The decorative elements of a classical building derive directly, and indirectly from the Greeks and the aim is to achieve a demonstrable harmony of parts by the use of proportions. Classical Architecture is built on a traditional system of the five orders, the Doric, Tuscan, Ionic, Corinthian and Composite orders.

*(figure available in print form)*

The development of Architecture reached the point where great buildings required the services a Specialist designer. The sheer power of Gothic Architecture indicates the product of an increasingly secular society, the mathematical and building knowledge and the talents and skills of a master-mason and a team of highly skilled specialist craftsmen. Gothic buildings represent a transition point in history between the church-dominated early Middle Ages and the period referred to as the Renaissance.

The basic elements of Architecture are the elements of construction and the elements of space. The elements of construction include brick or stone, columns (post), beams (entablature), walls, roof, platform (foundation), rafters, joist and pedestal. Elements of space include windows, doorways, pergola (covered walkway), aedicule, vents, intercolumnation, courtyard, fire court, rooms (dinning room, bedroom etc.), cortile (enclosed courtyard inside building) atrium (room which functions as a fire protector between two buildings) hallways, and pediment.

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The design vocabulary include the following: plan, elevation, section, solid void, figure, ground, proportion (the ratio of parts to the whole), scale (the ratio to an external unit measure), and grids. Since we will be analyzing the facade of the New Haven Center Church, which is purely Gothic on the interior. I will just comment briefly on the style. Gothic architecture is considered a phase of classical architecture. It had tentative beginnings in the 18th century and was firmly established in 1834 in England. It became accepted as an appropriate style for the country houses of the wealthy, romantic and eccentric. The style was not particularly popular in the Unites States, where there was a strong Classical tradition. However, there are many such buildings in New Haven and Center Church on the Green is unique in the use of that style for its ornaments.

The discipline inherent in the proportions and patterns of a building highlights the relatedness of parts. The Pythagorean triangle is frequently found in the building pattern and is used to approximate the relations of the heights of the steeples to the other parts of the building. By completing the following activities, students will not only be taught Math but will be forced to consider architectural designs and pattern making.

## Lesson Plan Sample 1

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***Theme:***
Ratio and Proportion

***Objective:***
To create an experience of ratio and proportion.

To solve problems using proportion.

***Material:***
Scissors, construction paper in different colors, pencils, rulers, question sheet.

***Key Question:***

> —How does Mathematics work in construction?
> —How much Mathematics will the Plumber, Cement Masons, Architects and other construction workers need in order to be proficient and successful in their jobs?

***Background Information:***
Construction workers must make careful plans in the form of budgets, time schedules, and blue print (scale drawing of projects). They need to know how to measure and they must have a working knowledge of ratios and proportion and scales. Cement Masons are faced with the job of ordering, mixing, and pouring cement to complete projects. They need to know the area of the cement work to be completed.

They much decide how much cement is needed for the job? How many cement trucks must be scheduled for the job? They must also decide the ratio of men to cement trucks for the job. They also need to have a working knowledge of ratio and proportions. The customers interested in the overall cost of the job. He needs to know Mathematics to decide if the estimates given by the various construction workers are cost effective.

***Method:***

Simulation to encourage brainstorming for imagination, and innovation.

***Management Suggestions:***
Divide class into groups of three or four.

***Activity:***

—Each group will be given a pair of scissors, 6 sheets of construction paper in 6 different colors and one question sheet.
—Instruct group to divide the task.
—Each group must fold, then cut the construction paper in the following way:

*(figure available in print form)*
Yellow is folded/cut into two equal pieces.

Green is folded/cut into four equal pieces.

Red is folded/cut into three equal pieces.

Orange is folded/cut into six equal pieces.

Blue is folded/cut into eight equal pieces.

White is left whole.

Each group prepares the pieces, then uses them to answer the questions of the question sheet.

**Sample Questions from Activity Sheet**

1. Write a fraction that compares one piece of green to the total number of green pieces.
2. Write a fraction that compares two pieces of blue to the total number of blue pieces.
3. Are these fractions equivalent?

When each group had completed the assignment, teacher leads out in a sharing discussion which should highlight the following concepts:

(1) A ratio is the comparison of two numbers by division e.g. numbers of yellow pieces to orange pieces is 2:6 or 2/6

(2) A proportion is an equation that equates tow ratios e.g. 1/4(green)=2/8 (blue)

Follow up activity to test for comprehension will include questions that require students to write a proportion by setting the ratios equal, then solving that equation.

***Sample Questions:***

1. Two men can paint a house in five days, How long will it take one man to do the job if he works at the same rate?
2. If one out of five people use Colgate Cavity Fighting Toothpaste, how many people will be expected to use this brand in a city of 30,000 people?
3. Find the perimeter of the room shown on the scale drawing if 1 cm->5m.

*(figure available in print form)*

## Sample Lesson 2

***Theme:***
Ratio and Proportion and Architecture

***Objectives:***
To reinforce the concepts of ratio and proportion and show the relatedness of these concepts to Architecture

***Mathematical Skills:***
The accuracy in measuring

***Material:***
Ruler, pencil, worksheets, overhead projector

***Key Question:***
How does Math foster/hinder good Architectural designs?

***Activity:***

1. Lecture on Mathematical concepts utilized by Architects. Illustrate sample problems using overhead projector.
2. Students will be asked to work in pairs.
3. Each pair will be given one worksheet on which is the side and front view of a storage house. The scale or the drawing will also be given. Students will be required to measure the length, width, and height of the drawings and from these measurement give the actual dimensions of the storage house.

*(figure available in print form)*

—When students have completed assignment, teacher will lead discussion on the answers generated by the activity.
—Ask about the variation in the answers given. Why did each group arrive at different dimensions for their building?
—When you measured, what section of the ruler did you use? Fourths, eighths, or sixteenth of an inch? Look at your rulers again. Do you see where the error was made?

***Extended activity:***
Take students to visit Center church and other historic buildings in New haven.

## Sample Lesson 3

***Objectives:***
To show the relatedness of the concepts of ratio and proportion and accuracy in measuring to good architectural designs. Students will develop a better understanding of the concept of designing a structure and executing that design.

***Mathematical Skills:***
Accuracy and competency in measuring and calculating ratio and proportions.

***Goal:***
Design and build a storage room.

***Material:***
Ruler, pencil, glue, scissors, tape, cardboard or heavy paper, wooden tongue depressors.

***Key Question:***
Ask students to draw a plan of their bedroom. Have each student explain his plan.

***Activity:***

1. Students will then be asked to work in groups.
2. They will design a plan for a storage rooms. Each group member must have an input in the final design.
3. They must decide on a scale and work out certain proportions. For example, the ratio of the door to the ratio of the windows or door ration to wall ratio.
4. After designs have been assessed by teacher and modifications, if any, are completed students

will be asked to construct the storage room from the material mentioned above.
5. At the completion of this activity student must demonstrate an understanding of the unique life relation between the architectural and mathematical vocabulary concepts such as congruence, symmetry, similarity and proportion.

***Procedure:***

1. Make the models approximately 6” x 8”
2. Cut seven pieces (4 walls, a roof, a floor)
3. Put spaces in the walls for windows and doors
4. Use wooden tongue depressors to support your roof, for corner post and door post
5. Assemble your building. Finished product should look like this:

*(figure available in print form)*

When students have completed assignments teacher will examine the projects focusing attention on accuracy in measurements. Teacher will also lead discussion on questions generated by this activity and show students how to avoid errors in future construction of models.

***Culminating Activity:***

Brief discourse on how to use a ruler, deciding on the scale for a building and how the concepts of proportionality, ratio, and symmetry are key elements to making sense of a structure.

***Extended Activity:***

1. Invite architectural lecturer or architects from the business community, contractors, and construction workers to speak to students or give slide presentations on designing and construction; or let them speak on specific aspects of architecture and the importance of having a solid math foundation.
2. Allow students to visit school’s Mechanical and Technical drawing department to have first hand hands-on experience of the designing process.
3. Complete the roof replacement problem.

**Roof Replacement**

The roof of the community recreation center was recently blown off by a strong wind unusual for this area. All structural underpinnings and gable ends were also destroyed. The roof must be replaced and your group has been invited to bid on the project. The rec center measures 30 feet by 100 feet. The basic walls are 16 feet high, the old roof shape or height of the gable ends above 16 feet.

The following information about costs is available for your consideration:

—New roofing costs $16 per square foot.
—New Material for gable ends cost $10 per square foot. (A gable end is any end wall above the 30 by 16 rectangle).
—Annual repair costs related to the costs for a 900 square foot building.

If this task is intended for an individual:

Prepare a written bid for the rec center board of directors. Propose the new roof design including sketches and dimensions. Show the work you did in developing your bid as well as your final bid proposal. Write an argument to convince the board of directors to adopt your plan as opposed to others.

If this task is intended to be used by groups of students:

Each student will build a model of a roof style and sketch that model from several different perspectives.
Each group will select one model and investigate possible cost of construction. Prepare a written bid for the rec center board of directors. Propose the new roof design including sketches and dimensions. Show the work you did in developing your bid as well as your final bid proposal. Write an argument to convince the board of directors to adopt your plan as opposed to others.
Each student will write a letter to a member of the board of directors intended to convince that person to select the plan submitted by the individual’s group.

## Sample Lesson 4

***Objectives:***
To allow students to

(1) analyze the dimensions of a facade of a house from a drawing,

(2) develop a grid for that house and analyze the proportion.

***Material:***
Scale drawing of Chateau 1860-1890

Pencil, ruler, grid paper, tracing paper, scissors, tape.

***Goal:***
To determine the actual dimensions with at least 97% accuracy.

***Procedure***

Activity 1: Students will be asked to work individually on this project. They will be given a 121% enlarged copy of the facade of the Chateau. Using their rulers and pencils, they will measure the different sections of the facades to the nearest fraction of an inch. They much determine the scale used, then convert scale measure to actual measure. (See attached sample of completed activity)
Activity 2: After concluding discussion on reasons for the differences in solutions and offering suggestions on how to improve measuring skills, students will be asked to put house in a grid and determine the proportions.

***Procedure:***

1. Using copy of Chateau, cut all perimeter lines of house
2. Use tape and paste the figure to grid paper, centering house as shown by drawing an imaginary line through the center of the large windows
3. Photocopy result
4. Use your grid to find any required proportions (NOTE: It would be a better idea if a transparent grid was used so grid could be superimposed on house but that was not available for this project.

The idea here is to allow students to understand the principles of symmetry and congruence and to apply these principles to the solution of problems. By completing this activity, they will not only understand about proportionality in buildings but will realize that architectural order and beauty is obtained by the inter-relationship of ratios and good proportions; that balance and repose are attained by symmetry.

***Extended Activity:***

1. Identify elements o the Chateau that are symmetrical.
2. Write a report on the history of a famous building in New Haven Chateau Measurement:
   Scale: 1/8”=1 ft.

To calculate dimensions, it is assumed that the average height of a castle door was 10 feet, this dimension was used to calculate all other dimensions.

***Dimensions:***

| | |
|---|---|
| Door height | 10 ft. |
| Door width | 8 ft |
| Width at base | 43 1/2 ft |
| Width at top of first landing | 42 ft |
| Width at ceiling | 42 ft |
| Height of 1st flr. (Including foundation) | 17 ft |
| left (side with steps) | 18 ft |
| Thickness of decking between flrs | 3 ft |
| Height of 2nd flr. | 10 ft |
| height of left side (to eaves) 3 | 3 1/2 ft |
| Height of right side (eaves) | 29 ft |
| Roof height to base of ornament | 24 ft. |
| Height of roof ornament (end) | 3 1/2 ft. middle 2 1/2 ft |
| Chimney height | 27 ft. |

Window Dimensions:

| | |
|---|---|
| Bay windows | 8’ by 8’ |
| Small window beside bay | 8’ by 3’ |
| Window above door | 6’ by 5’ |
| Total height of facade (at center) | 60 ft. |
| Door dimension (without arch) | 8’ by 8’ which is proportionate to the large window dimensions. |

*(figure available in print form)*

*(figure available in print form)*

## Sample Lesson 5

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***Objectives:***

Students will successfully analyze the scaled proportions of the facade of Center Church on New Haven Green and determine the actual dimensions of the facade.

***Material:***
Scale drawing of the facade of Center Church, by Leoni W. Robinson, Ruler, pencil

***Motivation:***
Give each student a color copy of church.

***Procedure:***

1. Take students to visit church before doing this project.
2. Have each student to take notes on elements of structural design that they did not notice on their first visit.
3. Pass out rulers, pencils and copy of scale drawing.
4. They will work individually on this project and will be given at least 2 nights to complete the analysis of this building. Students may enlarge copy of design to suit themselves.
5. When students have completed assignments have a critique with students about analysis. Let individual students discuss their analysis and suggest ways in which analysis could be improved.
6. Display copies of the analysis in class, but allow students to keep a copy in their work portfolio. (See completed copy of a student's work) Note that the top of the steeple is missing from this drawing because of the percentage enlargement but the student estimated the remaining height from the copy that was given to him at the beginning of the class.

***Extended Activity:***

1. Have student put church in a grid using transparent grid paper.
2. Design and color motifs for a facade for a building.

*(figure available in print form)*

## Notes

1. Durand J.N.L., "Art & Science of Architecture" p 35.
2. Risebero Bill, "The Story of Western Architecture" p7.

## Annotated Bibliography

*1. Brown, Elizabeth Mills "New Haven: A Guide to Architecture and Urban Design," Yale University Press, 1976 New Haven, Connecticut.

Basically a tourist guide through the streets and districts of New Haven. It includes a capsule history of New Haven relating periods of architecture and individual architects to buildings in town. By far the best reference to New Haven architecture in existence.

*2. Downing, Andrew Jackson "The Architecture of Country Houses" Dover Publications, 1969 (originally 1950) New York City.

A patternbook of American domestic architecture published in 1850. If summarizes the basic American romantic vision of home, individualism and illustrates the values of a young republic politically liberated from the social traditions and authority of England and Europe.

3. Durand, J.N.L. "Art and Science of Architecture" Rizzoli International Publications Inc. 1990 New York.

*4. Dusen, Albert E. Van "Connecticut" Random House 1961 New York.

A capsule history of the emergent changes in architecture in Connecticut between the period 1700-1900's.

*5. Historic American Buildings Survey #9 "New Haven Architecture" U.S. Department of Interior, 1970 Washington D.C.

Book includes detailed plans, elevations, details, and ornaments of several distinguished buildings. It contains dates and facts about the history of the building each building.

6. Jackson, J.B. "The Necessity for Ruins" University of Massachusetts Press, 1980 Amherst, Massachusetts.

Volume discusses the significance given to a variety of settings including "sacred groves", Mainstreet gardens, basic focus is on vernacular American sense of the monument.

*7. Jones, Owen "The Grammer of Ornament" Van Nostrand Reinhold Co. 1972 (originally 1856) New York City.

A patternbook of World Ornament.

8. Kranz, Steward and Fisher, Robert "The Design Continuum An Approach to Understanding Visual Forms" Van Nostrand Reinhold 1966 New York.

9. Moore, Charles; Allen, Gerald and Lyndon Donlyn "The Place of Houses" Holt, Rinehart 1974 New York.

*10. Osterweis, Rollin G. "Three Centuries of New Haven 1638-1938" New Haven Colony Historic Society, New Haven & London Yale University Press 1953, New Haven, Connecticut.

*11. Risebero, Bill "The Story of Western Architecture" Charles Scribriers Son 1979 New York.

A clear, concise exposition on the evolution and growth of Western architectural forms.

12. Sammerson, John "The Unromantic Castle" Thames & Hudson Inc. 1990 New York.

Book contains essays about English architecture from 1940-1981. It describes problems of Elizabethean draftsmen, analyze English classical 18th century country houses and discusses aspects of Victorian architecture.

13. Seymour, George Dudley “New Haven” Tuttle, Moorehouse & Taylor Co. 1942 New Haven Connecticut.

*14. Summerson, John “The Classical Language of Architecture” M.I.T. Press 1976 (originally 1963) Cambridge, Massachusetts.

A review of the elements of the classical architecture of Western culture. Useful for both amateur and professional studies of architecture.

*15. Von Meiss, Pierre “Elements of Architecture” Van Nostrand Reinhold (International), 1989 New York.

A clear and concise explanation of the logic, science, and techno-science of architecture.

16. Milstein, Jeffrey Design of Chateau 1860-1890 Paperhouse Productions, 1983 New York.

* Readings from these are quite enjoyable and adaptable for student reading.