Yale-New Haven Teachers Institute

Curriculum Units by Fellows of the Yale-New Haven Teachers Institute
1996 Volume VI: Selected Topics in Astronomy and Space Studies

An Interdisciplinary Approach to Kepler's Laws

Curriculum Unit 96.06.11
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Introduction

The Universe is still the subject of great interest, mistries, intrieges and great wonderment. Astronomers, Astrologers, mathematicians and philosophers have since ancient times, given their views and interpretations of its origin, chemistry, physical features and life forms. Current space technology has aided recent astronomical discoveries which have drastically reshaped the science and philosophy of some aspects of the Universe.

Long before space travel, man used mathematical calculations to interpret and predict astronomical events. It is from this perspective that the idea arises that students in the seventh grade can experience some of the fascination of the applied side of mathematics in a "universal" setting.

My high school experience with Mathematics was one of boredom primarily because at that time the subject was seen in total isolation from my practical interest. My favorite subject then was Chemistry, which required a fair amount of mathematical skill. Suddenly the mathematical topics in the Chemistry came alive, because it was now seen as a vital tool in interpreting the science. Thus, mathematical topics like variation, change of subject and proportionality, assumed new meanings, for the apparent reason that they presented a clear and immediate relevance.

This unit intends to provide a similar effect—an interdisciplinary approach of some Astronomy and Mathematics. Hopefully, this technique will increase the students' knowledge base and foster an interest in Astronomy and its related subjects. The principal objective of this unit, is to offer a mathematical development which provides relevance and application.

This unit is primarily intended for a New Haven inner city population which consists mostly of Blacks and Latinos. Great care should be taken to ensure that the instruction begins at the students' existing academic level.

Basic Objectives

1. Students should be able to evaluate a mathematical formula by substitution.
2. Students should be able to evaluate using exponents.

3. Students should be able to find the square roots of given values.
4. Students should be able to provide working definitions of rotation, revolution, light year, Astronomical Unit and velocity. They should relate these facts to an astronomical setting.
5. Students should be familiar with the basic features of an ellipse. This information should be applied in determining planetary orbital patterns.
6. They should be able to use Kepler's laws to calculate the Sidereal Period of a planet and its distance from the Sun. 7. They should be able to use the basic knowledge of the orbital patterns of the planets and produce reasonable inferential judgments.

BASIC CONCEPTS

In the introduction of this unit, the students should be given the chance to express what they know about space, the planets, the Solar System or any idea relating to the Universe. From this point on, the discussion should be directed towards the Solar System. A working definition should be developed from the various inputs from the students which might be supplemented by a more formal definition which is more precise.

Distance

The measurement of distance in space requires serious attention. Because of the vastness of space, the conventional unit of measuring distance on Earth becomes too cumbersome if applied to distances in space. Thus, the application of light year and Astronomical Units, become an essential part of the discussion. A basic interpretation of the light year could be contrasted with a real life situation. For example, a car traveling at a speed of 1 mile per minute will travel a distance of 60 miles in one hour. In two hours at the same speed, it will travel a distance of 120 miles. Students could be asked to predict how far it would travel in ten hours if it travels at the same average speed. Next, students could be expected to imagine that the car never stops and it maintains the average speed of 60 miles an hour for one day. How far would it have traveled in one day? The result could be named "one car day". Students could then be required to calculate the distance traveled by the same car under the same conditions for one week, then one month, then one year. This would provide working definitions for a "car week, "car month " and a "car year".

The same situation is applied to the concept of a light year. Light travels at a speed of about 186,000 miles per second or about 300,000 kilometers per second. That distance could be immediately classified as one light second. The same distance multiplied by 60 would provide the distance light travels in one minute, which is about 11,160,000 miles or 18,000,000 kilometers. If students are required to calculate up to a day, they might have a greater appreciation of what is meant by one light year—the distance light travels in space (vacuum) for one year. The Sun is about 93,000,000 miles or about 150,000,000 kilometers from the Earth. Light from the sun takes about eight minutes to reach the Earth, a distance which could be rightly classified as eight light minutes. Bodies in the Universe are so far apart that the use of light year becomes a useful and practical unit for measuring distance.

Astronomical Units

The Earth's distance of about 93,000,000 miles from the Sun is also used as a unit for measuring distances in space, especially when we deal with distances in our Solar System. One Astronomical Unit (AU) is equivalent to the distance of 93,000,000 million miles or 150,000,000 kilometers. Students should be required to convert large distances to Astronomical Units. A meaningful exercise would be to convert the planetary distances from miles to Astronomical Units, AU.

Rotation

It is of primary importance the students clearly understand the concept of rotation and revolution. To simply explain the concept of rotation, a globe which has a base can be placed in a stationary position and the spherical portion is spun. The idea is that a rotation is more or less an object completing circular motions (360 degrees) in some what of a stationary position. *Revolution*

If the teacher picks up the globe and allow it to rotate but at the same time move around the desk, then the idea of a revolution could be introduced . The idea is that in a revolution, the object moves around another object. In this case, the globe is rotating and at the same time it is revolving around the desk: orbiting and revolving should be used interchangeably. The Earth makes one complete rotation approximately every 24 hours, which allows us to experience night and day. However, as it rotates, it orbits or revolves around the Sun. It takes the Earth about 365.25 days or one year to complete one orbit or one revolution around the Sun.

Velocity

Students should be made to understand that velocity is somewhat like speed. Speed tells us how far an object travels in a certain time—60 miles an hour , 60 miles in one hour. Velocity also tells us how far the object travels in a certain time but it has a direction. However, at the seventh grade level, speed and velocity can be used inter-changeably.

Ellipse

At the seventh grade level, it might be safe to represent an ellipse as an oval shape. Students could be asked to identify objects which they are familiar with, which have oval shapes (a football, an egg). The basic features of an ellipse are the major axis, minor axis, center and foci. The students could be given several drawn ellipses, and as the instruction proceeds, they could be required to put in the elements. It would be fair to elicit from the students definitions of the various elements of the ellipse. This information they should be able to extract from the diagrams they have been working with. These definitions should be refined to fit the formal concept.

The maximum diameter or the major axis would represent the line drawn from the two farthest points on the ellipse which passes through the center. The minimum diameter or the minor axis would represent the line drawn at right angle to the major axis passing through the center of the ellipse. The two points, one on either side of the center of the major axis, are called the foci. These points determine the shape of the ellipse. The further apart the foci are, the more elongated the ellipse will be. The opposite is true if the two points get closer. If both points coincide, then there is a perfect circle. The distance between the foci compared to the major axis is called the eccentricity of the ellipse. When it is zero, it is a circle. Between zero and one, it is an ellipse. Whenever it is less than zero or greater than one, it produces some shapes other than ellipses.

There is a constant term which is associated with ellipses. If both foci A and B are identified, and a point C is chosen on the rim of the ellipse, the sum of the distances, AC + BC is equal to the length of the major axis. This is true for any point on an ellipse.

Kepler's Laws

This is a classic example of the application of a mathematical solution to a theory which had existed for centuries. The common belief that the Earth was the center of the Universe, was dispelled convincingly by Kepler, who was mathematician and an ardent astronomer.

Kepler's First Law

Kepler's first law stated that the planetary orbits around the Sun are elliptical, contrary to the long held belief that they were perfect circles. This law can be demonstrated by constructing an ellipse with the Sun at one focus. At a point on the ellipse, the orbiting planet could be drawn. Here, the students could from observation infer that at some points on the orbit, the planet is closer to the Sun, and at other times, it is farther away.

Kepler's Second Law

Kepler's second law states that when an orbiting body is closest to the Sun, its orbiting velocity will increase so as to sweep through an equal area within the same time interval as when it was farther away. This means that if it takes X days for the planet to cover an area of B square units on a portion of the orbit which is farther away from the Sun, then when the planet gets to the section of the orbit closest to the Sun, for it to cover the same area of sweep in the same time of X days, it will have to increase its velocity. This law further reinforces the principle that the orbital path of the planet is elliptical.

Students could have a clearer image of this law if it is sketched on grid paper. Here they will be able to develop and practice their estimating skills. Instead of trying to formally calculate the area of sweep on an ellipse, students could count the number of squares which would be needed to match the given time of orbit.

Kepler's Third Law

The time that the planet takes to complete on orbit around the Sun is called the Sidereal Period. Kepler stated that the square of the Sidereal Period of a planet is proportional to the cube of the mean distance from the Sun, $P^2 = D^3$. Where P is equal to the sidereal period and D is the average distance of the Sun from the Earth in Astronomical Units, AU. If the Sidereal Period of a planet is known, its distance from the Sun can be derived from the same law. Similarly, the Sidereal Period can be calculated from the known distance from the Sun.

Here students are provided with the interesting opportunity of learning and practicing the art of substituting in a formula. Firstly, there must be a clear understanding of what the variables represent. The cube of a number should be expressed in an expanded form to provide clarity. At times, some students think that it is a trivial mistake to express exponential functions as merely the exponent times the base. Therefore, the expression five to the third power is 5 x 5 x 5 = 125 and not 3 x 5 = 15.

Given the grade level, it might be more productive to give the formula where P is equal to the square root of the cube of D. And D is equal to the cube root of P squared.

BACKGROUND INFORMATION

The Universe consists of billions of systems of stars called galaxies. The galaxy to which we belong is called The Milky Way. It is a family of stars which form a great disc, which is about 100,000 light years in diameter, which bears a thickness of about 250 light year near our Sun. Our Sun is about 30,000 light years from the center of the Galaxy. The stars that we see with the naked eye in the nights are at a large range of distances from the Solar System, from a few light years to hundreds of light years

The bodies in the Universe are constantly moving through space, which implies that the galaxies are changing positions. Our Sun, which is traveling at about 500,000 miles per hour, completes an orbital path around the center of the Galaxy in about 230 million years. This movement through The Milky Way, may result in the Sun with its satellites, passing through areas of dense clouds of cold dust. Some scientists believe that this occurance could be responsible for significant climatic changes on Earth

. The stars or the suns in the galaxies form systems called solar systems. At the center of each system is the star. Our Solar System has the Sun at the center, and revolving around it, nine planets, five of which are visible with the naked eye. These planets which are of varying sizes, occupy varying elliptical orbits at different distances around the Sun. Some of these planets have satellites like the Moon and other large objects in orbit around them. There are about 50,000 smaller bodies called Planetoids or Asteroids, and countless smaller objects which are found in our Solar System.

The question is often asked "Why do the planets orbit the Sun?" The same is applicable to all orbiting bodies. This question which puzzled scientists for thousands of years was eventually solved by Isaac Newton's Laws of motion. Newton stated that if a body was in a state of rest or uniform motion, it would continue in that state unless it was disturbed by another force . That is considered the reason why the planets instead of following straight lines of path, they move in elliptical orbits. He further stated that every particle of matter in the Universe attracts every particle with a force that varies directly as the product of their masses and inversely as the square of the distance between them. Therefore, the greater the masses of the bodies, the greater is the mutual attraction. Since the distance between the attracting bodies affects the force between them, it means that the farther the bodies are apart, the lesser the attraction. The converse is true, the lesser the distance apart, the greater the attraction. As the planets orbit the Sun, the Sun which is much larger in mass, exerts a tremendous pull on it. This could result in the planet falling into the Sun. However, because of the tendency for the planet to keep in a straight path, a compromise between the two extremes is reached. This results in the planet taking an elliptical path.

Historical View of Planetary Motion

Astronomy is the science which involves the study of the universe. It is considered to be the oldest science. In early times, the movement of the Sun and the Moon were of particular interest for reasons that they caused changes in light and darkness. In addition to the natural events that the movement and positioning of these bodies effected, some societies attached social, political and religious significants.

Men in particular, priests and magicians, could be considered the forerunners of the practitioners of the study of Astronomy. They devoted inordinate amounts of time in the study of planetary movements. Their persistence and commitment provided the world with astonishing accuracy of recorded observations. That attitude continued even with the rise and fall of great civilizations in the Middle East, Greece and Rome.

Among the earlier astronomers were the Egyptians who discovered important patterns of solar eclipses. Any one place on Earth with a similar size of Egypt would witness a total eclipse once every eight years. This was an indication that the Egyptians must have devoted a great number of years of observations in developing those particular astronomical predictions.

With the decline of the Egyptians civilization, the art was lost for thousands of years. Later on, the existing knowledge of the Egyptians was utilized by the Greeks, who later became the torch bearers of astronomical knowledge. The Arabs continued the practice after the Greek civilization fell and were credited for providing the modern terminologies of Astronomy. After the Arabs, the interest in the science declined until the Middle Ages when the Renaissance produced the so-called fathers of modern Astronomy.

The persisting central concept for over two thousand years was the belief that the Earth was the center of the Universe. The prevailing thought was that the Earth was stationary and the other celestial bodies moved around it in uniform circular patterns.

Claudius Ptomley(90 AD—168 AD) a Greek astronomer, was credited with compiling and developing a great body of work on Greek Astronomy. He published a book called "The Almagest" which compiled ideas of the old philosophers and from the then new astronomical developments. His work provided impressive models for the planetary motions which were useful for the next thousand years in predicting planetary positions. One of his core ideas was the existing concept that the Earth was the center of the Universe. One of the serious difficulties that he encountered was the observation that some planets showed a retrograde motion—an apparent reversal in their orbital path. This was a marked contrast to the to the existing idea of uniform orbital patterns.

A Polish mathematician, canon and astronomer, Nicholas Copernicus, born in 1473, initiated what was regarded as the start of modern Astronomy. As an astronomer, he compiled a table of planetary motions which was useful up until the end of the sixteenth century. Quite noticeably, he produced a central shift in the core of the existing astronomical concepts that the Earth was the center of the Universe. He then suggested that it was the Sun which was the center of the Universe around which the planetary bodies orbit. He claimed that it was that principle which was responsible for the seemingly irregular planetary motions.

Tycho Brahe was born in 1546, three years after the death of Copernicus whose idea he viewed with suspicion. He was considered the father of observational astronomy; he fundamentally improved the designs of observational instruments. His greatest contribution in understanding the universe was the precision of his recorded observations. Prior to his death, he passed on to his assistant, Johnnes Kepler, the observational measurements he had collected.

Johnnes Kepler was born on December 27, 1571 in Weil Der Stadt, Germany, to a father who was a tavern keeper, and a mother who was a herbalist. In his youthful years he was plagued with bad digestion, boils, a stint of smallpox, crippled hands and various skin disorders. Kepler entered the University of Tubingen as a teenager, and graduated at the age of twenty. He had the ambition of becoming a Lutheran minister, instead, he was encouraged to pursue the study of Astronomy. He devoted nearly ten years of his life working with Tycho Brahe's data, and the Copernicus theory that the planet Mars, orbits in a perfect circular pattern. As a mathematician, he employed his full mathematical skills in finding a calculation that would fit. Reluctantly, he accepted his own finding that the only mathematical fit was an elliptical orbit. From this, Kepler established the three laws of planetary motions.

Kepler's First Law

Firstly, he dispensed with the concept of circular orbit. In his work he discovered that the orbit of Mars matched best with a curve called an ellipse. He stated that *the orbit of a planet is an ellipse with the Sun at one focus.*

Kepler's Second Law

"The line joining the sun to the planet(called the radius vector) sweeps through an equal area in any given interval of time". This law implies that when an orbiting planet is closest to the Sun, its orbiting velocity must increase in order to sweep through an equal area within the same interval of time as when it was farther away.

Kepler's Third Law

The square of the period of rotation about the sun (the sidereal period) is proportion to the cube of the mean distance, $P^2 = D^3$. Where P represents the sidereal period, and D is the average distance of the planet from the sun in Astronomical Units".

Later, Isaac Newton's laws of motion allowed for the derivation of Kepler's laws of elliptical planetary motion as a general consequence of gravitational force acting on the bodies in the Solar System.

LESSON PLAN

Lesson # 1 Grade 7

Aim : How to define and apply the units of Light Year and Astronomical Units.

Instructional Objectives : Students should be able to provide a working definition of the units of Light Year and Astronomical Units. They should be able to use these units in defining distances in space.

Activity : Use a globe and a small light bulb on a stand to demonstrate rotation and revolution. Students should be asked to tell the difference between a rotation and a revolution. Students should be given calculators to assist in mathematical calculations.

Development : If there were no obstruction and we turn on a very powerful light here in New Haven, how long would it take for if to get to California (students should be allowed to determine the necessary information they need to get the answer)?

If light travels at a speed of 186,000 miles per (150,000 kilometers per second), how far would it travel in (a) 10 seconds (b) 20 seconds (c) 1 minute (d) 1 hour (e) 1 day?

What is a light second?

What is a light hour?

What is a light day? What is a light year When do we use light year? (students should be provided with vast distances in the Universe where light year is used).

How far is the Sun from the Earth?

What name could we give for a distance that is equal to the distance of the sun from the Earth?

What is the distance of 3 AU in miles?

What would be the same distance in Kilometers?

Application : Students should be given various values in light years and they should be able to estimate the distances in light years They should be given various distances which they should be able to convert to AU or from AU to miles and kilometers.

Lesson # 2 Grade 7

Aim : How to evaluate a formula *Instructional Objectives* : Students should be able to substitute numerical values for the variables or symbols in a formula. They should be able to define and calculate velocity, distance and time. *Material* : Calculator

Development : The recipe of a cake can be used to demonstrate the nature of a formula. Here the cake represents the subject of the recipe or formula. Average velocity V,is equal to the distance d, divided by the time t: V = *d t*

What is the subject of the formula?

If d is 60 miles and t is 2 hours how would we know the average velocity?

The earth orbits at a velocity of about 66,000 miles an hour. What does that statement really mean? How far a distance would the earth travel in two hours?

What did we do to get that answer?

How could we put that in a formula where V is velocity, d is distance and t is time?.

If the earth travels at a Velocity of 66,000 miles per hour how long will it take to travel (a) 33,000 miles (b) 213,000 miles? Using the same letters: What formula can we develop for finding the time?

Application : Using the developed formulas student should be given problems requiring them to find velocity, distance and time.

Lesson # 3 Grade 7

Aim : How to find the square root of a number

Instructional Objectives : Student should be able to find the square root of a given value.

Activities : Students can use grid paper to draw several squares of varying dimensions. The dimensions can then be expressed in an exponential form 2 x 2 equals two to the second power; 3 x 3 is three to the second power.

Students are then given examples of perfect squares and are required to figure out the dimensions which produce the squares. This is later defined as the root of the number. The perfect squares can then be expressed in the form of a mathematical statement: d to the second power is 81 what is the value of d?

Students are then introduced to the square root sign: M = 144. What is M? *Application* :

1. Students should provide examples of perfect squares and their roots.
2. Students should calculate the square roots of given values using a calculator.

Lesson # 4 Grade 7

Aim : To draw ellipses of various shapes. *Instructional Objective* : students should be able to draw ellipses of various shapes by manipulating the foci. They should be able to identify the basic elements of an ellipse.

Background Information:

Students should be provided with the information of Kepler’s discovery of the elliptical orbits of the planets. *Activities* : Students could work in a cooperative setting. They should make a loop with a piece of string which is about six inches long. Draw a horizontal line of about 10 inches in length on a sheet of construction paper. Stick two thumb tacks about one inch apart on the line. Place the string around both tacks and use a pencil to gently pull the string while using the loop as a guide to draw a closed curve.

Students should label the points of the thumb tacks as the foci. Half way between the foci is the center. The major and minor axes should be draw and identified.

Application : Students should use the same loop to draw several ellipses by moving the foci further apart. Students should then draw several ellipses by bringing the foci closer and closer together.

Evaluation : When the foci move further and further apart, what effect did it have on the shape of the curve? When the foci were moved closer to each other, how did it affect the shape of the curve?

What comment would you make about a circle?

Lesson # 5 Grade 7

Aim : How to estimate area *Instructional Objective* : Applying Kepler’s second law students should be able to estimate the area of sweep in a planetary orbit for a constant time period.

Activities : Students could work in a cooperative setting.

1. Student will draw on a grid paper a planetary orbit (an ellipse) with the Sun as one focus. The Sun will be closer to one section of the orbit.

Development : Why do the planets move around the Sun? Which planets would you expect to move faster, the ones closer in or the ones farther out?

Why ?

As the planet orbits in its elliptical orbit what kind of effect will the sun have on its velocity?

Application : Shade in a section from the area nearest to the sun. Shade in an equal area in the section furthest from the sun. Repeat this activity on an ellipse of a different size.

Lesson # 6 Grade 7

Aim : To calculate the period of rotation and the distance from the Sun

Instructional Objectives : Students should be able to apply Kepler's third law in finding the orbital period and the distance of the planet from the Sun.

Definitions : Sidereal Period, Astronomical Unit *Material* : Calculators, copy of the planetary orbits of the Solar System.

Development : Which planet would you expect to have the shortest orbital period ?

Why would that be true?

Which planet would you expect to have the longest orbital period?

If the distance of a planet from the Sun is twice the distance of the Earth from the Sun, how many AU would it be?

If a planet takes three Earth years to orbit the Sun, how many Sidereal periods does it represent?

Application : Use the formula to calculate the Sidereal Period: $P^2 = D^3$; where P represents the Sidereal Period and D is the distance from the sun in Astronomical Units.

Use the formula $D = {}^{3}P^{2}$ to find the distance of the planet from the Earth in AU

Teacher's Note

Mathematics is one of the core academic subjects which is generally considered to be more difficult. Students complain that it is boring, difficult, and to some extent, irrelevant to their needs. However, the feeling is, educators and the adult public recognize its vital importance. The challenging task is to devise approaches in order to convince and interest our students into devoting more time and effort to the study and practice of Mathematics.

This unit is intended for the seventh grade level. It has not been tested. The choice of an interdisciplinary approach promises to sneak in the mathematics through the door of Astronomy. Students will have the opportunity to develop a broader knowledge base in Astronomy, and maybe, an interest in space travel.

In this unit, there is enough room for a teacher to adjust and tailor the lessons to fit his or her particular class. It is of great importance, that a part of the instruction be given to establishing the basic concepts of the science. This will lead to a better understanding of the relevance of the mathematics.

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